Points missed: _____ Student's Name: _____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 1900 (Tarnoff) – Discrete Structures
TEST 2 for Summer Semester, 2005

Read this before starting!

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

A short list of some tautologies:

1. $(p \wedge q) \Rightarrow p$
2. $(p \wedge q) \Rightarrow q$
3. $p \Rightarrow (p \vee q)$
4. $q \Rightarrow (p \vee q)$
5. $\sim p \Rightarrow (p \Rightarrow q)$
6. $\sim(p \Rightarrow q) \Rightarrow p$
7. $((p \Rightarrow q) \wedge p) \Rightarrow q$
8. $((p \vee q) \wedge \sim p) \Rightarrow q$
9. $((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$
10. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

Mathematical induction:

If P(n_0) is true and assuming P(k) is true implies P(k+1) is true, then P(n) is true for all $n \geq n_0$

Permutations and Combinations:

$$ {}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{r!(n-r)!} $$

Properties of operations for propositions

Commutative Properties

1. $p \vee q \equiv q \vee p$
2. $p \wedge q \equiv q \wedge p$

Associative Properties

3. $p \vee (q \vee r) \equiv (p \vee q) \vee r$
4. $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive Properties

5. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
6. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Idempotent Properties

7. $p \vee p \equiv p$
8. $p \wedge p \equiv p$

Properties of Negation

9. $\sim(\sim p) \equiv p$
10. $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
11. $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

Short answers – 2 points each unless otherwise noted

For problems 1 through 4, indicate whether the phrase is a statement or not.

1. "Study hard!" ☐ Statement ☐ Not a statement
2. "How hard did you study for this test?" ☐ Statement ☐ Not a statement
3. "42 is the answer." ☐ Statement ☐ Not a statement
4. "2 is an odd number." ☐ Statement ☐ Not a statement
5. Give the negation of the statement "2 + 7 = 9."

6. Give the negation of the statement "It will rain tomorrow or it will snow tomorrow." (3 points)

For problems 7 and 8, find the truth value of each proposition if ***p* and *q* are true** and ***r* is false**.

7. $p \wedge \sim q \wedge \sim r$ ☐ True ☐ False
8. $p \wedge (r \vee q)$ ☐ True ☐ False

For problems 9 and 10, convert the sentence given to an expression in terms of p, q, r, and logical connectives ($\sim$, $\vee$, $\wedge$, $\Leftrightarrow$, and $\Rightarrow$) if p: I'm rich; q: I work hard; and r: I'm lucky.

9. I'm not lucky, but I work hard. Answer: _____
10. If I work hard and I'm lucky, then I will be rich. Answer: _____

Each of the following six arguments uses one of the tautologies listed on the coversheet. (See table under the heading, "a short list of some tautologies.") For each of the four arguments, identify which tautology was used from this list ***by entering a value 1 through 10 in the space provided***.

11. If Tarnoff is lecturing, I'm fascinated
If I'm fascinated, then I'm awake
If Tarnoff is lecturing, then I'm awake

Answer: _____

12. If I studied, I will pass this test
I studied
I will pass this test

Answer: _____

13. If I like country music, then I like Alan Jackson
I don't like Alan Jackson
I must not like country music

Answer: _____

14. I have a child that is a girl or a boy
I do not have a daughter
I must have a son

Answer: _____

15. If I finish studying, I will go watch a movie
I finished studying
I will go watch a movie

Answer: _____

16. This test is easy
Either this test is easy or I studied

Answer: _____

For the next four arguments, ***indicate which are valid and which are invalid***.

17. If I walk to school, I am tired
I am tired

I must have walked to school

☐ Valid ☐ Invalid

18. If it rains, I will not walk to school
I walked to school

It isn't raining

☐ Valid ☐ Invalid

19. If I win the lottery, I will invest wisely
If I invest wisely, I will be rich

I won the lottery, therefore, I am rich

☐ Valid ☐ Invalid

20. If I own a dog, it will be a male dog
Spot is my pet

Spot is a male dog

☐ Valid ☐ Invalid

The following seven problems present seven situations where r items are selected from a set of n items. **Select the formula, n^r, ${}_nP_r$, ${}_nC_r$, or ${}_{(n+r-1)}C_r$,** that will compute the number of different, valid sequences **and identify the values of *r* and *n***. (4 points each)

21. Compute the number of possible four letter words using the English alphabet including the nonsensical ones like "zzyq".

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

22. How many ways can you create a 5 person committee from a group of 27 employees?

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

23. How many subsets are there of the set B = {1, 2, 3, 4, 5, 6}?

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

24. How many 7-digit numbers are there in base-3? Assume leading zeros are included as digits.

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

25. How many ways can 35 drivers finish first, second, and third in a race in a specific order? The rest of the finishing order is unimportant.

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

26. How many ways can 35 drivers finish first, second, and third in a race in any order? The rest of the finishing order is unimportant.

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

27. How many ways can 4 types of chocolate be used to fill a box with slots for 8 pieces of chocolate?

a.) n^r b.) ${}_nP_r$ c.) ${}_nC_r$, d.) ${}_{(n+r-1)}C_r$ $n =$ _____ $r =$ _____

28. True or false: r must always be less than or equal to n when determining the number of ways r items can be selected from a set of n items when order matters and duplicates are allowed.

29. True or false: ${}_nP_1 = {}_nC_1 = n^1$.

30. Assume that a computer can come configured with one of 3 different processors and one of 5 different memory configurations. Which of the following expressions describes how to calculate the number of ways that this computer can be configured?

a.) ${}_{(3+5-1)}C_2$ b.) $3 \cdot 5$ c.) ${}_8P_2$ d.) ${}_8C_2$ h.) None of the above

31. Remember that the Powerball lottery game consists of selecting 5 numbers from 53 and 1 Powerball number from 42. Select which of the following expressions describes how to calculate the number of ways that exactly 4 of the 5 numbers have been selected correctly and the Powerball number was selected incorrectly.

a.) ${}_{48}C_1 \cdot 41$ b.) ${}_{48}P_1 \cdot 41$ c.) ${}_{53}C_1 \cdot 42$ d.) ${}_{53}P_1 \cdot 42$
e.) ${}_{48}C_1 \cdot {}_5C_1 \cdot 41$ f.) ${}_{48}P_1 \cdot {}_5P_1$ g.) ${}_{53}C_1 \cdot {}_5C_1 \cdot 42$ h.) None of the above

32. What is the probability that you will get a heads from a single flip of a quarter? (Assume all outcomes are equally likely.)

a.) 0 b.) 1/4 c.) 1/3 d.) 1/2 e.) 1 f.) None of the above

33. What is the probability that you will get ***at least one*** heads from three flips of a quarter? (Assume all outcomes are equally likely.)

a.) 1/3 b.) 2/3 c.) 2/4 d.) 3/4 e.) 5/8 f.) 7/8 g.) 1

34. What is the probability that you will get a '5' from a single roll of a single six-sided die? (Assume all outcomes are equally likely.)

a.) 0 b.) 1/5 c.) 5/6 d.) 1/3 e.) 1/6 f.) None of the above

35. What is the probability that you will get an even number from a single roll of a single six-sided die? (Assume all outcomes are equally likely.)

a.) 1/6 b.) 3/5 c.) 4/6 d.) $6/6^2$ e.) 3/6 f.) 5/6 g.) 1

Medium answers – 4 points each unless otherwise noted

36. Use truth tables to show that $\sim(p \Rightarrow q) \Rightarrow p$ is a tautology. Show ***<u>all</u>*** intermediate steps. Be sure to label columns.

37. Use truth tables to show that $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ is a tautology. Show ***all*** intermediate steps. Be sure to label columns.

Mathematical induction problem – 7 points

38. ***Select only one*** of the following statements to prove true using mathematical induction.

a.) $1 + 2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$

b.) $2 + 4 + 6 + \ldots + 2n = n(n + 1)$

c.) $5 + 10 + 15 + \ldots + 5n = \dfrac{5n(n + 1)}{2}$