CSCI 1900
Discrete Structures

Trees
Reading: Kolman, Section 7.1

Brain Teaser

Assume you are driving with your child, and he/she is screaming for the milk out of his happy meal. You haven't cleaned out the car in a while, so there are 9 milk bottles rolling around on the floorboards under your feet, one full, 8 half full and a bit foul.

Assuming you can pick up more than one bottle at a time in each hand, using your hands as balance scales, how many "balances" will it take for you to find the full bottle.

Trees – Their Definition

Let *A* be a set and let *T* be a relation on *A*. We say that *T* is a **tree** if there is a vertex v_0 with the property that there exists a unique path in *T* from v_0 to every other vertex in *A*, but no path from v_0 to v_0.

-Kolman, Busby, and Ross, p. 254

Trees – Our Definition

We need a way to describe a tree, specifically a "rooted" tree.

- First, a rooted tree has a single root, v_0, which is a vertex with absolutely no edges coming into it. (in-degree of $v_0 = 0$)
- Every other vertex, *v*, in the tree has exactly one path to it from v_0. (in-degree of $v = 1$)
- There may be any number of paths coming out from any vertex.
- Denoted (T,v_0)

Trees – Characteristics

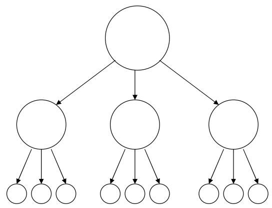
If *T* is a relation that is also a tree, then *T* must have the following characteristics:

- There are no cycles in T
- V_0 is the only root of T

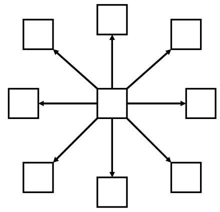
Examples of Trees

Using the definitions given above, determine which of the following examples are trees and which are not.

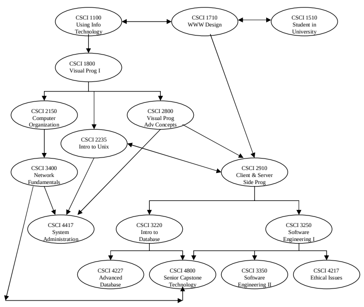
Is this a tree?



Is this a tree?



Is this a tree?



Is this a tree?

Microsoft’s troubleshooting wizard.

Is this a tree?

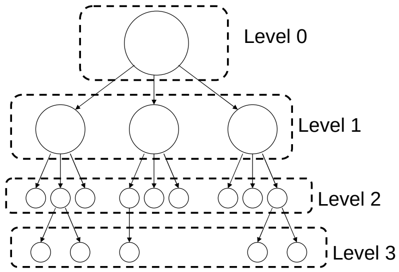
```
for i = 0 to 256
    for j = 0 to 16
        array[i,j] = 1000*i + j
    next j
next i
```


Is this a tree?

```
for i = 0 to 256
    for j = 0 to 16
        array[i,j] = i*j
    next j
next i
```

Definitions

Levels – all of the vertices located *n*-edges from v_0 are said to be at level *n*.



More Definitions

- A vertex, *v*, is considered the **parent** of all of the vertices connected to it by edges leaving *v*.
- A vertex, *v*, is considered the **offspring** of the vertex connected to the single edge entering *v*.
- A vertex, *v*, is considered the **sibling** of all vertices at the same level with the same parent.

More Definitions

- A vertex v_2 is considered a **descendant** of a vertex v_1 if there is a path from v_1 to v_2.
- The **height** of a tree is the number of the largest level.
- The vertices of a tree that have no offspring are considered **leaves**.
- If the vertices of a level of a tree can be ordered from left to right, then the tree is an **ordered tree**.

More Definitions

- If every vertex of a tree has at most n offspring, then the tree is considered an **n-tree**.
- If every vertex of a tree with offspring has exactly n offspring, then the tree is considered a **complete n-tree**.
- When n=2, this is called a **binary tree**.

Examples

1. If the set A = {a, b, c, d, e} represents all of the vertices for a tree T, what is the maximum height of T? What is the minimum height of T?
2. If the set A = {a, b, c, d, e} represents all of the vertices for a tree T and T is a complete binary tree, what is the maximum height of T?

More Examples

3. If every path from the root of a complete 4-tree has 3 levels, how many leaves does this tree have?
4. What is *n* for the following *n*-tree?

```
for i = 0 to 256
    for j = 0 to 16
        array[i,j] = 1000*i + j
    next j
next i
```

One More Example

5. Let T be a complete n-tree with 125 leaves.

   a.) What are the possible values of n

   b.) What are the possible values for the height of T.