Points missed: _____   Student's Name: _____

Total score: _____/100 points

Read this before starting!

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!)
  Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need
  arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University
  Faculty Handbook, June 1, 2001:

A short list of some tautologies:

1. $(p \wedge q) \Rightarrow p$ 　　　　　　2. $(p \wedge q) \Rightarrow q$
3. $p \Rightarrow (p \vee q)$ 　　　　　　4. $q \Rightarrow (p \vee q)$
5. $\sim p \Rightarrow (p \Rightarrow q)$ 　　　　　　6. $\sim(p \Rightarrow q) \Rightarrow p$
7. $((p \Rightarrow q) \wedge p) \Rightarrow q$ 　　　　　　8. $((p \vee q) \wedge \sim p) \Rightarrow q$
9. $((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$ 　　　　　　10. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

Mathematical induction:

If $P(n_0)$ is true and assuming $P(k)$ is true implies $P(k+1)$ is true, then $P(n)$ is true for all $n \geq n_0$

Permutations and Combinations:

$$_nP_r = \frac{n!}{(n - r)!} \qquad\qquad _nC_r = \frac{n!}{r!(n - r)!}$$

Properties of Relations:

- A relation is reflexive if $a \ R \ a$, for all a $\in$ A.
- A relation is irreflexive if $a \ \not{R} \ a$, for all a $\in$ A.
- A relation is symmetric if whenever $a \ R \ b$, then $b \ R \ a$.
- A relation is asymmetric if whenever $a \ R \ b$, then $b \ \not{R} \ a$.
- A relation is antisymmetric if whenever $a \ R \ b$ and $b \ R \ a$, then a = b.
- A relation is transitive if whenever $a \ R \ b$ and $b \ R \ c$, then $a \ R \ c$.
- A relation is called an equivalence relation if it is reflexive, symmetric, and transitive.

Each of the following six arguments uses one of the tautologies listed on the coversheet. (See table under the heading, "a short list of some tautologies.") For each of the four arguments, identify which tautology was used from this list *by entering a value 1 through 10 in the space provided*. (2 points each)

1. If it is thundering, then there is lightning
    There is thunder
    There is lightning

    Answer: _____

2. It is either raining or snowing
    It isn't raining
    It must be snowing

    Answer: _____

3. Either Ed is short or Ed is tall
    Ed is not short
    Ed is tall

    Answer: _____

4. If I drive to school, I will be late to class
    I am on time for class
    I didn't drive to school

    Answer: _____

5. This test is easy
    Either I studied well or this test is easy

    Answer: _____

6. Matthew is my son
    Matthew is a child of mine

    Answer: _____

For the next four arguments, *indicate which are valid and which are invalid*. (2 points each)

7. If I publish a novel, I will be famous
    If I am famous, I will be happy
    I am happy, therefore, I published a novel

    ☐ Valid   ☐ Invalid

8. If I drive to school, I will be late to class
    I was late to class
    I drove to school

    ☐ Valid   ☐ Invalid

9. If I try hard, then I will succeed
    If I succeed, then I will be happy
    I am not happy, therefore, I didn't try hard

    ☐ Valid   ☐ Invalid

10. Pete is the name of my pet
    The only dogs I own are black labs
    Pete is a black lab

    ☐ Valid   ☐ Invalid

The following seven problems present seven situations where *r* items are selected from a set of *n* items. **Select the formula, n^r, $_nP_r$, $_nC_r$, or $_{(n+r-1)}C_r$,** that will compute the number of different, valid sequences **and identify the values of *r* and *n*.** (4 points each)

11. Compute the number of 4-digit ATM PINs where duplicate digits are allowed.

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

12. Compute the number of different 5 card hands can be drawn from a deck of 52 cards.

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

13. How many committees of 5 people can be created from a group of 8 people?

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

14. How many ways can the letters in the word MICHAEL be arranged?

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

15. Assume you need to buy 10 bottles of soda from a selection of {Coke, Pepsi, Dr. Pepper, and Sprite}. How many ways could you do this?

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

16. How many three-digit numbers are there in base-5? Assume leading zeros are included as digits.

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

17. How many different ways can 2 six-sided dice come up? There is no order, e.g., 3 & 4 are the same as 4 & 3.

    a.) n^r b.) $_nP_r$ c.) $_nC_r$,   d.) $_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

18. True or false: In selecting r items from a set of n items where order doesn't matter and duplicates are allowed, r may be greater than n. (2 points)

19. Which of the following expressions describes how to calculate the number of available license plate combinations of the format "ABC 123"? (2 points)

    a.) $_{26}C_3 \cdot {}_{10}C_3$ b.) $_{(26 + 10 - 1)}C_{10}$ c.) $_{26}P_3 \cdot {}_{10}P_3$ d.) $_{(26 + 10 - 1)}P_6$
    e.) $_{(36 + 6 - 1)}C_6$ f.) $26^3 \cdot 10^3$ g.) $_{26}C_3 \cdot 10^3$ h.) None of the above

20. Assume we have a lottery where you first pick 5 from a group of 60 then pick one powerball option from a group of 45? What is the ratio of picking the wrong powerball to picking the right powerball? (2 points)

    a.) 45:1   b.) 44:1   c.) $_{60}C_5 \cdot 44 : {}_{60}C_5 \cdot 45$ d.) $_{45}C_1 : 1$ f.) None of the above

21. Let $A = $ {a, b} and $B = $ {1, 2, 3}. List all of the elements in $A \times B$. (3 points)

22. If $|A| = 5$ and $|B| = 10$, then the cardinality of $A \times B$ is: (2 points)

    a.) $5 \cdot 10$ b.) 10^5 c.) 5^{10} d.) $_5C_{10}$ d.) $_5P_{10}$ f.) None of the above

The next 5 problems represent relations across the Cartesian product $A \times A$ where $A = $ {a, b, c, d}. The relations are represented either as subsets of $A \times A$, matrices, or digraphs. For each problem, determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, and/or transitive. ***Check all that apply.*** (4 points each)

23. R = {(a,b), (a,c), (a,d), (b,c) , (b,d) , (c,d)}

☐ reflexive   ☐ irreflexive   ☐ symmetric   ☐ asymmetric   ☐ antisymmetric   ☐ transitive
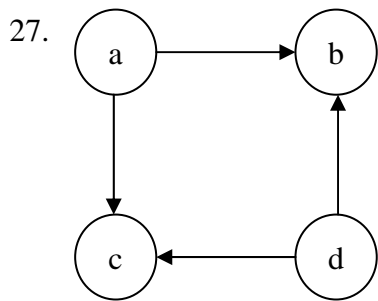
24. R = $A \times A$

☐ reflexive   ☐ irreflexive   ☐ symmetric   ☐ asymmetric   ☐ antisymmetric   ☐ transitive

25.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

☐ reflexive   ☐ irreflexive   ☐ symmetric   ☐ asymmetric   ☐ antisymmetric   ☐ transitive

26.
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

☐ reflexive   ☐ irreflexive   ☐ symmetric   ☐ asymmetric   ☐ antisymmetric   ☐ transitive
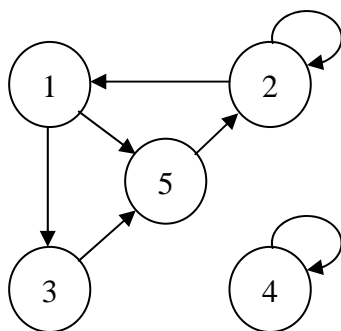
27.

☐ reflexive   ☐ irreflexive   ☐ symmetric   ☐ asymmetric   ☐ antisymmetric   ☐ transitive

The next three problems represent relations across the Cartesian product A × A where A = {1, 2, 3, 4, 5}.

28. Write the set of ordered pairs represented by the relation matrix (4 points)

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

29. Convert the following digraph to a matrix. (3 points)

30. Fill out the table below listing the in-degree and out degree of each element for the digraph of the previous problem. (4 points)

	1	2	3	4	5
In-Degree					
Out-Degree					

31. Create the digraph of the relation $R = A \times A$ for the set $A = \{a, b, c\}$. (3 points)

32. ***Select only one*** of the following statements to prove true using mathematical induction. (7 points)

a.) $2 + 4 + 6 + \ldots + 2n = n(n + 1)$

b.) $1 + 2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$

c.) $1 + a^1 + a^2 + a^3 + \ldots + a^{n-1} = \dfrac{a^n - 1}{a - 1}$