

Points missed: _____

Student's Name: _____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 1900 (Tarnoff) – Math for Computer Science
TEST 1 for Summer Term, 2005

Read this before starting!

- This test is closed book and closed notes
- You may **NOT** use a calculator
- All answers must have a box drawn around them. This is to remove any ambiguity during grading. Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- ***If not otherwise identified, every integer is assumed to be represented using base 10.***
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

"Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarizing, the changing or falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

Algebraic properties of sets:

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cup A = A$
- $\overline{A \cap A} = A$
- $\overline{(\overline{A})} = A$
- $\overline{A} \cup A = U$
- $\overline{A} \cap A = \emptyset$
- $\overline{\emptyset} = U$
- $\overline{U} = \emptyset$
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$
- $\overline{A \cap B} = \overline{A} \cup \overline{B}$
- $A \cup U = U$
- $A \cap U = A$
- $A \cup \emptyset = A$ or $A \cup \{ \} = A$
- $A \cap \emptyset = \emptyset$ or $A \cap \{ \} = \{ \}$

Addition principle of cardinality:

- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Properties of characteristic functions:

- $f_{A \cap B} = f_A f_B$; that is $f_{A \cap B}(x) = f_A(x) f_B(x)$ for all x .
- $f_{A \cup B} = f_A + f_B - f_A f_B$; that is $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) f_B(x)$ for all x .
- $f_{A \oplus B} = f_A + f_B - 2f_A f_B$; that is $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2f_A(x) f_B(x)$ for all x .

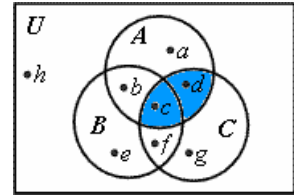
Properties of integers:

- If n and m are integers and $n > 0$, we can write $m = qn + r$ for integers q and r with $0 \leq r < n$.
- Properties of divisibility:
 - If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$ If $a \mid b$ or $a \mid c$, then $a \mid bc$
 - If $a \mid b$ and $a \mid c$, where $b > c$, then $a \mid (b - c)$ If $a \mid b$ and $b \mid c$, then $a \mid c$
- Every positive integer $n > 1$ can be written uniquely as $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} p_4^{k_4} \dots p_s^{k_s}$ where $p_1 < p_2 < p_3 < p_4 < \dots < p_s$ are distinct primes that divide n and the k 's are positive integers giving the number of times each prime occurs as a factor of n .

Problems 1 through 4 refer to the Venn Diagram shown to the right. (2 points each)

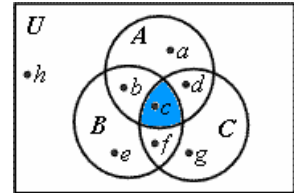
1. True or False: $d \in A \cap C$

The shaded area of the Venn Diagram to the right represents $A \cap C$, and since d is contained in this area, the answer is TRUE.



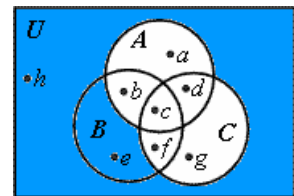
2. True or False: $f \in A \cap B \cap C$

The shaded area of the Venn Diagram to the right represents $A \cap B \cap C$, and since c is not contained in this area, the answer is FALSE.



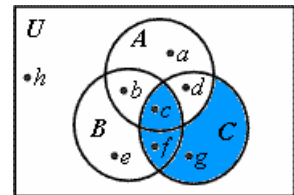
3. True or False: $g \in \overline{(A \cup C)}$

The shaded area of the Venn Diagram to the right represents $\overline{(A \cup C)}$, and since g is not contained in this area, the answer is FALSE.

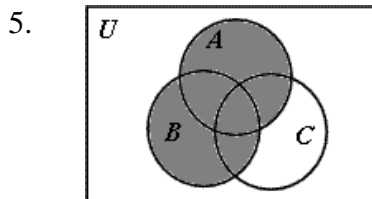


4. True or False: $f \in C - (A - B)$

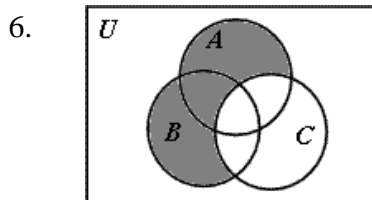
The shaded area of the Venn Diagram to the right represents $C - (A - B)$, and since f is contained in this area, the answer is TRUE.



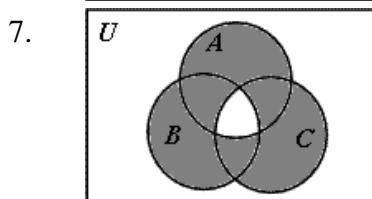
For problems 5, 6, and 7, use unions, intersections, and complements of the sets A , B , and C , to write an expression to describe the set represented by the shaded area in the given Venn Diagram. (3 points each)



Answer for 5: $A \cup B$



Answer for 6: $(A \cup B) - C$



Answer for 7: $(A \cup B \cup C) - (A \cap B \cap C)$

8. If $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d, e, f\}$, compute the set represented by $A \cap B$. (2 points)

$A \cap B = \{a, e\}$ – this is the set of all letters that are elements of BOTH A and B .

9. If $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d, e, f\}$, compute the set represented by $A \oplus B$. (2 points)

$A \oplus B = \{i, o, u, b, c, d, f\}$ – this is the set of all letters that are elements of A or B , but not both.

10. If $A = \{a \mid a \text{ is an integer divisible by 2 or 3}\}$ and $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, compute the set represented by \bar{A} . (3 points)

$\bar{A} = \{1, 5, 7\}$ – this is the set of all elements of U that are not elements of A .

11. If set A has 10 elements, set B has 8 elements, and they have 5 elements in common, how many elements are a member of $A - B$? (3 points)

If $|A| = 10$, $|B| = 8$, and $|A \cap B| = 5$, then $|A - B| = 10 - 5 = 5$

12. If set A has 10 elements, set B has 8 elements, and they have 5 elements in common, how many elements are a member of $A \cap B$? (3 points)

It should be clear from the wording that the number of elements in both A and B , i.e., $A \cap B$, is $|A \cap B| = 5$

13. Give the corresponding set to the sequence *thisiseasy*. (3 points)

The set being asked for in this problem is the letters that make up the string "thisiseasy" without order or duplicates. Therefore, the answer is $\{t, h, i, s, e, a, y\}$.

14. Write a **recursive** formula for the n -th term of the sequence 2, -2, 2, -2, 2, -2, 2, -2, ... (3 points)

First, note that the set is simply the number two with alternating signs, i.e., the n -th value is equal to the negative of the $(n - 1)$ -th. This gives us the formula; $a_n = -a_{n-1}$. Since the recursive expression requires one previous value, i.e., we only go back to $n-1$, then one initial value must be defined. This gives us the following answer.

$$a_1 = 2$$

$$a_n = -a_{n-1} \text{ for } n \geq 2$$

For problems 15 and 16, determine if the sequence defined by the formula is explicit or recursive. In addition, identify the value of the 3rd element in the sequence. (2 points each)

15. $a_1 = 0$; $a_n = a_{n-1} + 5$ explicit recursive $a_3 = \underline{10}$

$$a_1 = 0; a_2 = a_1 + 5 = 5; a_3 = a_2 + 5 = 10$$

16. $b_n = 5(n - 1)$ explicit recursive $b_3 = \underline{10}$

For problems 17, 18, and 19, tell whether or not the string on the left belongs to the regular set corresponding to the regular expression on the right. (2 points each)

17. string: 010101010101 regular expression: $0(0 \vee 1)^*1$ Belongs Doesn't belong

The regular expression $(0 \vee 1)$ defines the set $\{0, 1\}$. The regular expression $(0 \vee 1)^*$ defines the set of any possible string that can be derived from the set $\{0, 1\}$, i.e., any sequence of 1's and 0's. Therefore, the regular expression $0(0 \vee 1)^*1$ defines the set of any sequence of 1's and 0's where the first digit is a 0 and the last digit is a 1. This includes 010101010101.

18. string: Λ (empty string) regular expression: $(0 \vee 1 \vee 2)^*$ Belongs Doesn't belong

The regular expression $(0 \vee 1 \vee 2)$ defines the set $\{0, 1, 2\}$. The regular expression $(0 \vee 1 \vee 2)^*$ defines the set of any possible string that can be made from the elements of the set $\{0, 1, 2\}$ including the empty string.

19. string: 1111100 regular expression: $1^*0^*1^*$ Belongs Doesn't belong

The regular expression 1^* defines the set of all strings of 1's including the empty string while the regular expression 0^* defines the set of all strings of 0's including the empty string. The set $1^*0^*1^*$ therefore defines any length string of 1's followed by any length string of 0's followed by any length string of 1's. 1111100 is a string of 1's of length 5 followed by a string of 0's of length 2 followed by a string of 1's of length 0. Therefore, 1111100 is an element of the set defined by $1^*0^*1^*$.

20. Write three elements that are members of the regular set corresponding to the regular expression $a(b^*)b$. (3 points)

The regular expression b^* defines the set of all strings of b's including the empty string. The regular expression $a(b^*)b$ takes every element from the set defined by b^* and puts an 'a' in front and a 'b' at the end. This gives us a set that looks like the following:

$\{ab, abb, abbb, abbbb, abbbbbb, abbbbbb, abbbbbb, abbbbbb, \dots\}$

21. Give the regular expression corresponding to the regular set $\{c, ac, aac, aaac, aaaac, aaaaac, aaaaaac, aaaaaaac, \dots\}$. (3 points)

First, let's try to use words to come up with a description of all of the elements of the above set. First, they all end in 'c'. Therefore, our regular expression should end in a 'c'. Second, the 'c' is preceded by a string of a's of length 0 (empty string) on up. That sounds like a^* . Catenating the two strings gives us:

a^*c

22. True or False: 40394787484731 is divisible by 5. (2 points)

To be divisible by 5, a number must end in 0 or 5. This number ends in 1 so the answer is FALSE.

23. True or False: 234555432 is divisible by 3. (2 points)

To be divisible by 3, the sum of all of the digits in the number must also be divisible by 3. $2 + 3 + 4 + 5 + 5 + 5 + 4 + 3 + 2 = 33$ which is divisible by 3. Therefore, the answer is TRUE.

24. True or False: 45239456603 is divisible by 11. (2 points)

To be divisible by 11, the alternating sum and subtraction of the digits must be zero or divisible by 11. $4 - 5 + 2 - 3 + 9 - 4 + 5 - 6 + 6 - 0 + 3 = 11$ which is divisible by 11. Therefore, the answer is TRUE.

25. True or False: 2342324₅ is a valid base 5 number. (2 points)

To be a valid base 5 number, all of the digits of the number must be from the set {0, 1, 2, 3, 4}. This is the case for 2342324₅, therefore, the answer is TRUE.

26. Write the expansion in base 5 of 254₁₀. (4 points)

By generating the remainders from repetitive applications of $m = q \cdot n + r$ where m starts out as 254₁₀ and n is 5, we can generate in reverse order the digits of the base 5 expansion of 254₁₀.

$$\begin{array}{l} 254 = \boxed{50} \cdot 5 + \boxed{4} \\ \swarrow \\ 50 = \boxed{10} \cdot 5 + \boxed{0} \\ \swarrow \\ 10 = \boxed{2} \cdot 5 + \boxed{0} \\ \swarrow \\ 2 = \boxed{0} \cdot 5 + \boxed{2} \end{array}$$

The remainders in reverse order are 2-0-0-4. Therefore, $2004_5 = 254_{10}$. To verify this, simply add the powers of 5 with their respective multipliers.

$$2 \cdot 5^3 + 0 \cdot 5^2 + 0 \cdot 5^1 + 4 \cdot 5^0 = 2 \cdot 125 + 0 \cdot 25 + 0 \cdot 5 + 4 \cdot 1 = 254$$

27. Convert the base 5 number 2211₅ to base 10. Just write out the formula with the correct values; do not worry about crunching the numbers for a final calculation. (4 points)

Any of the following answers would have been fine:

$$\begin{array}{l} 2 \cdot 5^3 + 2 \cdot 5^2 + 1 \cdot 5^1 + 1 \cdot 5^0 \\ 2 \cdot 125 + 2 \cdot 25 + 1 \cdot 5 + 1 \cdot 1 \\ 250 + 50 + 5 + 1 \\ 306_{10} \end{array}$$

28. How many possible answers/results are there from the mod-24 function? (2 points)

Remember that the mod- n function is the remainder from a division by n . A division by 24 generates a remainder from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}. There are 24 elements in this set, so the answer is $\boxed{24}$.

29. What is the result of $f(33)$ if f is the mod-4 function, i.e., calculate $33 \bmod 4$. (2 points)

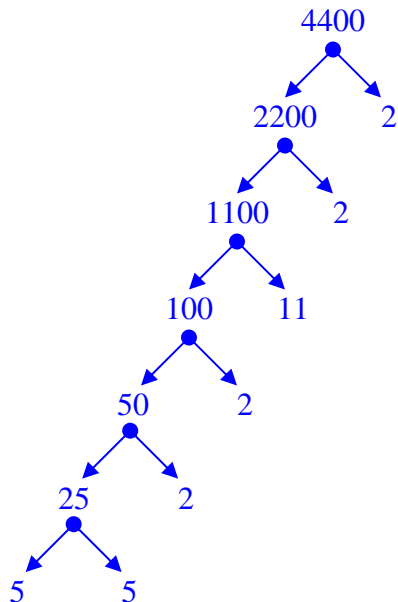
33 divided by 4 equals 8 with a remainder of 1. Therefore, $\boxed{f(33) = 1}$.

30. If $m = 65$ and $n = 6$, determine the values of q and r that satisfy the expression $m = q \cdot n + r$ such that $0 \leq r < n$. (2 points)

$$65 = \boxed{10} \cdot 6 + \boxed{5}$$

$$q = \underline{\quad 10 \quad} \quad r = \underline{\quad 5 \quad}$$

31. Write the integer 4400_{10} as a product of powers of primes. (4 points)



$4400 = 2^4 \cdot 5^2 \cdot 11$

32. Use any method you wish to find the greatest common divisor of 180 and 126. (4 points)

There are a couple of ways to do this. First, you could break both 180 and 126 down to their multiples of prime numbers. This would give us:

$$180 = 2^2 \cdot 3^2 \cdot 5^1$$

$$126 = 2^1 \cdot 3^2 \cdot 7^1$$

This shows us that 126 and 180 have common prime number divisors of 2^1 and 3^2 . Multiplying these together gives us the GCD:

$$2^1 \cdot 3^2 = 18$$

Another way to do this is to use the Euclidian Algorithm which is based on a repetitive application of the principle of integers $m = qn + r$ using m and n to hold the values we are trying to find for the GCD where $m \geq n$.

$$180 = \underline{1} \cdot 126 + \underline{54}$$

$$126 = \underline{2} \cdot 54 + \underline{18}$$

$$54 = \underline{3} \cdot 18 + \underline{0}$$

The remainder of zero in the last expression shows that the $n=18$ represents our GCD.

Answer: 18_{10}

33. Use any method you wish to find the least common multiple of 180 and 126. (4 points)

Once again, there are a couple of ways to do this. The first way uses the prime number expansion of 180 and 126. By taking the maximum powers for each prime and multiplying them together, we generate the least common multiple:

$$\text{LCM}(180,126) = 2^2 \cdot 3^2 \cdot 5^1 \cdot 7^1$$

You could have left the answer in the form shown above or multiplied it out to get 1260.

Another way to do this is to use the GCD, then substitute it into the equation $\text{GCD}(m,n) \cdot \text{LCM}(m,n) = m \cdot n$. Substituting the GCD found in problem 32 into the equation for the LCM gives us:

$$\text{LCM}(180,126) = 180 \cdot 126 \div \text{GCD}(180,126) = 180 \cdot 126 \div 18 = 126 \cdot 10 = 1260$$

Answer: **1260₁₀**

34. True or False: The matrix A has the same dimensions as A^T , i.e., A^T has the same number of rows as A and the same number of columns as A . (2 points)

The way to show that this is false is to simply come up with a matrix A where this wouldn't work. For example, if we define A as:

$$A = \begin{bmatrix} 25 & 19 \\ 50 & 91 \\ 14 & 45 \end{bmatrix}$$

Now take the transpose and we get:

$$A^T = \begin{bmatrix} 25 & 50 & 14 \\ 19 & 91 & 45 \end{bmatrix}$$

The different number of rows and columns indicates that the answer to this problem is FALSE.

35. True or False: If A , B , and C are matrices, then $(A + B)C = AC + BC$. (2 points)

Theorem 2 in our book states that this is so, so the answer is TRUE. Let's do an example:

$$A = \begin{bmatrix} 1 & 4 \\ -3 & 2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 \\ 5 & 0 \\ 4 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 1+2 & 4-3 \\ -3+5 & 2+0 \\ 1+4 & 0+4 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 5 & 4 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 3 \cdot 2 + 1 \cdot 4 & 3 \cdot 3 + 1 \cdot 0 \\ 2 \cdot 2 + 2 \cdot 4 & 2 \cdot 3 + 2 \cdot 0 \\ 5 \cdot 2 + 4 \cdot 4 & 5 \cdot 3 + 4 \cdot 0 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ 12 & 6 \\ 26 & 15 \end{bmatrix}$$

Now for the other way:

$$AC+BC = \begin{bmatrix} 1 & 4 \\ -3 & 2 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 5 & 0 \\ 4 & 4 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} 1 \cdot 2 + 4 \cdot 4 & 1 \cdot 3 + 4 \cdot 0 \\ -3 \cdot 2 + 2 \cdot 4 & -3 \cdot 3 + 2 \cdot 0 \\ 1 \cdot 2 + 0 \cdot 4 & 1 \cdot 3 + 0 \cdot 0 \end{bmatrix} + \begin{bmatrix} 2 \cdot 2 + -3 \cdot 4 & 2 \cdot 3 + -3 \cdot 0 \\ 5 \cdot 2 + 0 \cdot 4 & 5 \cdot 3 + 0 \cdot 0 \\ 4 \cdot 2 + 4 \cdot 4 & 4 \cdot 3 + 4 \cdot 0 \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} 18 & 3 \\ 2 & -9 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -8 & 6 \\ 10 & 15 \\ 24 & 12 \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} 18 - 8 & 3 + 6 \\ 2 + 10 & -9 + 15 \\ 2 + 24 & 3 + 12 \end{bmatrix} + \begin{bmatrix} 10 & 9 \\ 12 & 6 \\ 26 & 15 \end{bmatrix}$$

Well, at least the example worked!

For problems 36 through 39, use the matrices A , B , and C shown below.

$$A = \begin{bmatrix} 2 & 1 & 8 \\ 3 & 2 & 9 \\ 1 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}$$

36. True or False: It is possible to compute $B^T + C$. (2 points)

B has three rows and one column, therefore, B^T has one row and three columns. Since C also has one row and three columns, B^T can be added to C : TRUE.

37. True or False: The result of BC is matrix with 3 rows and 3 columns. (2 points)

B has three rows and one column, and C has one row and three columns. The matrix resulting from the multiplication of B and C will have the number of rows from B and the number of columns from C . Therefore, it will have three rows and three columns, and the answer is TRUE.

38. True or False: It is possible to compute $CA + B^T$. (2 points)

C has one row and three columns and A has three rows and three columns. Therefore, the matrix resulting from the multiplication of C and A will have the number of rows from C and the number of columns from A , i.e., it will have one row and three columns. B has three rows and one column, therefore, B^T has one row and three columns. Since CA and B^T have the same number of rows and columns, they can be added and the answer is TRUE.

39. Calculate AB . Show all work. (4 points)

$$AB = \begin{bmatrix} 2 & 1 & 8 \\ 3 & 2 & 9 \\ 1 & 1 & 3 \end{bmatrix} * \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 + 1 \cdot 5 + 8 \cdot 0 \\ 3 \cdot 1 + 2 \cdot 5 + 9 \cdot 0 \\ 1 \cdot 1 + 1 \cdot 5 + 3 \cdot 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 \\ 13 \\ 6 \end{bmatrix}$$