CSCI 1900 Discrete Structures

Properties of Relations

Reading: Kolman, Section 4.1-4.2

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Cartesian Product

- If $A_1, A_2, ..., A_m$ are nonempty sets, then the *Cartesian Product* of them is the set of all ordered m-tuples $(a_1, a_2, ..., a_m)$, where $a_i \in A_i$, i = 1, 2, ... m.
- Denoted $A_1 \times A_2 \times ... \times A_m = \{(a_1, a_2, ..., a_m) \mid a_i \in A_i, i = 1, 2, ... m\}$

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Cartesian Product Example

- If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, find $A \times B$
- A × B = {(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)}

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Using Matrices to Denote Cartesian Product

- For Cartesian Product of two sets, you can use a matrix to find the sets.
- Example: Assume A = {1, 2, 3} and B = {a, b, c}.
 The table below represents A x B.

	а	b	С
1	(1, a)	(1, b)	(1, c)
2	(2, a)	(2, b)	(2, c)
3	(3, a)	(3, b)	(3, c)

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Cardinality of Cartesian Product

The cardinality of the Cartesian Product equals the product of the cardinality of all of the sets:

$$|A_1 \times A_2 \times ... \times A_m| = |A_1| \cdot |A_2| \cdot ... \cdot |A_m|$$

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Subsets of the Cartesian Product

- Many of the results of operations on sets produce subsets of the Cartesian Product set
- Relational database
 - Each column in a database table can be considered a set
 - Each row is an m-tuple of the elements from each column or set
 - No two rows should be alike

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Relations

- A relation, R, is a subset of a Cartesian
 Product that uses a definition to state whether
 an m-tuple is a member of the subset or not
- Terminology: Relation R from A to B
- $R \subset A \times B$
- Denoted "x R y" where x ∈ A and y ∈ B and x has a relation with y
- If x does not have a relation with y, denoted

x∤ky

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Relation Example

- A is the set of all students and B is the set of all courses
- A relation R may be defined as the course is required

Paul Giblock R CSCI 2710
Danny Camper R CSCI 2710

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Relations Across Same Set

- Relations may be from one set to the same set, i.e., A = B
- Terminology: *Relation R on A* $R \subseteq A \times A$

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Relation on a Single Set Example

- · A is the set of all courses
- A relation R may be defined as the course is a prerequisite
- CSCI 2150 R CSCI 3400
- R = {(CSCI 2150, CSCI 3400), (CSCI 1710, CSCI 2910), (CSCI 2800, CSCI 2910), ...}

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Example: Features of Digital Cameras

- Megapixels = {<2, 3 to 4, >5}
- battery life = {<200 shots, 200 to 400 shots, >400 shots}
- optical zoom = {none, 2X to 3X, 4X or better}
- storage capacity = {<32 MB, 32MB to 128MB, >128MB}
- price = Z+

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Digital Camera Example (continued)

Possible relations might be:

- · Priced below \$X
- · above a certain megapixels
- a combination of price below \$X and optical zoom of 4X or better

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Theorems of Relations

- Let R be a relation from A to B, and let A₁ and A₂ be subsets of A
 - If $A_1 \subseteq A_2$, then $R(A_1) \subseteq R(A_2)$
 - $-R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$
 - $-R(A_1 \cap A_2) \subseteq R(A_1) \cap R(A_2)$
- Let R and S be relations from A to B. If R(a) = S(a) for all a in A, then R = S.

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Matrix of a Relation

- We can represent a relation between two finite sets with a matrix
- $M_R = [m_{ii}]$, where

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R \\ 0 \text{ if } (a_i, b_j) \notin R \end{cases}$$

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Example of Using a Matrix to Denote a Relation

Using the previous example where A = {1, 2, 3} and B = {a, b, c}. The matrix below represents the relation R = {(1, a), (1, c), (2, c), (3, a), (3, b)}.

	а	b	С
1	1	0	1
2	0	0	1
3	1	1	0

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Digraph of a Relation

- · Let R be a relation on A
- · We can represent R pictorially as follows
 - Each element of A is a circle called a vertex
 - If a_i is related to a_j, then draw an arrow from the vertex a_i to the vertex a_i
- In degree = number of arrows coming into a vertex
- Out degree = number of arrows coming out of a vertex

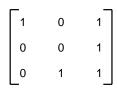
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Representing a Relation

The following three representations depict the same relation on $A = \{1, 2, 3\}$.

$$R = \{(1, 1), (1, 3), (2, 3), (3, 2), (3, 3)\}$$



2), (3, 3))

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