Points missed: _____ Student's Name: _____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2710 (Tarnoff) – Discrete Structures
TEST 2 for Spring Semester, 2005

**Read this before starting!**

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

"Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

***A short list of some tautologies:***

1. $(p \wedge q) \Rightarrow p$
2. $(p \wedge q) \Rightarrow q$
3. $p \Rightarrow (p \vee q)$
4. $q \Rightarrow (p \vee q)$
5. $\sim p \Rightarrow (p \Rightarrow q)$
6. $\sim(p \Rightarrow q) \Rightarrow p$
7. $((p \Rightarrow q) \wedge p) \Rightarrow q$
8. $((p \vee q) \wedge \sim p) \Rightarrow q$
9. $((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$
10. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

***Mathematical induction:***

If P($n_0$) is true and assuming P($k$) is true implies P($k$+1) is true, then P($n$) is true for all $n \geq n_0$

***Permutations and Combinations:***

$$ {}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{r!(n-r)!} $$

***Properties of operations for propositions***

Commutative Properties

1. $p \vee q \equiv q \vee p$
2. $p \wedge q \equiv q \wedge p$

Associative Properties

3. $p \vee (q \vee r) \equiv (p \vee q) \vee r$
4. $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive Properties

5. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
6. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Idempotent Properties

7. $p \vee p \equiv p$
8. $p \wedge p \equiv p$

Properties of Negation

9. $\sim(\sim p) \equiv p$
10. $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
11. $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

***Short answers – 2 points each unless otherwise noted***

For problems 1 through 4, indicate whether the phrase is a statement or not.

1. "Does class begin at 9:45 AM?" ☐ Statement ☐ Not a statement
2. "5 is greater than 25." ☐ Statement ☐ Not a statement
3. "Read the section before coming to class." ☐ Statement ☐ Not a statement
4. "It snowed more than usual this past February." ☐ Statement ☐ Not a statement
5. Give the negation of the statement "$24 \leq 5$."

6. Give the negation of the statement "I will exercise and eat right." (3 points)

For problems 7 and 8, find the truth value of each proposition if ***p* is false** and ***q* and *r* are true**.

7. $p \vee \sim q$ Answer: _____
8. $\sim(p \vee r) \wedge q$ Answer: _____

For problems 9 and 10, convert the sentence given to an expression in terms of p, q, r, and logical connectives if p: I drove; q: I found parking; and r: I am on time.

9. I am on time and I found parking. Answer: _____
10. I am on time if and only if I don't drive. Answer: _____

Each of the following six arguments uses one of the tautologies listed on the coversheet. (See table under the heading, "a short list of some tautologies.") For each of the four arguments, identify which tautology was used from this list ***by entering a value 1 through 10 in the space provided***.

11. Either this is easy or I studied
This isn't easy
---
I must have studied

Answer: _____

12. If I bought a Model T, it is black
I bought a Model T
---
My Model T is black

Answer: _____

13. If I work hard, I will succeed
I didn't succeed
---
I must not have worked hard

Answer: _____

14. After March 31, ETSU turns off the heat
The heat is still on at ETSU
---
It must be before March 31

Answer: _____

15. It is cold and it is snowing
---
It is snowing

Answer: _____

16. It's hot in here
---
Either its hot in here or I'm tired

Answer: _____

For the next four arguments, ***indicate which are valid and which are invalid***.

17. Casey is the name of my pet
I only own dogs for pets
---
Casey must be a dog

☐ Valid ☐ Invalid

18. If you are driving, I am walking
If I am walking, then I am on time
---
I'm not on time, therefore, you didn't drive

☐ Valid ☐ Invalid

19. If I win the lottery, I will invest wisely
If I invest wisely, I will be rich
---
I am rich, therefore, I won the lottery

☐ Valid ☐ Invalid

20. If I live in DC, driving is a hassle
Driving is a hassle
---
I must live in DC

☐ Valid ☐ Invalid

The following seven problems present seven situations where $r$ items are selected from a set of $n$ items. **Select the formula, $n^r$, ${}_nP_r$, ${}_nC_r$, or ${}_{(n+r-1)}C_r$,** that will compute the number of different, valid sequences **and identify the values of *r* and *n***. (4 points each)

21. Compute the number of possible license plates with 6 digits that can be either letters or numbers.

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

22. Compute the number of combinations of 5 marbles you could pull from a bag containing 10 different colored marbles.

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

23. How many subsets are there of the set A = {a, b, c, d, e}?

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

24. How many five-digit numbers are there in base-16? Assume leading zeros are included as digits.

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

25. How many shades of color can be created by mixing 5 parts from red, green, and blue?

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

26. How many ways can the letters in the word "COMPUTER" be arranged without omitting or duplicating a letter?

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

27. How many different dominos are there in a package? (Note: Each domino is a pair of numbers from the values 0, 1, 2, 3, 4, 5, and 6. A number can be paired with itself, e.g., 3 and 3 is allowed, but there is no order, e.g., a 3 paired with a 4 is the same as a 4 paired with a 3.)

a.) $n^r$ b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n$ = _____ $r$ = _____

28. True or false: $r$ must always be less than or equal to $n$ when determining the number of ways $r$ items can be selected from a set of $n$ items when order matters and duplicates are not allowed.

29. True or false: ${}_nC_1$ is always equal to ${}_nC_{(n-1)}$.

30. Which of the following expressions describes how to calculate the number of ways that a committee of 2 students and 4 faculty members can be formed from sets of 10 students and 16 faculty members?

| | | | |
|---|---|---|---|
| a.) ${}_{10}C_2 \cdot {}_{16}C_4$ | b.) ${}_{(10+16-1)}C_6$ | c.) ${}_{10}P_2 \cdot {}_{16}P_4$ | d.) ${}_{(10+16-1)}P_6$ |
| e.) ${}_{(26+6-1)}C_6$ | f.) $10^2 \cdot 16^4$ | g.) $(10! \cdot 16!) \div (2! \cdot 4!)$ | h.) None of the above |

31. Which of the following expressions describes how to calculate the number of ways that drawing 5 cards from a deck of 52 can result in 4 of a kind with any other card being the fifth card?

| | | | |
|---|---|---|---|
| a.) ${}_{52}C_5 \div 13$ | b.) ${}_{52}P_4 \cdot {}_{48}P_1$ | c.) ${}_{52}C_1 \cdot {}_{48}C_1$ | d.) ${}_{52}C_4 \cdot {}_{48}C_1$ |
| e.) ${}_{13}C_1 \cdot {}_{48}C_1$ | f.) ${}_{13}C_1 \cdot {}_4P_1 \cdot {}_{48}C_1$ | g.) ${}_{52}P_4 \cdot {}_{48}C_1$ | h.) None of the above |

***Medium answers – 4 points each unless otherwise noted***

32. Assume that a lottery allows you to pick 5 numbers from a group of 62. What is the probability that you will pick all five right? Don't bother performing multiplications or divisions. Just leave expanded.

33. What is the probability that you will get a royal flush (four possible ways to do this) from drawing 5 cards from a deck of 52? Don't bother performing multiplications or divisions. Just leave expanded.

34. Use truth tables to show that $p \Rightarrow (p \vee q)$ is a tautology. Show ***all*** intermediate steps. Be sure to label columns.

35. Use truth tables to show that $(p \Leftrightarrow q) \Leftrightarrow ((q \Rightarrow p) \wedge (p \Rightarrow q))$ is a tautology. Show ***all*** intermediate steps. Be sure to label columns.

***Mathematical induction problem – 7 points***

36. ***Select only one*** of the following statements to prove true using mathematical induction.

a.) $2 + 4 + 6 + \ldots + 2n = n(n + 1)$

b.) $1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = \dfrac{n(2n + 1)(2n - 1)}{3}$

c.) $5 + 10 + 15 + \ldots + 5n = \dfrac{5n(n + 1)}{2}$