Points missed: _____ Student's Name: _____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2710 (Tarnoff) – Discrete Structures
TEST 3 for Spring Semester, 2005

Read this before starting!

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

QUESTIONS BEGIN HERE!

Problems 1, 2, and 3 represent relations across the Cartesian product A×A where A = {a, b, c, d}. The relations are represented either as subsets of A×A, matrices, or digraphs. For each problem, determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, and/or transitive. <u>Check all that apply.</u> (3 points each)

1. $R = A \times A$

☐ reflexive ☐ irreflexive ☐ symmetric ☐ asymmetric ☐ antisymmetric ☐ transitive

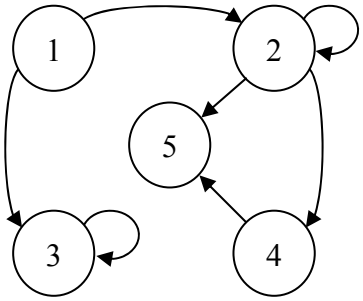
2. $R = \{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, c), (d, d)\}$

☐ reflexive ☐ irreflexive ☐ symmetric ☐ asymmetric ☐ antisymmetric ☐ transitive

3.
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

☐ reflexive ☐ irreflexive ☐ symmetric ☐ asymmetric ☐ antisymmetric ☐ transitive

4. The digraph below represents a relation ***R*** on ***A*** = {1, 2, 3, 4, 5}. Convert the digraph to a matrix. (3 points)



5. Fill out the table below listing the in-degree and out-degree of each element for the relation of the previous problem. (3 points)

	1	2	3	4	5
In-Degree					
Out-Degree					

For problems 6, 7, and 8, let A = {a, b, c, d} and B = {1, 2, 3, 4}. Determine whether the each of the relations R from A to B in these problems is a function. (2 points each)

6. R = A × B ☐ Function ☐ Not a function

7. R = {(a, 2), (c, 1), (d, 1), (b, 2)} ☐ Function ☐ Not a function

8. R = {(a, 4), (b, 3), (c, 2), (c, 1)} ☐ Function ☐ Not a function

For problems 9 and 10, determine the domain and range of the function f. In other words, if f(a) = b, then what values of 'a' make sense for f (Dom(f)) and what values of 'b' make sense for f (Ran(f))? By the way, please stick to subsets of real numbers. (3 points each)

9. $f(a) = +\sqrt{a}$ Dom(f) = _____ Ran(f) = _____

10. $f(a) = a \pmod 5$ where a is an integer Dom(f) = _____ Ran(f) = _____

For problems 11 and 12, let the universal set U = Z^+ (the set of positive integers). Given the subset A, determine the output of the given characteristic or membership function f_A. (1 point each)

11. A = {n | n = even positive integer} $f_A(234)$ = _____

12. A = {0, 5, 10, 15, … 5n} n = 0, 1, 2, … $f_A(234)$ = _____

For problems 13 and 14, let f be the mod-100 function. Compute the output for each of the problems. (2 points each)

13. $f(34)$ = _____ 14. $f(222)$ = _____

Each relation R in problems 15 through 17 is defined on A = {a, b, c, d, e}. In each case, determine if R is a rooted tree, and if it is, what is the root? If there is no root, leave that space blank. (3 points ea.)

15. ***R*** = {(a, c), (a, d), (b, e), (b, c)}

☐ ***R*** is a rooted tree ☐ ***R*** is not a rooted tree
If ***R*** is a rooted tree, the root is: _____

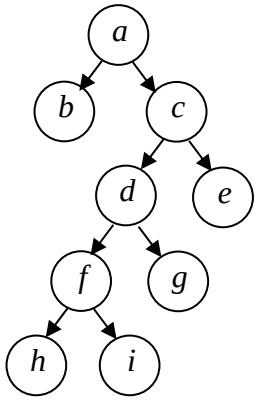
16. ***R*** = {(c, a), (b, c), (e, d), (d, b)}

☐ ***R*** is a rooted tree ☐ ***R*** is not a rooted tre
If ***R*** is a rooted tree, the root is: _____

17. ***R*** = {(e, a), (e, c), (c, d), (c, b), (d, b)}

☐ ***R*** is a rooted tree ☐ ***R*** is not a rooted tree
If ***R*** is a rooted tree, the root is: _____

For problems 18 through 24, use the rooted tree T shown in the figure to the right. (2 points each)

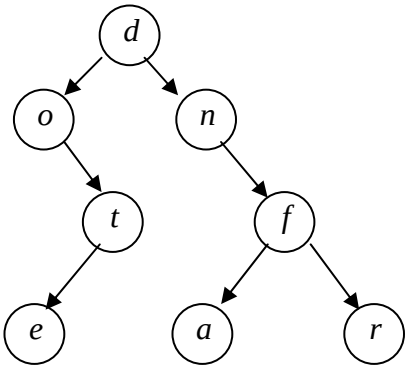


18. What is the height of *T*? _____
19. *T* is an *n*-tree. What is the value of *n*? _____
20. List all of the leaves of *T*. _____
21. List all of the siblings of *d*. _____
22. List all of the offspring of *d*. _____
23. List all of the descendants of *d*. _____
24. True or false: T is a ***complete*** *n*-tree? _____
25. Construct the tree of the algebraic expression **((*a* ÷ 3) + 2) × (*b* – 6)**. (4 points)

26. The following doubly linked list represents a binary positional labeled tree. Construct the digraph of this tree with each vertex labeled as indicated. (6 points)

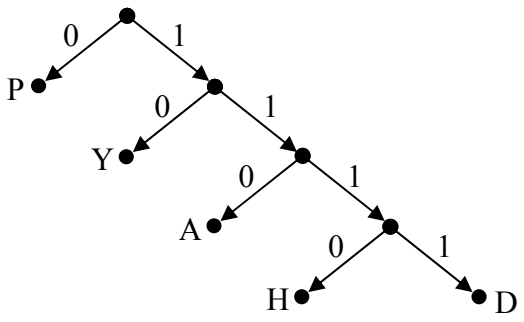
index	left	data	right
1	8		0
2	0	N	0
3	0	T	6
4	0	U	0
5	0	H	0
6	0	S	2
7	5	A	10
8	7	M	3
9	0	F	0
10	9	I	4

27. Fill in the LEFT and RIGHT arrays in the table to the left for the tree shown below. (6 points)



index	left	data	right
1	5		0
2		*t*	
3		*e*	
4		*r*	
5		*d*	
6		*f*	
7		*n*	
8		*o*	
9		*a*	

28. Use the Huffman code tree shown to the right to find the string of 0's and 1's that represents the word **PAYDAY**. (4 points)



29. Use the Huffman code tree shown to the right to decode the message **11101100010111111010**. (3 points)

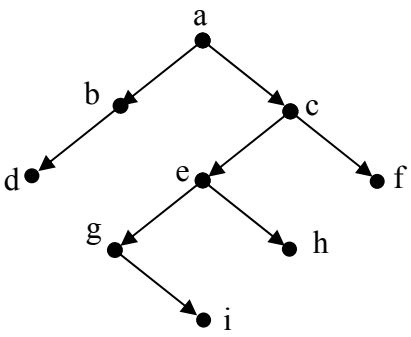
30. The expression shown below is written in Polish (prefix) notation. Evaluate it to the final integer result. Note that all of the numbers are single digit integers. (3 points)

– ÷ × 4 2 – 6 2 1

31. The expression shown below is written in reverse Polish (postfix) notation. Evaluate it to the final integer result. Note that all of the numbers are single digit integers. (3 points)

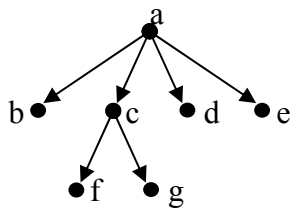
5 3 – 4 5 + 3 ÷ ×

32. List the vertices in the order that they are visited in a preorder search of the tree shown to the right. (3 points)



33. List the vertices in the order that they are visited in an inorder search of the same tree from problem 32. (3 points)

34. In the space to the right, convert the tree shown below to a binary positional tree. (4 points)



35. Use any method you wish to determine the minimal spanning tree for the connected graph shown below and to the left. Draw the connections of the minimal spanning tree using the vertices shown to the right. (5 points)