Points missed: _____ Student's Name: ___________________________________

Total score: _____ /100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2150 (Tarnoff) – Computer Organization
TEST 1 for Fall Semester, 2005

Read this before starting!

- The total possible score for this test is 100 points.
- This test is ***closed book and closed notes***
- You may ***NOT*** use a calculator. Leave all numeric answers in the form of a formula.
- You may use one sheet of scrap paper that you must turn in with your test.
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer. Example:

$$\boxed{32F1_{16}}$$

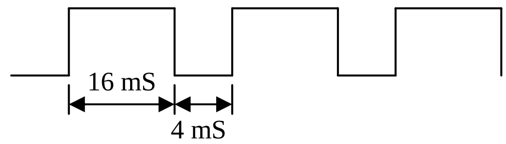
- **1 point will be deducted** per answer for missing or incorrect units when required. **No** assumptions will be made for hexadecimal versus decimal, so you should always include the base in your answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

Basic Rules of Boolean Algebra:	**1.** $A + 0 = A$	**7.** $A \cdot A = A$
	2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
	3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
	4. $A \cdot 1 = A$	**10.** $A + AB = A$
	5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
	6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$
DeMorgan's Theorem:	$\overline{(AB)} = (\overline{A} + \overline{B})$	$\overline{(A + B)} = (\overline{A} \bullet \overline{B})$

Short-ish Answer (2 points each unless otherwise noted)

1. What is the frequency of the periodic signal in the figure shown to the right? ***Be sure to include your units!*** (Note: mS means milliseconds = 10^{-3} seconds)



First, be sure to use the correct value for the period, i.e., it is one full cycle of the signal from high to low and back to high. This becomes 16 mS + 4 mS = 20 mS. Next, we need to invert the period in order to calculate the frequency.

$$\text{frequency} = \frac{1}{\text{period}} = \frac{1}{20\times10^{-3}\text{ sec.}} = 50 \text{ Hertz}$$

You could have left your answer as $1/(20\times10^{-3})$ Hz if you wanted to. I also needed to see the units of Hertz to be sure that you knew what the frequency was measured with.

2. What is the duty cycle of the periodic signal from problem 1?

Remember that the duty cycle is the ratio of the duration of a logic one to the duration of the period. The logic one duration is 16 mS while the period is 20 mS. Therefore, the duty cycle, with its units of % is:

$$\text{Duty cycle} = \frac{t_h}{T} * 100\% = \frac{16}{20} * 100\% = 80\%$$

3. How many combinations of 1's and 0's can a 9-bit number (i.e., 9 binary variables) have?

a.) 2^{9-1} b.) $2^9 - 1$ c.) 2^{9+1} d.) 2^9 e.) $2^{9-1} - 1$ f.) None of the above

4. For each of the following binary representations, give the decimal expression for the smallest/lowest value that can be represented using **7** bits. (2 points each)

a.) unsigned binary: Since unsigned binary cannot represent negative values, the smallest value it can represent is **0**.

b.) 2's complement: The formula for the lowest 2's complement value of n-bits is $-(2^{(n-1)})$. In the case of 7 bits, this is $-(2^6) = -64_{10}$. Another way of doing this is to know that the most negative 2's complement value of 7 bits is 1000000_2. Converting this to decimal involves first figuring out what the unsigned binary pattern is.

$$\begin{array}{rl} 1000000_2 & \rightarrow \text{2's complement representation} \\ 0111111_2 & \rightarrow \text{1's complement representation} \\ +\quad\quad\quad 1_2 & \\ \hline 1000000_2 & \rightarrow \text{unsigned representation of positive value} \end{array}$$

In unsigned representation, $1000000_2 = 64_{10}$. Therefore, in 2's complement representation, $1000000_2 = -64_{10}$.

c.) signed magnitude: The formula for the lowest signed magnitude value of n-bits is $-(2^{(n-1)} - 1)$. In the case of 7 bits, this is $-(2^6 - 1) = -63_{10}$. Another way of doing this is to know that the most negative signed magnitude value of 7 bits is all ones or 1111111_2. Converting this to decimal involves first figuring out what the unsigned binary pattern is.

1111111_2 → 2's complement representation
0111111_2 → unsigned representation of positive value

In unsigned representation, $0111111_2 = 63_{10}$. Therefore, in signed magnitude representation, $1111111_2 = -63_{10}$.

5. True or False: The number 1110101100101100111 is a valid BCD number.

Below is the hex to binary conversion table. This is the same table that is used for BCD conversion where the letters A, B, C, D, E, and F are disallowed.

binary				hex
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	A
1	0	1	1	B
1	1	0	0	C
1	1	0	1	D
1	1	1	0	E
1	1	1	1	F

Therefore, the number 0111 0101 1001 0110 0111 (broken into nibbles) is the BCD value 75967_{10}. Since this is a decimal value, 1110101100101100111 is a valid BCD value.

6. True or False: The inversion bar (NOT operation) in boolean algebra is equivalent to an algebraic negative sign.

7. True or False: True or False: The binary floating-point number 01101101101011001101000011001001 is negative.

The MSB is zero, and therefore it is a positive value.

8. Write the complete truth table for a 2-input XOR gate (3 points).

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Remember that the XOR only outputs a 1 when an odd number of 1's are present at the input.

The following two questions are based on the 8-bit binary addition shown below.

```
              1  11100010
Carry out       +00110010
                 00010100
```

9. True or False: If the addition above is considered 8-bit 2's complement, an overflow has occurred.

   False. You cannot have an overflow by adding a negative number to a positive number. The only way a 2's complement can have an overflow is if the sign bits of the numbers that are being added are the same as each other, but different from the sign bit of the result.

10. True or False: If the addition above is considered 8-bit unsigned, an overflow has occurred.

    True. If a carry occurs in unsigned addition, an overflow has occurred.

11. Multiply 00010100_2 by 4. Leave your answer in binary. (Hint: There is a shortcut.) (3 points)

    Remember that a multiply by a power of two is equivalent to shifting the bits of the value left once for each multiple of two. Since $4 = 2^2$, then two left shifts of 00010100_2 will be equivalent to a multiply by four.

$$00010100_2 \times 4_{10} = 01010000_2$$

12. If an entire digital system could be created using only one type of gate, which gate should it be?

    a.) AND b.) OR c.) NOT d.) XOR e.) NAND f.) NOR g.)Exclusive NOR

    This problem was thrown out due to poor wording and general problems with the question.

13. Circle the function that would first be performed in the following expression.

$$A \cdot B \cdot (C \cdot D + E)$$

Medium-ish Answer (4 points each unless otherwise noted)

14. Convert the BCD number 100101010001 to decimal.

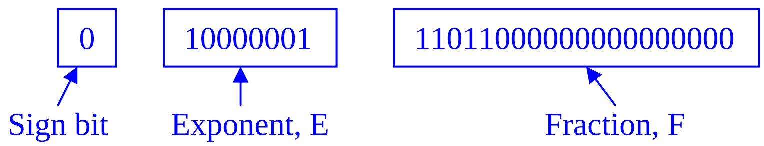
    Begin by dividing the number into nibbles (groups of four bits) starting at the right hand side. (I wasn't tricky this time. You didn't need to add leading zeros.)

1001 0101 0001

Using the table from problem 5, convert each nibble to its decimal value. This gives us $\mathbf{951_{10}}$.

15. Convert the floating-point number 01000000111011000000000000000000 to its binary exponential format, e.g., 1.1010110×2^{-12}, (which, by the way, is not even close to the right answer).

First, break the 32-bit number into its components.



A sign bit of 0 means that this will be a positive number.

The exponent, E, will be used to determine the power of two by which our mantissa will be multiplied. To use it, we must first convert it to a decimal integer using the unsigned method.

$$\begin{aligned} \text{Exponent, E} &= 10000001_2 \\ &= 2^7 + 2^0 \\ &= 128 + 1 \\ &= 129_{10} \end{aligned}$$

Substituting these components into our equation for floating point values gives us:

$$\begin{aligned} (\underline{+})1.F \text{ x } 2^{(E-127)} &= +1.11011000000000000000000 \text{ x } 2^{(129-127)} \\ &= +1.11011 \times 2^2 \end{aligned}$$

16. Convert 0101.0100 to decimal. (You may leave your answer in expanded form if you wish.)

Remember that binary digits to the right of the point continue in descending order relative to the 2^0 position. Therefore, the powers of two are in order to the right of the point $2^{-1} = 0.5$, $2^{-2} = 0.25$, $2^{-3} = 0.125$, and $2^{-4} = 0.0625$.

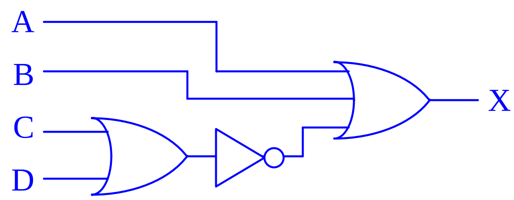
2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
0	1	0	1	0	1	0	0

Therefore, the answer is:

$$2^2 + 2^0 + 2^{-2} = 4 + 1 + 1/4 = 5.25$$

You could have left your answer in any of these forms in order to receive full credit.

17. Draw the circuit ***exactly*** as it is represented by the Boolean expression $A + B + \overline{C + D}$



18. Convert 1011101111010101001_2 to hexadecimal.

Begin by dividing the number into nibbles (groups of four bits) starting at the right hand side. (This time I was tricky, and you needed to add a single leading zero.)

0101 1101 1110 1010 1001

Using the table from problem 5, convert each nibble to its hexadecimal value. This gives us **$5DEA9_{16}$**.

19. Use any method you wish to prove rule 10: $A + A \cdot B = A$. Show all steps.

The easiest way is to prove it using a truth table:

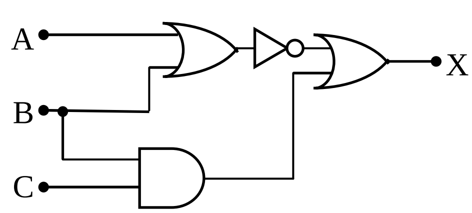
A	B	A·B	A + A·B
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Since the first column equals the last column, then $A + A \cdot B = A$.

There was also a simple proof of the formula:

```
A + A·B = A(1 + B)          Distributive Law
        = A·1               Rule 2
        = A                 Rule 4
```

20. In the space to the right, create the truth table for the circuit shown below. (5 points)



A	B	C	A + B	^(A + B)	B·C	X = ^(A+B)+ BC
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	1	0	0	0
0	1	1	1	0	1	1
1	0	0	1	0	0	0
1	0	1	1	0	0	0
1	1	0	1	0	0	0
1	1	1	1	0	1	1

21. Write the Boolean expression for the circuit shown in the previous problem. ***Do not simplify!***

$$X = \overline{(A + B)} + B \cdot C$$

22. If an 8-bit binary number is used to represent an analog value in the range from –5 to 5, how large an analog value does a single binary increment represent? In other words, if the binary number is incremented by one, how much change in the analog range is represented? (Leave your answer in the form of a fraction.)

A single increment = range/number of increments

$= (5 - (-5))/(2^8 - 1)$

$= 10/255$

$= 0.039216$

You could have left your answer in any of the last three forms.

23. Use DeMorgan's Theorem to distribute the inverse of the expression $\overline{A \bullet B + C \bullet D}$ to the individual input terms. ***Do not simplify!***

$\overline{A \cdot B + C \cdot D}$

$(\overline{A \cdot B}) \cdot (\overline{C \cdot D})$

$(\overline{A} + \overline{B}) \cdot (\overline{C} + \overline{D})$

First, distribute inverse across OR

Next, distribute inverse across A·B and C·D

This gives us the final answer. Note the importance of the parenthesis. Without them, the order of precedence would AND the inverses of B and C first which would be wrong.

Longer Answers (Points vary per problem)

24. Mark each boolean expression as ***true*** or ***false*** depending on whether the right and left sides of the equal sign are equivalent. Show all of your work to receive partial credit for incorrect answers. (3 points each)

a.) $(A+B)\cdot(B+\overline{A})=B$ Answer: **True**

$A\cdot B + A\cdot\overline{A} + B\cdot B + B\cdot\overline{A}$	(Using "F-O-I-L")
$A\cdot B + 0 + B\cdot B + B\cdot\overline{A}$	(Using Rule 8)
$A\cdot B + B\cdot B + B\cdot\overline{A}$	(Using Rule 1)
$A\cdot B + B + B\cdot\overline{A}$	(Using Rule 7)
$B + A\cdot B + B\cdot\overline{A}$	(Using Commutative Law)
$B + B\cdot(A + \overline{A})$	(Using Distributive Law)
$B + B\cdot 1$	(Using Rule 6)
$B + B$	(Using Rule 4)
B	(Using Rule 5)

b.) $A\cdot\overline{B}\cdot C+\overline{A}\cdot B\cdot C+A\cdot B\cdot C+\overline{A}\cdot\overline{B}\cdot C=C$ Answer: **True**

$C\cdot(A\cdot\overline{B} + \overline{A}\cdot B + A\cdot B + \overline{A}\cdot\overline{B})$	(Using Distributive Law)
$C\cdot(A\cdot\overline{B} + A\cdot B + \overline{A}\cdot B + \overline{A}\cdot\overline{B})$	(Using Commutative Law)
$C\cdot(A\cdot(\overline{B} + B) + \overline{A}\cdot(B + \overline{B}))$	(Using Distributive Law)
$C\cdot(A\cdot 1 + \overline{A}\cdot 1)$	(Rule 6)
$C\cdot(A + \overline{A})$	(Rule 4)
$C\cdot 1$	(Rule 6)
C	(Rule 4)

c.) $\overline{A} \cdot C + \overline{(A + \overline{C})} \cdot B = \overline{A} \cdot C$ Answer: **True**

$\overline{A} \cdot C + \overline{A} \cdot C \cdot B$	(DeMorgan's Theorem)
$\overline{A} \cdot C \cdot (1 + B)$	(Distributive Law)
$\overline{A} \cdot C \cdot 1$	(Rule 2)
$\overline{A} \cdot C$	(Rule 4)

25. Fill in the blank cells of the table below with the correct numeric format. ***For cells representing binary values, only 8-bit values are allowed!*** If a value for a cell is invalid or cannot be represented in that format, write "X". Use your scrap paper to do your work. (2 points per cell)

Decimal	2's complement binary	Signed magnitude binary	Unsigned binary
106	**01101010**	**01101010**	**01101010**
–74	**10110110**	**11001010**	**X**
91	**01011011**	**01011011**	**01011011**

In the top row, all three binary representations are the same for positive numbers, and since the MSB of the signed magnitude value is 0, it is a positive value. Converting to decimal gives us $2^6 + 2^5 + 2^3 + 2^1 = 64 + 32 + 8 + 2 = 106$.

In the second row, we can put an X in the last column as unsigned binary has no negative values. To convert from decimal to any of the other signed binary representation, we must first figure out what the unsigned binary representation of the positive value of +74 is. To do this, we successively pull out the largest powers of 2 that we can from 74 beginning with $2^6 = 64$. The result we get is:

$$74 = 64 + 8 + 2 = 2^6 + 2^3 + 2^1$$

Therefore, the unsigned binary representation of 74 has bits 1, 3, and 6 set:

$$74_{10} = 01001010_2$$

To convert this to -74 in 2's complement, we can use the short cut which says starting on the right side, copy all digits up to and including the first 1 we come to. After that, invert the remaining bits to the left. The work below shows what happens with the values in red being copied straight down and the values in blue inverted.

0	1	0	0	1	0	1	0
1	0	1	1	0	1	1	0

Therefore, the 2's complement representation of –74 is 10110110_2.

To convert to signed magnitude, simply take the original value for +74 and flip the MSB. This gives us the signed magnitude representation of –74 is 11001010_2.

For the third row, we have a similar situation as the top row, specifically that for positive values of 7-bits or less, the 8-bit value is the same for all representations. (Note that I say 7-bits or less because both signed representations use the MSB as a sign bit.) Once again, we successively pull out the largest powers of 2 that we can from 91 beginning with $2^6 = 64$. The result we get is:

$$\begin{aligned} 91 &= 64 + 27 \\ &= 64 + 16 + 11 \\ &= 64 + 16 + 8 + 3 \\ &= 64 + 16 + 8 + 2 + 1 \\ &= 2^6 + 2^4 + 2^3 + 2^1 + 2^0 \end{aligned}$$

Therefore, the unsigned binary representation of 91 has bits 0, 1, 3, 4, and 6 set:

$$91_{10} = 01011011_2$$