ELTR 105 (DC 2), section 3

Recommended schedule

<u>Day 1</u>

Topics: *Inductance and inductors*
Questions: *1 through 20*
Lab Exercises: *Series inductances (question 81) and parallel inductances (question 82)*

<u>Day 2</u>

Topics: *Capacitance and capacitors*
Questions: *21 through 40*
Lab Exercises: *Series capacitances (question 83) and parallel capacitances (question 84)*

<u>Day 3</u>

Topics: *Time constants*
Questions: *41 through 60*
Lab Exercise: *RC discharge circuit (question 85)*

<u>Day 4</u>

Topics: *Time constant circuits*
Questions: *61 through 80*
Lab Exercise: *Time-delay relay (question 86)*

<u>Day 5</u>

Exam 3: *includes RC discharge circuit performance assessment*
Troubleshooting Assessment due: *Loaded voltage divider (question 87)*
Question 88: Troubleshooting log
Question 89: Sample troubleshooting assessment grading criteria

<u>Practice and challenge problems</u>

Questions: *90 through the end of the worksheet*

1

Skill standards addressed by this course section

B Technical Skills – DC circuits

B.03 Demonstrate an understanding of the meaning of and relationships among and between voltage, current, resistance and power in DC circuits.

B.07 Demonstrate an understanding of the physical, electrical characteristics of capacitors and inductors.

B.21 Understand principles and operations of DC RC and RL circuits.

B.22 Fabricate and demonstrate DC RC and RL circuits.

B.23 Troubleshoot and repair DC RC and RL circuits.

B Basic and Practical Skills – Communicating on the Job

B.01 Use effective written and other communication skills. *Met by group discussion and completion of labwork.*

B.03 Employ appropriate skills for gathering and retaining information. *Met by research and preparation prior to group discussion.*

B.04 Interpret written, graphic, and oral instructions. *Met by completion of labwork.*

B.06 Use language appropriate to the situation. *Met by group discussion and in explaining completed labwork.*

B.07 Participate in meetings in a positive and constructive manner. *Met by group discussion.*

B.08 Use job-related terminology. *Met by group discussion and in explaining completed labwork.*

B.10 Document work projects, procedures, tests, and equipment failures. *Met by project construction and/or troubleshooting assessments.*

C Basic and Practical Skills – Solving Problems and Critical Thinking

C.01 Identify the problem. *Met by research and preparation prior to group discussion.*

C.03 Identify available solutions and their impact including evaluating credibility of information, and locating information. *Met by research and preparation prior to group discussion.*

C.07 Organize personal workloads. *Met by daily labwork, preparatory research, and project management.*

C.08 Participate in brainstorming sessions to generate new ideas and solve problems. *Met by group discussion.*

D Basic and Practical Skills – Reading

D.01 Read and apply various sources of technical information (e.g. manufacturer literature, codes, and regulations). *Met by research and preparation prior to group discussion.*

E Basic and Practical Skills – Proficiency in Mathematics

E.01 Determine if a solution is reasonable.

E.02 Demonstrate ability to use a simple electronic calculator.

E.05 Solve problems and [sic] make applications involving integers, fractions, decimals, percentages, and ratios using order of operations.

E.06 Translate written and/or verbal statements into mathematical expressions.

E.12 Interpret and use tables, charts, maps, and/or graphs.

E.13 Identify patterns, note trends, and/or draw conclusions from tables, charts, maps, and/or graphs.

E.15 Simplify and solve algebraic expressions and formulas.

E.16 Select and use formulas appropriately.

E.17 Understand and use scientific notation.

E.18 Use properties of exponents and logarithms.

Common areas of confusion for students

Difficult concept: *Rates of change.*

When learning the relationships between voltage and current for inductors and capacitors, one must think in terms of how fast a variable is changing. The amount of voltage induced across an inductor is proportional to how *quickly* the current through it changes, not how strong the current is. Likewise, the amount of current "through" a capacitor is proportional to how *quickly* the voltage across it changes. This is the first hurdle in calculus: to comprehend what a rate of change is, and it is not obvious.

Common mistake: *Series and parallel relationships for capacitors.*

How inductors add and diminish in series and parallel (respectively) is easy to grasp because it resembles the relationships for resistors. Capacitors are "backwards" to both resistors and inductors, though, which causes confusion.

The best way I know how to overcome this confusion is to relate the series or parallel connection of capacitors to changes in physical dimension for a theoretical capacitor, and ask what change in capacitance such a change in dimension will yield. Connecting capacitors in series may be modeled by increasing the distance between plates of a theoretical capacitor, decreasing capacitance. Connecting capacitors in parallel is analogous to increasing the plate area of a theoretical capacitor, increasing capacitance.

Difficult concept: *The time-constant equation.*

Many students find the time-constant equation difficult because it involves exponents, particularly exponents of Euler's constant e. This exponent is often expressed as a negative quantity, making it even more difficult to understand. The single most popular mathematical mistake I see students make with this equation is failing to properly follow algebraic order of operations. Some students try to overcome this weakness by using calculators which allow parenthetical entries, nesting parentheses in such a way that the calculator performs the proper order of operations. However, if you don't understand order of operations yourself, you will not know where to properly place the parentheses. If you have trouble with algebraic order of operations, there is no solution but to invest the necessary time and learn it!

Beyond mathematical errors, though, the most common mistake I see students make with the time constant equation is mis-application. One version of this equation expresses increasing quantities, while another version expresses decreasing quantities. You must already know what the variables are going to do in your time-constant circuit before you know which equation to use! You must also be able to recognize one version of this equation from the other: not by memory, lest you should forget; but by noting what the result of the equation does as time (t) increases. Here again there will be trouble if you are not adept applying algebraic order of operations.

Question 1

As an electric current is passed through a coil of wire, it creates a magnetic field. If the magnitude of this current changes over time, so will the strength of the magnetic field.

We also know that a magnetic field flux that changes over time will induce a voltage along the length of a wire coil. Explain how the complementary principles of electromagnetism and electromagnetic induction manifest themselves simultaneously in the same wire coil to produce *self-induction*.

Also, explain how Lenz's Law relates to the polarity of the coil's self-induced voltage.

file 00263

Question 2

Inductance is a very important property in many types of electric circuits. Define what "inductance" is, and what causes it.

file 01136

Question 3

$\boxed{\int f(x)\, dx \quad \text{Calculus alert!}}$

Ohm's Law tells us that the amount of voltage dropped by a fixed resistance may be calculated as such:

$$E = IR$$

However, the relationship between voltage and current for a fixed inductance is quite different. The "Ohm's Law" formula for an inductor is as such:

$$e = L\frac{di}{dt}$$

What significance is there in the use of lower-case variables for current (i) and voltage (e)? Also, what does the expression $\frac{di}{dt}$ mean? Note: in case you think that the d's are variables, and should cancel out in this fraction, think again: this is no ordinary quotient! The d letters represent a calculus concept known as a *differential*, and a quotient of two d terms is called a *derivative*.
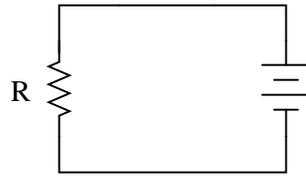
file 01381

Question 4

$\int f(x)\,dx$ *Calculus alert!*

In a simple resistor circuit, the current may be calculated by dividing applied voltage by resistance:


$$I = \frac{V}{R}$$

Although an analysis of this circuit probably seems trivial to you, I would like to encourage you to look at what is happening here from a fresh perspective. An important principle observed many times in the study of physics is that of *equilibrium*, where quantities naturally "seek" a state of balance. The balance sought by this simple circuit is equality of voltage: the voltage across the resistor must settle at the same value as the voltage output by the source:
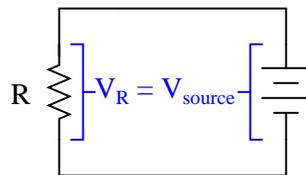
If the resistor is viewed as a source of voltage seeking equilibrium with the voltage source, then current *must* converge at whatever value necessary to generate the necessary balancing voltage across the resistor, according to Ohm's Law ($V = IR$). In other words, *the resistor's current achieves whatever magnitude it has to in order to generate a voltage drop equal to the voltage of the source.*

This may seem like a strange way of analyzing such a simple circuit, with the resistor "seeking" to generate a voltage drop equal to the source, and current "magically" assuming whatever value it must to achieve that voltage equilibrium, but it is helpful in understanding other types of circuit elements.

For example, here we have a source of DC voltage connected to a large coil of wire through a switch. Assume that the wire coil has negligible resistance ($0\,\Omega$):
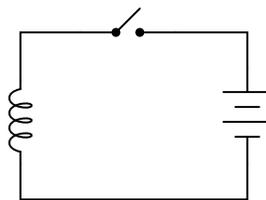
Like the resistor circuit, the coil will "seek" to achieve voltage equilibrium with the voltage source once the switch is closed. However, we know that the voltage induced in a coil is not directly proportional to current as it is with a resistor – instead, a coil's voltage drop is proportional to the *rate of change of magnetic flux over time* as described by Faraday's Law of electromagnetic induction:

$$v_{coil} = N\frac{d\phi}{dt}$$

5

Where,

v_{coil} = Instantaneous induced voltage, in volts
N = Number of turns in wire coil
$\frac{d\phi}{dt}$ = Instantaneous rate of change of magnetic flux, in webers per second

Assuming a linear relationship between coil current and magnetic flux (i.e. ϕ doubles when i doubles), describe this simple circuit's current over time after the switch closes.

file 01560

Question 5

The amount of inductance inherent in a wire coil may be calculated by the following equation:

$$L = \frac{N^2 A \mu}{l}$$

Where,

L = Inductance in Henrys
N = Number of wire "turns" wrapped around the core
μ = Permeability of core material (absolute, not relative)
A = Core area, in square meters
l = Length of core, in meters

Calculate how many turns of wire must be wrapped around a hollow, non-magnetic (air) core 2 cm in diameter and 10 cm in length in order to create an inductance of 22 mH. You may use the permeability of free space (μ_0) for the μ value of the air core.

Next, calculate the required number of turns to produce the same inductance with a solid iron core of the same dimensions, assuming that the iron has a relative permeability (μ_r) of 4000.

Finally, knowing that the formula for the area of a circle is πr^2, re-write the inductance equation so as to accept a value for inductor radius rather than inductor area. In other words, *substitute* radius (r) for area (A) in this equation in such a way that it still provides an accurate figure for inductance.

file 00211

Question 6

Draw the direction of current in this circuit, and also identify the polarity of the voltage across the battery and across the resistor. Then, compare the battery's polarity with the direction of current through it, and the resistor's polarity with the direction of current through it.
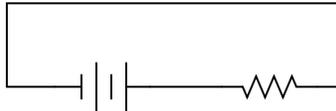
What do you notice about the relationship between voltage polarity and current direction for these two different types of components? Identify the fundamental distinction between these two components that causes them to behave differently.

file 01555

Question 7

Suppose an inductor is connected directly to an adjustable-current source, and the current of that source is steadily *increased* over time. We know that an increasing current through an inductor will produce a magnetic field of increasing strength. Does this increase in magnetic field constitute an *accumulation* of energy in the inductor, or a *release* of energy from the inductor? In this scenario, does the inductor act as a *load* or as a *source* of electrical energy?

Current *increasing*

Now, suppose the adjustable current source is steadily *decreased* over time. We know this will result in a magnetic field of decreasing strength in the inductor. Does this decrease in magnetic field constitute an *accumulation* of energy in the inductor, or a *release* of energy from the inductor? In this scenario, does the inductor act as a *load* or as a *source* of electrical energy?

Current *decreasing*
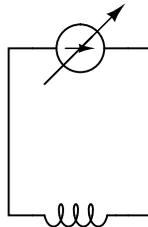
For each of these scenarios, label the inductor's voltage drop polarity.
file 00209

Question 8

Complete this statement by substituting the correct electrical variables (voltage, current, resistance, inductance):

Inductors oppose changes in (*fill-in-the-blank*), reacting to such changes by producing a (*fill-in-the-blank*).

file 00208

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Question 9

Many precision resistors utilize a *wire-wound* construction, where the resistance is determined by the type and length of wire wrapped around a spool. This form of construction allows for high precision of resistance, with low temperature sensitivity if certain metal alloys are used for the wire.

Unfortunately, though, wrapping wire around a spool forms a coil, which will naturally possess a significant amount of inductance. This is generally undesirable, as we would like to have resistors possessing *only resistance*, with no "parasitic" properties.

There is, however, a special way in which a wire coil may be wound so as to have almost no inductance. This method is called *bifilar* winding, and it is common in wire-wound resistor construction. Describe how bifilar winding works, and why it eliminates parasitic inductance.

file 00375

Question 10

Components soldered into printed circuit boards often possess "stray" inductance, also known as *parasitic* inductance. Observe this resistor, soldered in to a circuit board:
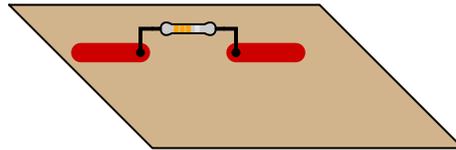
Where does the parasitic inductance come from? What is it about a resistor, mounted to a circuit board, that creates a (very) small amount of inductance? How is it possible to minimize this inductance, in case it is detrimental to the circuit's operation?

file 00374

Question 11

Many years ago, I decided to experiment with electromagnetism by making an electromagnet out of a spool of wire. I placed a steel bolt through the center of the spool so as to have a core of high permeability, and passed current from a battery through the wire to make a magnetic field. Not having any "jumper" wires, I held the wire ends of the spool in contact with the 9-volt battery terminals, one in each hand.

The electromagnet worked just fine, and I was able to move some steel paperclips with the magnetic field generated by it. However, when I broke the circuit by releasing one of the wire ends from the battery terminal it was touching, I received a small electric shock! Shown here is a schematic diagram of me, in the circuit:

Break in circuit
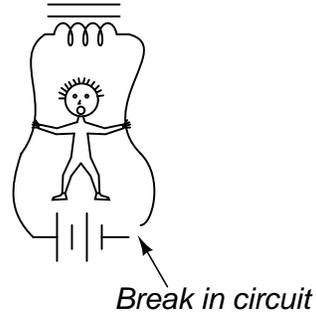
At the time, I didn't understand how inductance worked. I only understood how to make magnetism with electricity, but I didn't realize a coil of wire could generate (high voltage!) electricity from its own magnetic field. I did know, however, that the 9 volts output by the battery was much too weak to shock me (yes, I touched the battery terminals directly to verify this fact), so *something* in the circuit must have generated a voltage greater than 9 volts.

If you had been there to explain what just happened to me, what would you say?

file 00376

Question 12

$\boxed{\int f(x)\, dx \quad \text{Calculus alert!}}$

A 5 H inductor is subjected to an electric current that changes at a rate of 4.5 amps per second. How much voltage will be dropped by the inductor?
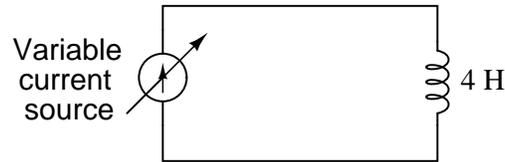
file 00372

Question 13

$\int f(x)\,dx$ *Calculus alert!*

Suppose an inductor is connected to a variable current source, where the current is steadily increased at a rate of 1.5 amps per second. How much voltage will the 4 Henry inductor drop, and what will be the polarity of that drop? Remember, the direction of the arrow in a current source symbol points in the direction of *conventional flow*, not electron flow!

In real life, an inductor will not drop the exact same amount of voltage that you will calculate here. Determine if the real voltage drop across such an inductor would be greater or less than predicted, and explain why.

file 00353

Question 14

Two 5 H inductors connected in series are subjected to an electric current that changes at a rate of 4.5 amps per second. How much voltage will be dropped across the series combination?

Now suppose that two 5 H inductors connected in parallel are subjected to the same total applied current (changing at a rate of 4.5 amps per second). How much voltage will be dropped by these inductors? Hint: the total current is divided evenly between the two inductors.
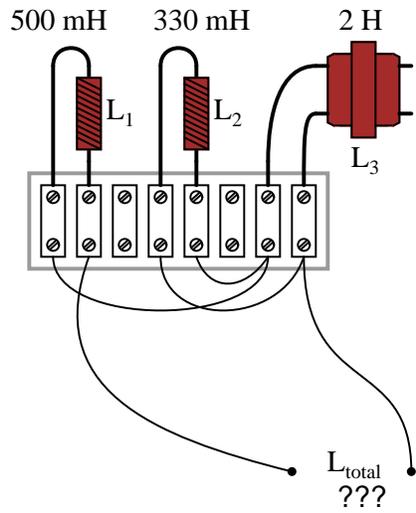
file 00373

Question 15

Suppose two 50 mH inductors are connected in parallel with each other. What will their combined inductance be, in Henrys? Explain your answer.

file 00378

Question 16

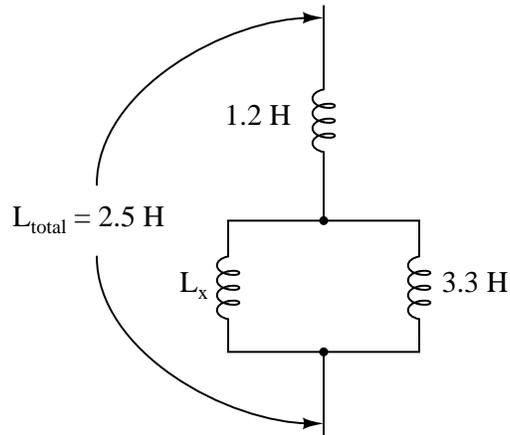Calculate the total inductance in this collection of inductors, as measured between the two wires:

Also, write a formula that calculates total inductance given the values of inductors L_1, L_2, and L_3.

file 00379

Question 17

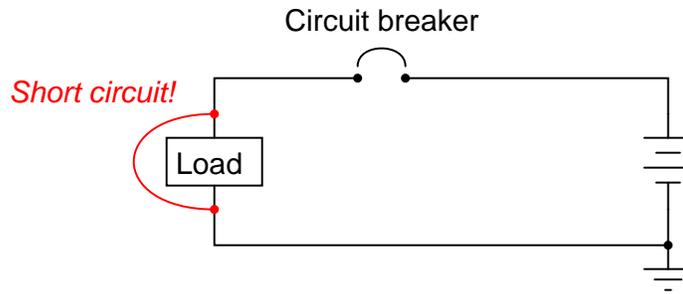How large must inductor L_x be in order to provide a total inductance of 2.5 H in this network of inductors?
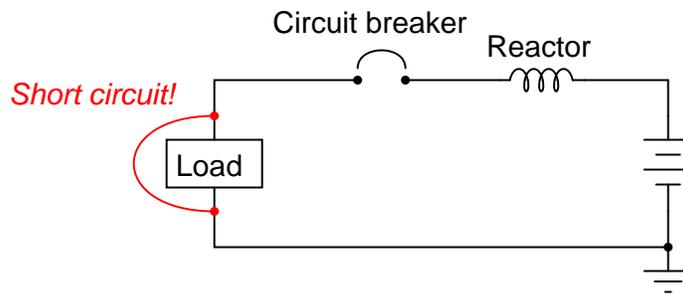

file 00381

Question 18

Suppose a short-circuit were to develop in this electric power system:

Circuit breaker

Short circuit!

Load

The purpose of the circuit breaker, of course, is to open the circuit automatically, to prevent damage to the power conductors. In a large electric power system, the magnitudes of such short-circuit currents can be enormous.

Large inductors, commonly called *reactors*, are often installed in series with power conductors in high-voltage power systems in order to "soften" the onset of short-circuit currents:

Circuit breaker

Reactor

Short circuit!

Load

Explain how the addition of a "reactor" helps to minimize the magnitude of the short-circuit current the breaker has to interrupt.

file 00380

Question 19

Small inductors often look like resistors, even to the extent that their values are designated by colored bands. Determine the values of the following inductors (express the tolerance as a percentage), based on their color codes:

- Red, Grn, Brn, Gld
- Wht, Org, Red, Sil
- Grn, Gry, Blk
- Vio, Blu, Org, Gld

file 00440

Question 20

When checked with an ohmmeter, how should a properly functioning inductor respond?

file 00439

Question 21

Suppose two wires, separated by an air gap, are connected to opposite terminals on a voltage source (such as a battery). An electric field will develop in the space between the two wires: an invisible web of interaction, similar in some ways to a magnetic field. In this diagram, draw the invisible "lines of flux" for this electric field, showing their physical range:
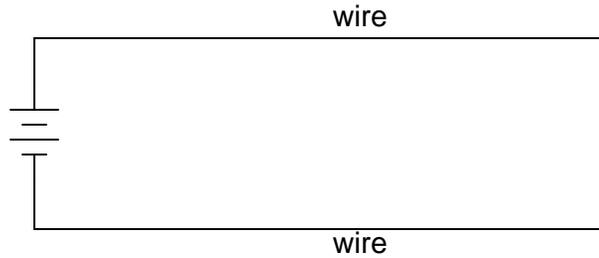
file 00189

Question 22

Electric fields may be described as "invisible webs" of interaction across space between electrically charged objects. Most people should be familiar with *magnetic* fields from playing with magnets as children: the forces of attraction or repulsion that act across open space between two or more magnetic objects. But electric fields are not the same as magnetic fields. The two different kinds of fields exert forces on entirely different objects.

Give an example of where an electric field manifests a tangible, physical force, like the magnetic fields we are all familiar with. Under what conditions are electric fields strong enough for human beings to detect without instruments?

file 00204

Question 23

Capacitance is a very important property in many types of electric circuits. Define what "capacitance" is, and what causes it.

file 01135

Question 24

The amount of capacitance between two conductors may be calculated by the following equation:

$$C = \frac{\epsilon A}{d}$$

Where,

C = Capacitance in Farads
ϵ = Permittivity of dielectric (absolute)
A = Conductor area, in square meters
d = Separation distance, in meters

How far away from each other would two metal plates, 2 square meters in area each, have to be in order to create a capacitance of 1 μF? Assume that the plates are separated by air.

file 00200

Question 25

Capacitance exists between any two conductors separated by an insulating medium. Given this fact, it makes sense that a length of two-conductor electrical cable will have capacitance distributed naturally along its length:

Two-conductor cable

Distributed capacitance
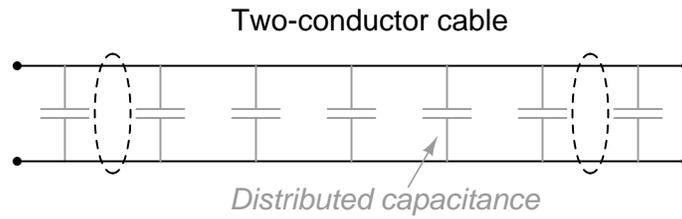
There should be a way to prove the existence of such "stray" capacitance in a substantial length of two-conductor cable. Devise an experiment to do this.

file 00203

Question 26

Electric fields, like all fields, have two fundamental measures: field *force* and field *flux*. In a capacitor, which of these field quantities is directly related to voltage between the plates, and which is directly related to the amount of charge (in coulombs) stored?

Based on this relationship, which electric field quantity changes when a sheet of glass is inserted between these two metal plates, connected to a source of constant voltage?
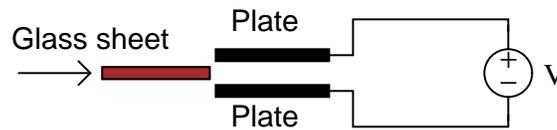
file 00191

Question 27

Write an equation describing the precise mathematical relationship between electric charge (Q), capacitance (C), and voltage (V).

file 01454

Question 28

Suppose a capacitor is connected directly to an adjustable-voltage source, and the voltage of that source is steadily *increased* over time. We know that an increasing voltage across a capacitor will produce an electric field of increasing strength. Does this increase in electric field constitute an *accumulation* of energy in the capacitor, or a *release* of energy from the capacitor? In this scenario, does the capacitor act as a *load* or as a *source* of electrical energy?

Voltage *increasing*
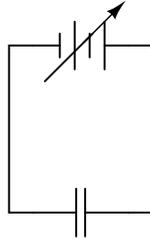
Now, suppose the adjustable voltage source is steadily *decreased* over time. We know this will result in an electric field of decreasing strength in the capacitor. Does this decrease in electric field constitute an *accumulation* of energy in the capacitor, or a *release* of energy from the capacitor? In this scenario, does the capacitor act as a *load* or as a *source* of electrical energy?

Voltage *decreasing*
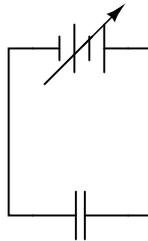
For each of these scenarios, label the direction of current in the circuit.

<u>file 00190</u>

15

Question 29

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Ohm's Law tells us that the amount of current through a fixed resistance may be calculated as such:

$$I = \frac{E}{R}$$

We could also express this relationship in terms of *conductance* rather than *resistance*, knowing that $G = \frac{1}{R}$:

$$I = EG$$

However, the relationship between current and voltage for a fixed capacitance is quite different. The "Ohm's Law" formula for a capacitor is as such:

$$i = C\frac{de}{dt}$$

What significance is there in the use of lower-case variables for current (i) and voltage (e)? Also, what does the expression $\frac{de}{dt}$ mean? Note: in case you think that the d's are variables, and should cancel out in this fraction, think again: this is no ordinary quotient! The d letters represent a calculus concept known as a *differential*, and a quotient of two d terms is called a *derivative*.

file 01380

Question 30

Complete this statement by substituting the correct electrical variables (voltage, current, resistance, capacitance):

Capacitors oppose changes in (*fill-in-the-blank*), reacting to such changes by producing a (*fill-in-the-blank*).

file 00207

Question 31

Identify some of the different types of capacitors, and their characteristics.

file 00197

Question 32

How are *electrolytic* capacitors constructed, and what is particularly noteworthy about their use?

file 00196

Question 33

A capacitor has a label on it saying, "100 WVDC". What does this label mean? What is the consequence of exceeding this rating?

file 00199

Question 34

Capacitors may pose an electric shock hazard, even in unpowered circuits. Explain why.

file 00202

Question 35

A 470 μF capacitor is subjected to an applied voltage that changes at a rate of 200 volts per second. How much current will there be "through" this capacitor?

Explain why I placed quotation marks around the word "through" in the previous sentence. Why can't this word be used in its fullest sense when describing electric current in a capacitor circuit?

file 00194

Question 36

Two 470 μF capacitors connected in series are subjected to a total applied voltage that changes at a rate of 200 volts per second. How much current will there be "through" these capacitors? Hint: the total voltage is divided evenly between the two capacitors.

Now suppose that two 470 μF capacitors connected in parallel are subjected to the same total applied voltage (changing at a rate of 200 volts per second). How much total current will there be "through" these capacitors?

file 00201

Question 37

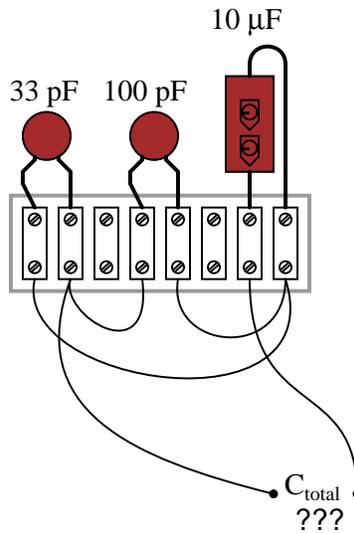Suppose two 33 μF capacitors are connected in series with each other. What will their combined capacitance be, in Farads? Explain your answer.

file 00198

Question 38

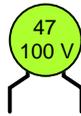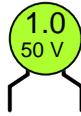Calculate the total capacitance in this collection of capacitors, as measured between the two wires:

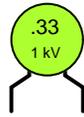file 00205

Question 39

Identify the following capacitor values and styles:

.33
1 kV

1.0
50 V

4700 MFD
36 V

103
25 V

47
100 V

4.7µ
16 V

Question 40
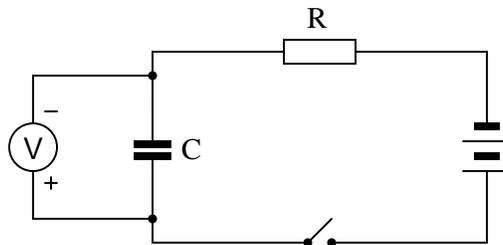
When checked with an ohmmeter, how should a properly functioning capacitor respond?

Question 41

Suppose this circuit were constructed, using a fully discharged capacitor (0 volts) and a voltmeter connected in parallel with it to measure its voltage:
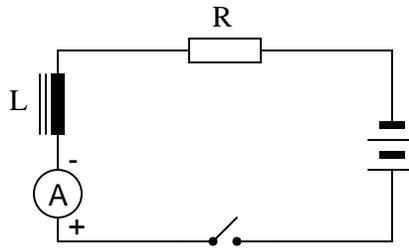
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+

C

What will happen to the capacitor's voltage after the switch is closed? Be as precise as you can with your answer, and explain why it does what it does.

Question 42

Suppose this circuit were constructed, using an inductor and an ammeter connected in series with it to measure its current:

R

L

A

What will happen to the inductor's current after the switch is closed? Be as precise as you can with your answer, and explain why it does what it does.

Question 43

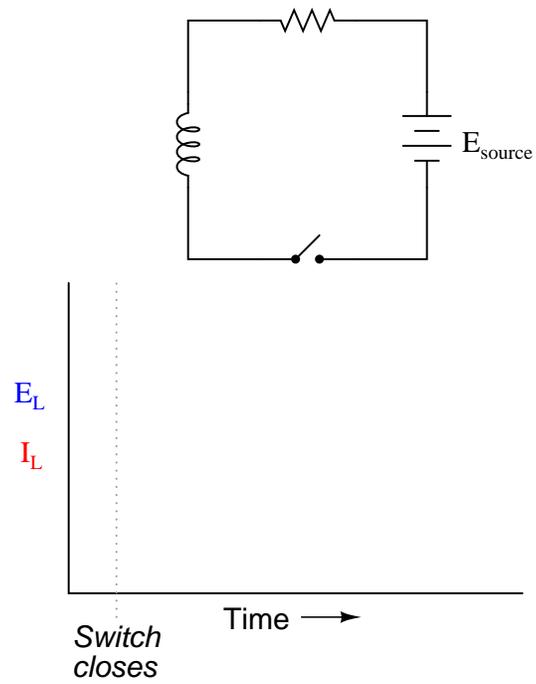Graph both the capacitor voltage (E_C) and the capacitor current (I_C) over time as the switch is closed in this circuit. Assume the capacitor begins in a complete uncharged state (0 volts):

E_{source}

E_C

I_C

Time

Switch closes
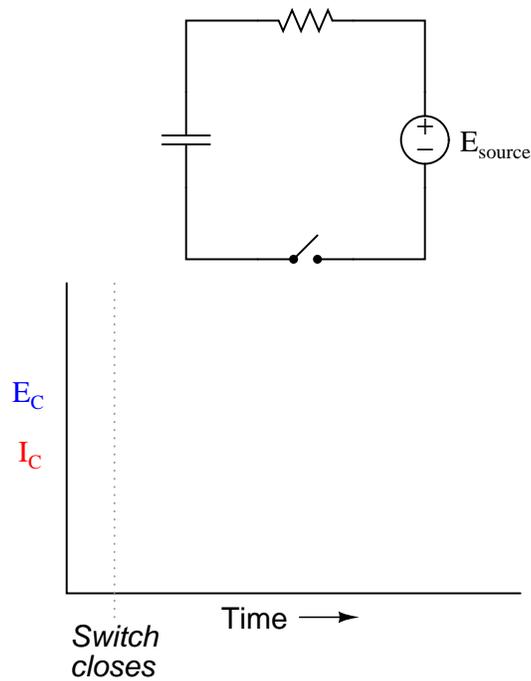
Graph both the inductor voltage (E_L) and the inductor current (I_L) over time as the switch is closed in this circuit:

Question 45

There are many, many processes in the natural sciences where variables either grow (become larger) or decay (become smaller) over time. Often, the rate at which these processes grow or decay is directly proportional to the growing or decaying quantity. Radioactive decay is one example, where the rate of decay of a radioactive substance is proportional to the quantity of that substance remaining. The growth of small bacterial cultures is another example, where the growth rate is proportional to the number of live cells.

In processes where the rate of decay is proportional to the decaying quantity (such as in radioactive decay), a convenient way of expressing this decay rate is in terms of time: how long it takes for a certain percentage of decay to occur. With radioactive substances, the decay rate is commonly expressed as *half-life*: the time it takes for exactly half of the substance to decay:
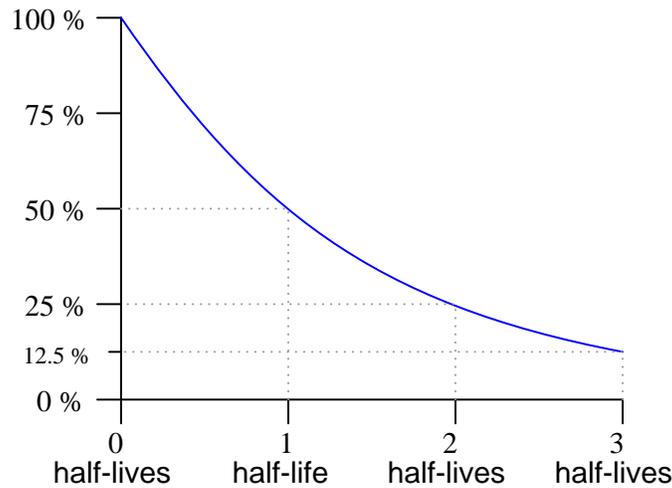
In RC and LR circuits, decay time is expressed in a slightly different way. Instead of measuring decay rate in units of *half-lives*, we measure decay rate in units of *time constants*, symbolized by the Greek letter "tau" (τ).

What is the percentage of decay that takes place in an RC or LR circuit after one "time constant's" worth of time, and how is this percentage value calculated? Note: it is not 50%, as it is for "half life," but rather a different percentage figure.

Graph the curve of this decay, plotting points at 0, 1, 2, and 3 time constants:
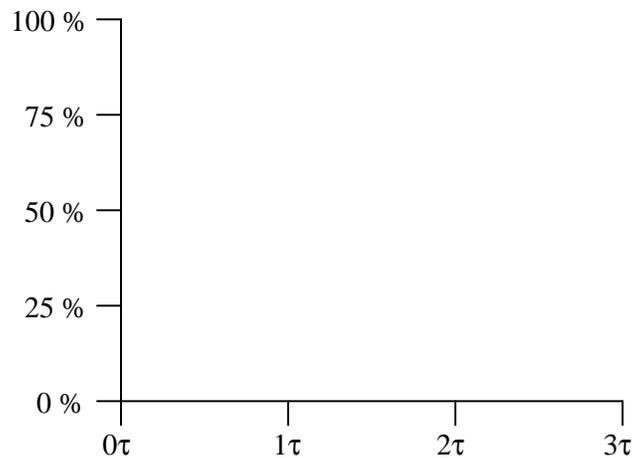

<u>file 01797</u>

Question 46

Capacitors tend to oppose change in voltage, and so they may be considered "temporary voltage sources." That is, they tend to hold a constant voltage over time, but they cannot do so indefinitely. Any motion of charge (current) will change the voltage of a capacitor.

Likewise, inductors may be considered "temporary current sources" because while they tend to hold current constant over time, they cannot do so indefinitely. Any application of voltage across a (perfect) inductor will alter the amount of current going through it.

Given the above characterizations, determine what resistance levels will result in the *fastest* discharge of energy for both the capacitive circuit and the inductive circuit. Considering each of the reactive components (C and L, respectively) as "temporary" power sources whose store of energy will drain over time, determine what value of R in each circuit will result in the quickest depletion of energy by making each source "work hardest:"
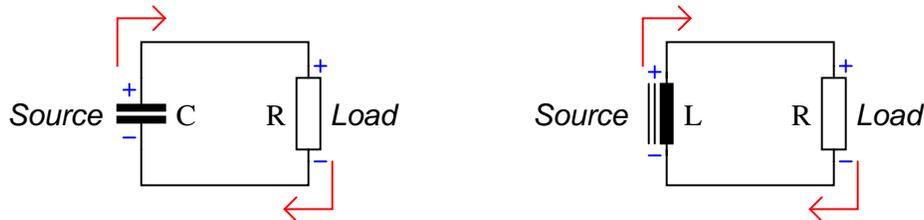
Current arrows pointing in the direction of "conventional" flow, not electron flow!

Based on your answer to this question, explain how circuit resistance (R) affects the time constants (τ) of RC and of LR circuits.

file 03526

Question 47

What value of resistor would need to be connected in series with a 33 μF capacitor in order to provide a *time constant* (τ) of 10 seconds? Express your answer in the form of a five-band precision resistor color code (with a tolerance of +/- 0.1%).
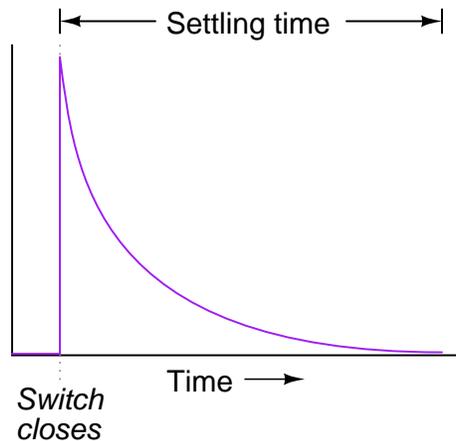
file 00436

Question 48

What value of resistor would need to be connected in series with a 75 mH inductor in order to provide a *time constant* (τ) of 20 microseconds? Express your answer in the form of a five-band precision resistor color code (with a tolerance of +/- 0.25%).

file 01799

Question 49

Generally speaking, how many "time constants" worth of time does it take for the voltage and current to "settle" into their final values in an RC or LR circuit, from the time the switch is closed?

Question 50

An electronic service technician prepares to work on a high-voltage power supply circuit containing one large capacitor. On the side of this capacitor are the following specifications:

$$3000 \text{ WVDC} \qquad 0.75 \mu\text{F}$$

Obviously this device poses a certain amount of danger, even with the AC line power secured (lockout/tag-out). Discharging this capacitor by directly shorting its terminals with a screwdriver or some other piece of metal might be dangerous due to the quantity of the stored charge. What needs to be done is to discharge this capacitor at a modest rate.

The technician realizes that she can discharge the capacitor at any rate desired by connecting a resistor in parallel with it (holding the resistor with electrically-insulated pliers, of course, to avoid having to touch either terminal). What size resistor should she use, if she wants to discharge the capacitor to less than 1% charge in 15 seconds? State your answer using the standard 4-band resistor color code (tolerance = +/- 10%).

23

Question 51

The circuit shown here is called a *relaxation oscillator*. It works on the principles of capacitor charging over time (an RC circuit), and of the *hysteresis* of a gas-discharge bulb: the fact that the voltage required to initiate conduction through the bulb is significantly greater than the voltage below which the bulb ceases to conduct current.

In this circuit, the neon bulb ionizes at a voltage of 70 volts, and stops conducting when the voltage falls below 30 volts:
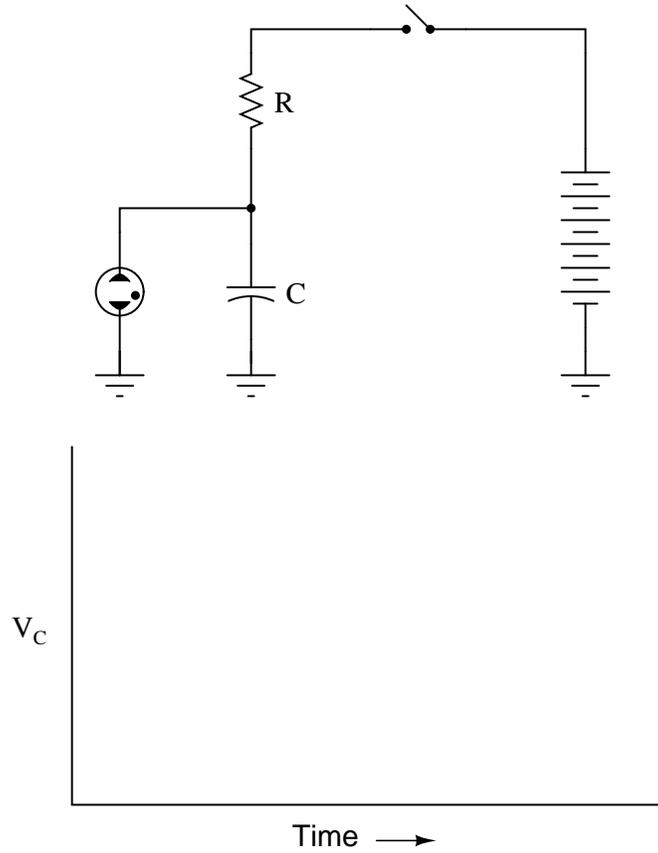
Graph the capacitor's voltage over time as this circuit is energized by the DC source. Note on your graph at what times the neon bulb is lit:

file 00430

24

Question 52

Replace the fixed-value resistor with a potentiometer to adjust the blinking rate of the neon lamp, in this *relaxation oscillator* circuit. Connect the potentiometer in such a way that clockwise rotation of the knob makes the lamp blink faster:
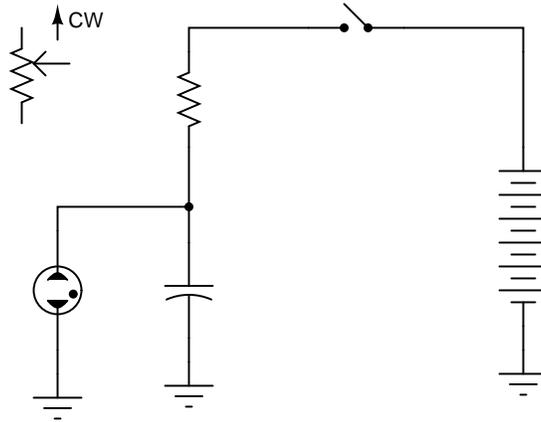
file 00431

Question 53

Design a circuit with a three-position switch, where one position charges a capacitor, one position holds the capacitor's voltage constant, and the last position discharges the capacitor. Provide independent, variable adjustments for charging and discharging time constants.

file 00443

Question 54

An *intervalometer* is a device that measures the interval of time between two events. Such devices are commonly used to measure the speed of projectiles, given a known distance between two sensors:

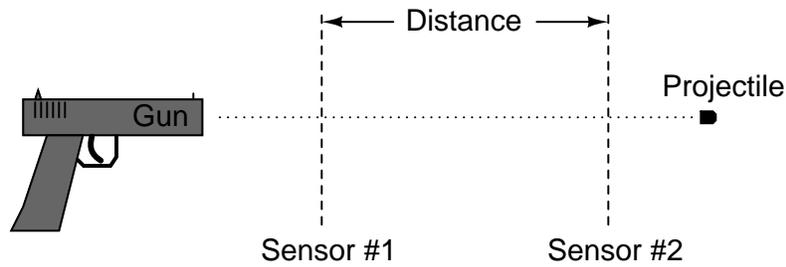A crude intervalometer may be constructed using two thin wires as sensors, which are broken by the passage of the projectile. The two wires are connected in an RC circuit as such:
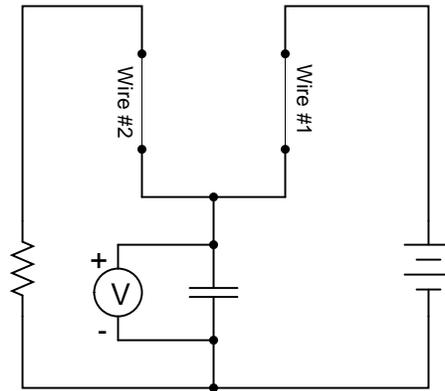
In order for this circuit to function properly as an intervalometer, which wire does the projectile need to break first? Explain why. Also, the voltmeter used in this instrument must be one with as high an input resistance as possible for best accuracy. Explain why this is necessary as well.

Which will produce a greater voltage indication after the test, a fast projectile or a slow projectile? Explain your answer.

file 00444

Question 55

Re-write this mathematical expression so that the exponent term $(-x)$ is no longer negative:

$$e^{-x}$$

Also, describe the calculator keystroke sequence you would have to go through to evaluate this expression given any particular value for x.

file 01803

At a party, you happen to notice a mathematician taking notes while looking over the food table where several pizzas are set. Walking up to her, you ask what she is doing. "I'm mathematically modeling the consumption of pizza," she tells you. Before you have the chance to ask another question, she sets her notepad down on the table and excuses herself to go use the bathroom.

Looking at the notepad, you see the following equation:

$$\text{Percentage} = \left(e^{-\frac{t}{6.1}}\right) \times 100\%$$

Where,

$t = $ Time in minutes since arrival of pizza.

The problem is, you don't know whether the equation she wrote describes the percentage of pizza eaten or the percentage of pizza remaining on the table. Explain how you would determine which percentage this equation describes. How, exactly, can you tell if this equation describes the amount of pizza already eaten or the amount of pizza that remains to be eaten?

file 03549

The following two expressions are frequently used to calculate values of changing variables (voltage and current) in RC and LR timing circuits:

$$e^{-\frac{t}{\tau}} \qquad \text{or} \qquad 1 - e^{-\frac{t}{\tau}}$$

One of these expressions describes the percentage that a changing value in an RC or LR circuit *has gone* from the starting time. The other expression describes how far that same variable *has left to go* before it reaches its ultimate value (at $t = \infty$).

The question is, which expression represents which quantity? This is often a point of confusion, because students have a tendency to try to correlate these expressions to the quantities by rote memorization. Does the expression $e^{-\frac{t}{\tau}}$ represent the amount a variable has changed, or how far it has left to go until it stabilizes? What about the other expression $1 - e^{-\frac{t}{\tau}}$? More importantly, *how can we figure this out so we don't have to rely on memory?*
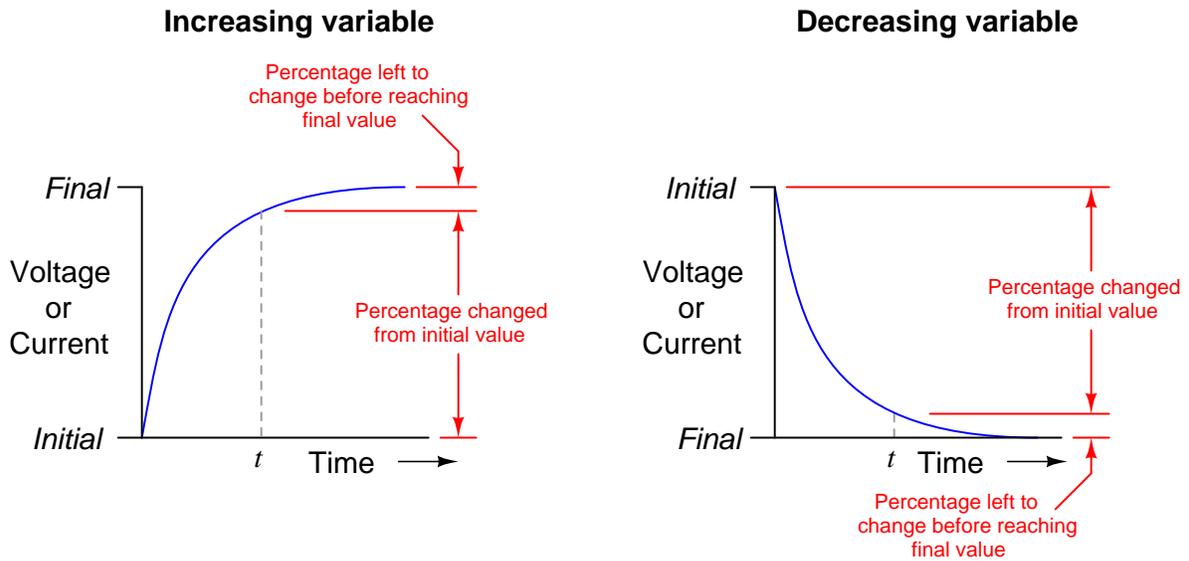
file 03117

Question 58

Graph both the capacitor voltage (V_C) and the capacitor current (I_C) over time as the switch is closed in this circuit. Assume the capacitor begins in a complete uncharged state (0 volts):
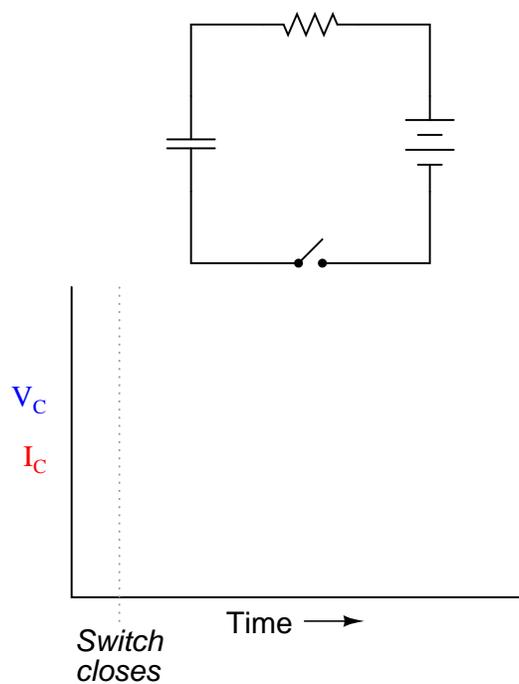
Then, select and modify the appropriate form of equation (from below) to describe each plot:

$$e^{-\frac{t}{\tau}} \qquad\qquad 1 - e^{-\frac{t}{\tau}}$$

file 03550

Question 59

The following circuit allows a capacitor to be rapidly charged and slowly discharged:
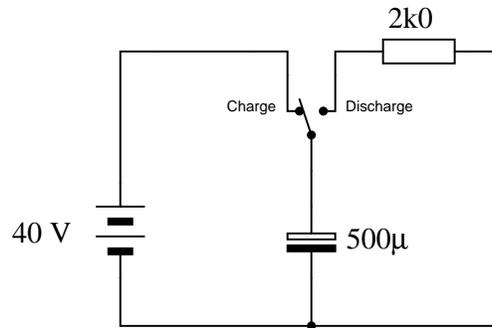
Suppose that the switch was left in the "charge" position for some substantial amount of time. Then, someone moves the switch to the "discharge" position to let the capacitor discharge. Calculate the amount of capacitor voltage and capacitor current at exactly 3 seconds after moving the switch to the "discharge" position.

$V_C = $ _____ @ $t = 3$ seconds

$I_C = $ _____ @ $t = 3$ seconds

Also, show the direction of discharge current in this circuit.
file 03552

Question 60

The following circuit allows a capacitor to be rapidly discharged and slowly charged:
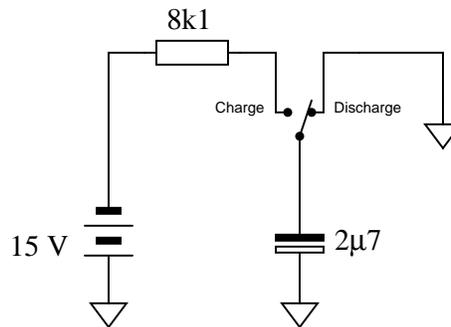
Suppose that the switch was left in the "discharge" position for some substantial amount of time. Then, someone moves the switch to the "charge" position to let the capacitor charge. Calculate the amount of capacitor voltage and capacitor current at exactly 45 milliseconds after moving the switch to the "charge" position.

$V_C = $ _____ @ $t = 45$ ms

$I_C = $ _____ @ $t = 45$ ms
file 03557

Qualitatively determine the voltages across all components as well as the current through all components in this simple RC circuit at three different times: (1) just before the switch closes, (2) at the instant the switch contacts touch, and (3) after the switch has been closed for a long time. Assume that the capacitor begins in a completely discharged state:
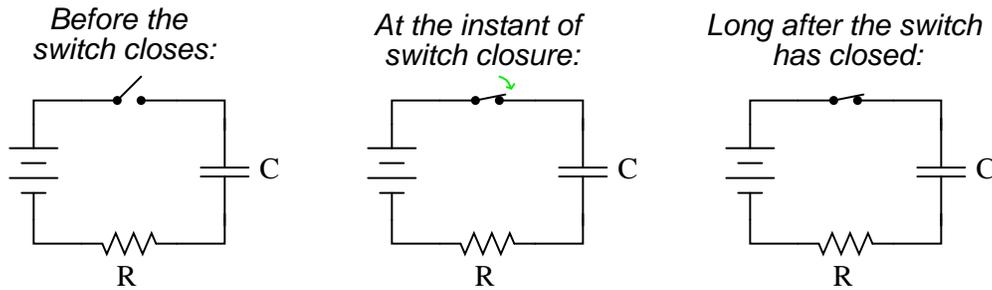
Express your answers qualitatively: "maximum," "minimum," or perhaps "zero" if you know that to be the case.

Before the switch closes:

$V_C =$

$V_R =$

$V_{switch} =$

$I =$

At the instant of switch closure:

$V_C =$

$V_R =$

$V_{switch} =$

$I =$

Long after the switch has closed:

$V_C =$

$V_R =$

$V_{switch} =$

$I =$

Hint: a graph may be a helpful tool for determining the answers!

file 01811

Question 62

Qualitatively determine the voltages across all components as well as the current through all components in this simple LR circuit at three different times: (1) just before the switch closes, (2) at the instant the switch contacts touch, and (3) after the switch has been closed for a long time.

Before the
switch closes:

At the instant of
switch closure:
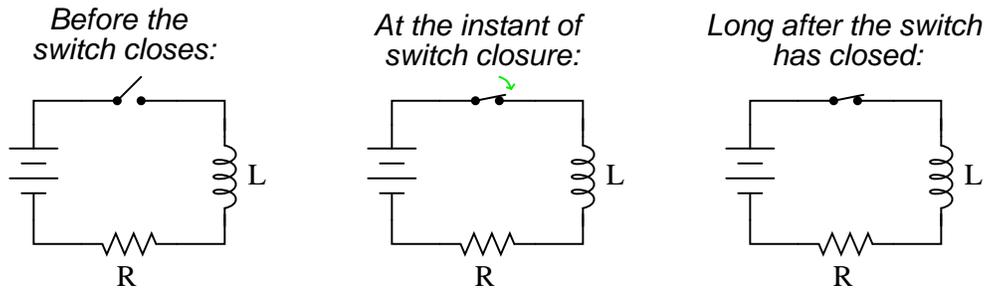
Long after the switch
has closed:

Express your answers qualitatively: "maximum," "minimum," or perhaps "zero" if you know that to be the case.

Before the switch closes:
$V_L =$
$V_R =$
$V_{switch} =$
$I =$

At the instant of switch closure:
$V_L =$
$V_R =$
$V_{switch} =$
$I =$

Long after the switch has closed:
$V_L =$
$V_R =$
$V_{switch} =$
$I =$

Hint: a graph may be a helpful tool for determining the answers!
file 01812

Question 63

Calculate the final value of current through the inductor with the switch in the left-hand position (assuming that many time constants' worth of time have passed):
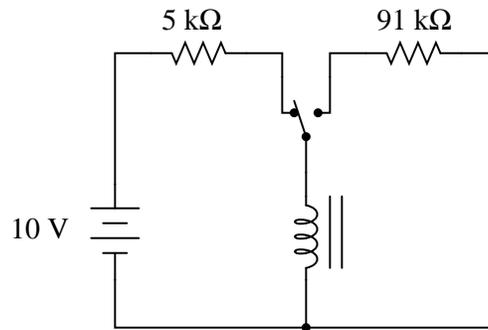
Now, assume that the switch is *instantly* moved to the right-hand position. How much voltage will the inductor initially drop?
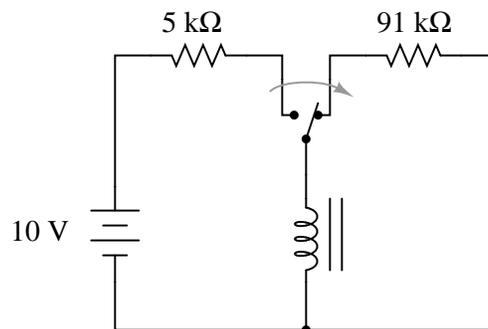
Explain why this voltage is so very different from the supply voltage. What practical uses might a circuit such as this have?

file 01808

Question 64

An unfortunate tendency that many new students have is to immediately plug numbers into equations when faced with a time-constant circuit problem, before carefully considering the circuit. Explain why the following steps are very wise to follow *before* performing any mathematical calculations:

- Step 1: Identify and list all the known ("given") quantities.
- Step 2: Draw a schematic of the circuit, if none is given to you.
- Step 3: Label components in the schematic with all known quantities.
- Step 4: Sketch a rough plot of how you expect the variable(s) in the circuit to change over time.
- Step 5: Label starting and final values for these graphed variables, wherever possible.

file 03553

Question 65

Suppose a capacitor is charged to a voltage of exactly 100 volts, then connected to a resistor so it discharges slowly. Calculate the amount of voltage remaining across the capacitor terminals at the following points in time:

- 1 time constant (τ) after connecting the resistor:
- 2 time constants (2τ) after connecting the resistor:
- 3 time constants (3τ) after connecting the resistor:
- 4 time constants (4τ) after connecting the resistor:
- 5 time constants (5τ) after connecting the resistor:

file 03551

Question 66

Calculate the voltage across a 470 μF capacitor after discharging through a 10 kΩ resistor for 9 seconds, if the capacitor's original voltage (at $t = 0$) was 24 volts.

Also, express this amount of time (9 seconds) in terms of how many *time constants* have elapsed.

file 00452

Question 67

Calculate the current through a 250 mH inductor after "charging" through a series-connected resistor with 100 Ω of resistance for 6 milliseconds, powered by a 12 volt battery. Assume that the inductor is perfect, with no internal resistance.

Also, express this amount of time (6 milliseconds) in terms of how many *time constants* have elapsed.

file 00453

Question 68

Determine the capacitor voltage at the specified times (time $t = 0$ milliseconds being the exact moment the switch contacts close). Assume the capacitor begins in a fully discharged state:
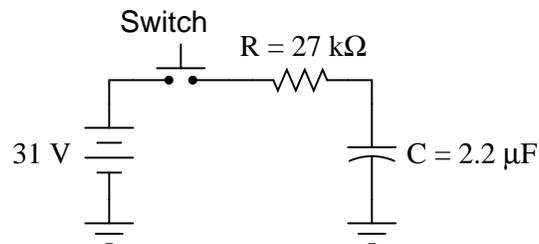
Time	V_C (volts)
0 ms	
30 ms	
60 ms	
90 ms	
120 ms	
150 ms	

file 03555

Question 69

Plot the capacitor voltage and the capacitor current over time after the switch closes in this circuit, for at least 4 time constants' worth of time:
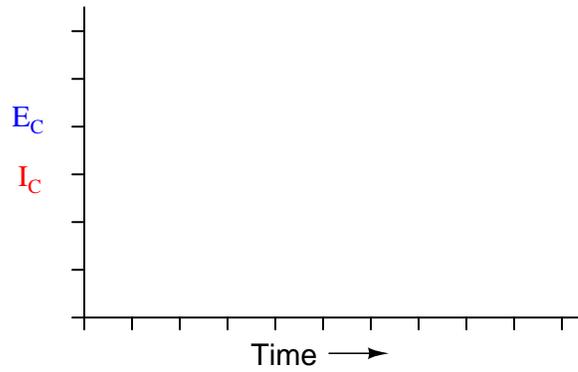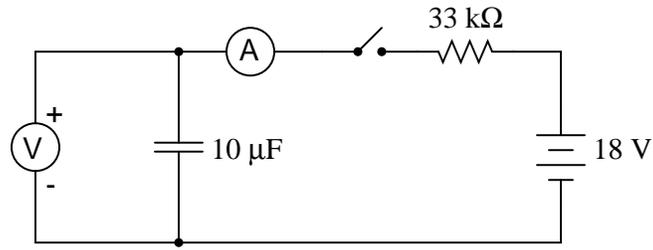
Be sure to label the axes of your graph!

file 00456

Question 70

Plot the inductor voltage and the inductor current over time after the switch closes in this circuit, for at least 4 time constants' worth of time:
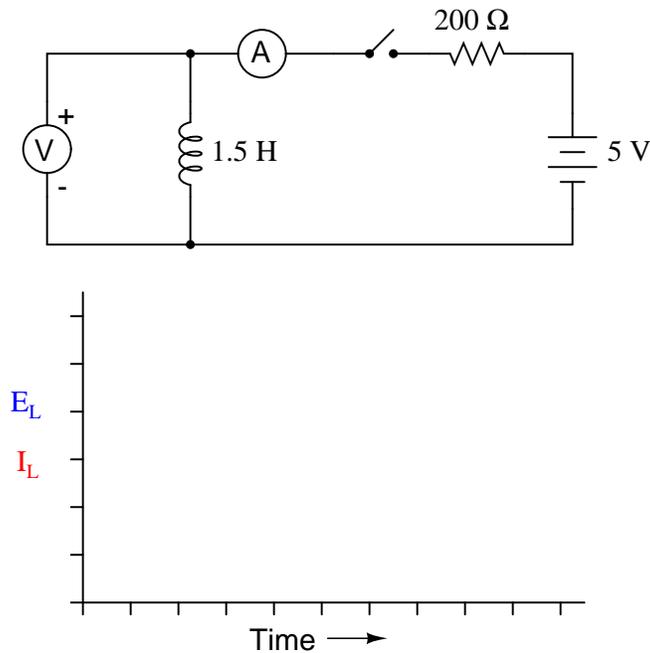
Be sure to label the axes of your graph!

<u>file 01806</u>

Question 71

The decay of a variable over time in an RC or LR circuit follows this mathematical expression:

$$e^{-\frac{t}{\tau}}$$

Where,

e = Euler's constant (≈ 2.718281828)

t = Time, in seconds

τ = Time constant of circuit, in seconds

For example, if we were to evaluate this expression and arrive at a value of 0.398, we would know the variable in question has decayed from 100% to 39.8% over the period of time specified.

However, calculating the amount of time it takes for a decaying variable to reach a specified percentage is more difficult. We would have to manipulate the equation to solve for t, which is part of an exponent.

Show how the following equation could be algebraically manipulated to solve for t, where x is the number between 0 and 1 (inclusive) representing the percentage of original value for the variable in question:

$$x = e^{-\frac{t}{\tau}}$$

Note: the "trick" here is how to isolate the exponent $\frac{-t}{\tau}$. You will have to use the natural logarithm function!

<u>file 02001</u>

Question 72

Calculate the amount of time it takes for a 33 μF capacitor to charge from 0 volts to 20 volts, if powered by a 24 volt battery through a 10 kΩ resistor.

file 01814

Question 73

Determine the amount of time needed after switch closure for the capacitor voltage (V_C) to reach the specified levels:
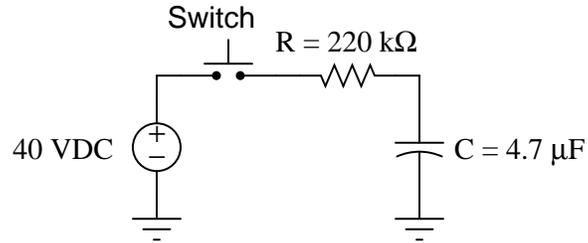
V_C	Time
0 volts	
10 volts	
20 volts	
30 volts	
40 volts	

Trace the direction of electron flow in the circuit, and also mark all voltage polarities.

file 02942

Question 74

A simple *time-delay relay circuit* may be built using a large capacitor connected in parallel with the relay coil, to temporarily supply the relay coil with power after the main power source is disconnected. In the following circuit, pressing the pushbutton switch sounds the horn, which remains on for a brief time after releasing the switch:
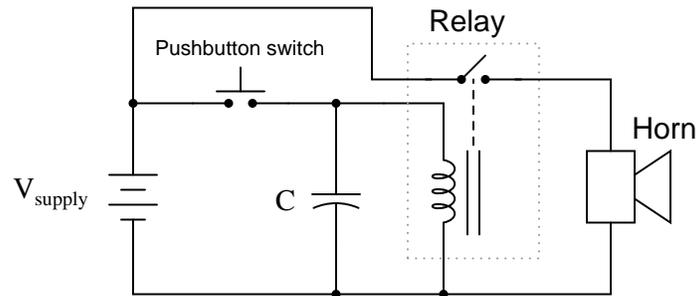
To calculate the amount of time the horn will remain on after the pushbutton switch is released, we must know a few things about the relay itself. Since the relay coil acts as a resistive load to the capacitor, we must know the coil's resistance in ohms. We must also know the voltage at which the relay "drops out" (i.e. the point at which there is too little voltage across the coil to maintain a strong enough magnetic field to hold the relay contacts closed).

Suppose the power supply voltage is 24 volts, the capacitor is 2200 μF, the relay coil resistance is 500 Ω, and the coil drop-out voltage is 6.5 volts. Calculate how long the time delay will last.

file 03554

Question 75

Design an experiment to calculate the size of a capacitor based on its response in a time-constant circuit. Include in your design an equation that gives the value of the capacitor in farads, based on data obtained by running the experiment.

file 00455

38

Question 76

A helpful technique for analyzing RC time-constant circuits is to consider what the *initial* and *final* values for circuit variables (voltage and current) are. Consider these four RC circuits:

Figure 1
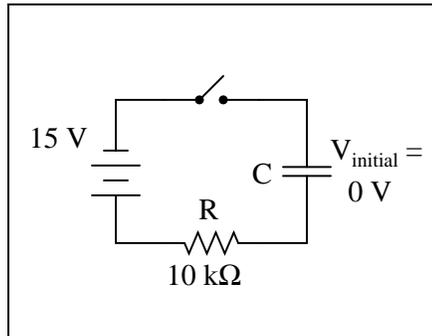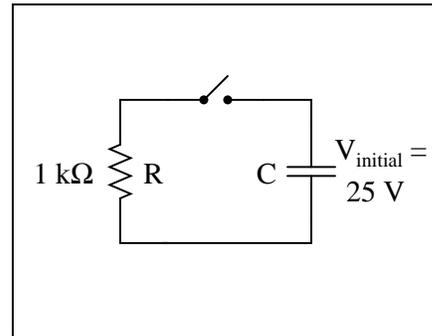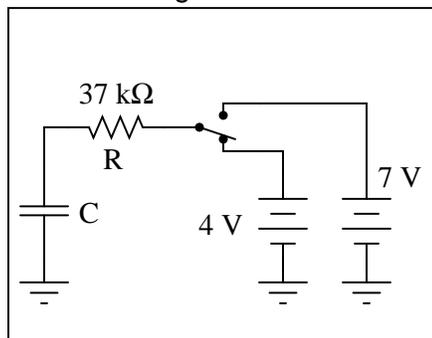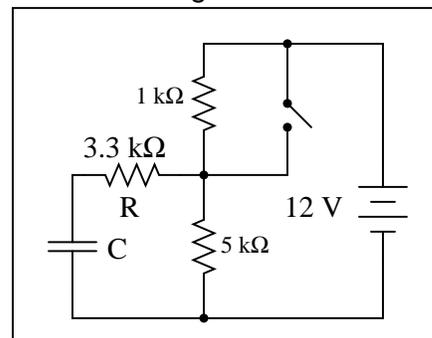
Figure 2

Figure 3

Figure 4

In each of these circuits, determine what the initial values will be for voltage across and current through both the capacitor and (labeled) resistor. These will be the voltage and current values at the very first instant the switch changes state from where it is shown in the schematic. Also, determine what the final values will be for the same variables (after a large enough time has passed that the variables are all "settled" into their ultimate values). The capacitor's initial voltage is shown in all cases where it is arbitrary.

file 01809

Question 77

A helpful technique for analyzing LR time-constant circuits is to consider what the *initial* and *final* values for circuit variables (voltage and current) are. Consider these four LR circuits:

Figure 1
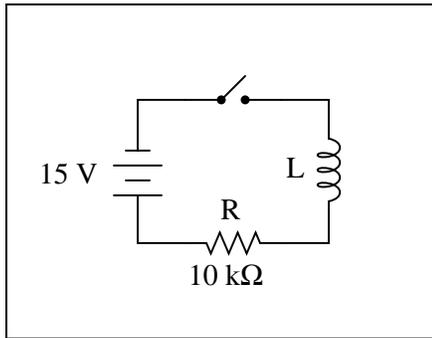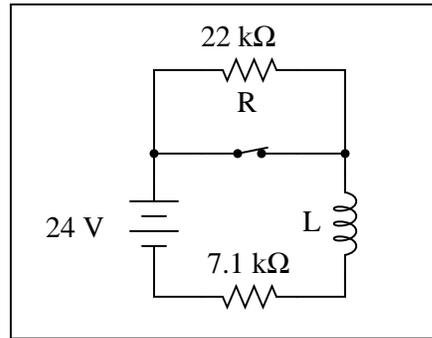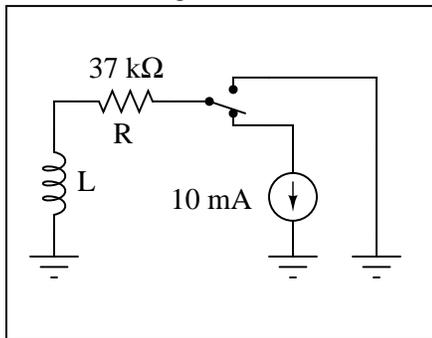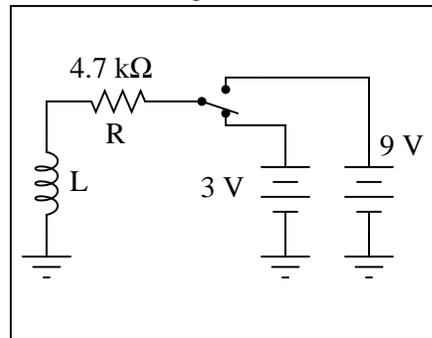
Figure 2

Figure 3

Figure 4

In each of these circuits, determine what the initial values will be for voltage across and current through both the inductor and (labeled) resistor. These will be the voltage and current values at the very first instant the switch changes state from where it is shown in the schematic. Also, determine what the final values will be for the same variables (after a large enough time has passed that the variables are all "settled" into their ultimate values).

Assume all inductors are ideal, possessing no coil resistance ($R_{coil} = 0\,\Omega$).

file 01810

A formula I like to use in calculating voltage and current values in either RC or LR circuits has two forms, one for voltage and one for current:

$$V(t) = (V_f - V_0)\left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right) + V_0 \qquad \text{(for calculating voltage)}$$

$$I(t) = (I_f - I_0)\left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right) + I_0 \qquad \text{(for calculating current)}$$

The "0" subscript represents the condition at time $= 0$ (V_0 or I_0, respectively), representing the "initial" value of that variable. The "f" subscript represents the "final" or "ultimate" value that the voltage or current would achieve if allowed to progress indefinitely. Obviously, one must know how to determine the "initial" and "final" values in order to use either of these formulae, but once you do you will be able to calculate *any* voltage and *any* current at *any* time in either an RC or LR circuit.

What is not so obvious to students is how each formula works. Specifically, what does each portion of it represent, in practical terms? This is your task: to describe what each term of the equation means in your own words. I will list the "voltage" formula terms individually for you to define:

$$V(t) =$$

$$(V_f - V_0) =$$

$$\left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right) =$$

file 01813

Question 79

Determine the voltage across the capacitor three seconds after the switch is moved from the upper position to the lower position, assuming it had been left in the upper position for a long time:
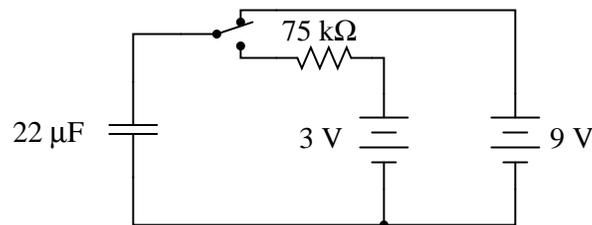
file 00457

Question 80

Calculate the voltage across the switch contacts the exact moment they open, and 15 milliseconds after they have been opened:
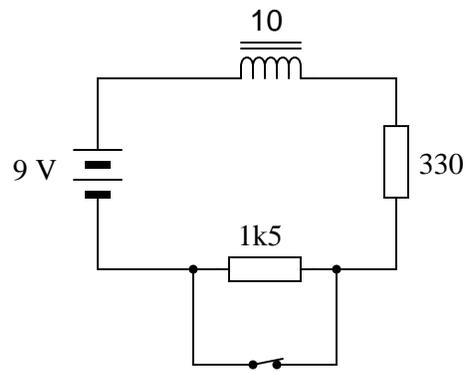
Competency: **Series inductances**	Version:

Schematic

$$L_1 \quad\quad L_2 \quad\quad L_3$$

Given conditions

$L_1 =$ $\quad\quad\quad$ $L_2 =$ $\quad\quad\quad$ $L_3 =$

Parameters

	Predicted	Measured
L_{total}		

Analysis

Equation used to calculate L_{total} :

file 01650

43

Competency: **Parallel inductances**   Version:

Schematic

L_1

L_2

L_3

Given conditions

$L_1 =$ $L_2 =$ $L_3 =$

Parameters

        Predicted   Measured

L_{total}

Analysis

Equation used to calculate L_{total} :

file 01651

44

Competency: **Series capacitances**   Version:

Schematic

C_1 C_2 C_3

Given conditions

$C_1 =$ $C_2 =$ $C_3 =$

Parameters

        Predicted   Measured

C_{total}

Analysis

Equation used to calculate C_{total} :

file 01652

Competency: **Parallel capacitances**   Version:

Schematic

C_1

C_2

C_3

Given conditions

$C_1 =$ $C_2 =$ $C_3 =$

Parameters

Predicted   Measured

C_{total}

Analysis

Equation used to calculate C_{total} :

Competency: **RC discharge circuit**   Version:

Schematic

Pushbutton switch

V_{supply} C_1 R_1 (V) Meter
+
-

Given conditions

$V_{supply} =$ $C_1 =$ $R_1 =$

$t_1 =$ $t_2 =$ $t_3 =$

Parameters

Predicted   Measured

V_{t1}

V_{t2}

V_{t3}

Calculations

Competency: **Time-delay relay**   Version:

Schematic

CR$_1$

Pushbutton switch

V$_{supply}$

C$_1$

Given conditions

V$_{supply}$ =   C$_1$ =   R$_{coil}$ =   V$_{dropout}$ =

Parameters

              Predicted   Measured

t$_{delay}$

Calculations

Competency: **Loaded voltage divider**   Version:

Schematic

Given conditions

$V_{supply} =$ $R_1 =$ $R_2 =$ $R_3 =$ $R_{load1} =$ $R_{load2} =$

Parameters

	Predicted	Measured		Predicted	Measured
I_{supply}			I_{load1}		
V_A			I_{load2}		
V_B			I_{bleed}		

Fault analysis

Suppose component [   ] fails   ☐ open   ☐ other _____

☐ shorted

What will happen in the circuit?

Troubleshooting log

Actions / Measurements / Observations (i.e. *What I did and/or noticed . . .*)	**Conclusions** (i.e. *What this tells me . . .*)

file 03933

Question 89

NAME: _____   **Troubleshooting Grading Criteria**

You will receive the highest score for which *all* criteria are met.

<u>100 %</u> *(Must meet or exceed all criteria listed)*
  A. Absolutely flawless procedure
  B. No unnecessary actions or measurements taken

<u>90 %</u> *(Must meet or exceed these criteria in addition to all criteria for 85% and below)*
  A. No reversals in procedure (i.e. changing mind without sufficient evidence)
  B. Every single action, measurement, and relevant observation properly documented

<u>80 %</u> *(Must meet or exceed these criteria in addition to all criteria for 75% and below)*
  A. No more than one unnecessary action or measurement
  B. No false conclusions or conceptual errors
  C. No missing conclusions (i.e. at least one documented conclusion for action / measurement / observation)

<u>70 %</u> *(Must meet or exceed these criteria in addition to all criteria for 65%)*
  A. No more than one false conclusion or conceptual error
  B. No more than one conclusion missing (i.e. an action, measurement, or relevant observation without a corresponding conclusion)

<u>65 %</u> *(Must meet or exceed these criteria in addition to all criteria for 60%)*
  A. No more than two false conclusions or conceptual errors
  B. No more than two unnecessary actions or measurements
  C. No more than one undocumented action, measurement, or relevant observation
  D. Proper use of all test equipment

<u>60 %</u> *(Must meet or exceed these criteria)*
  A. Fault accurately identified
  B. Safe procedures used at all times

<u>50 %</u> *(Only applicable where students performed significant development/design work – i.e. not a proven circuit provided with all component values)*
  A. Working prototype circuit built and demonstrated

<u>0 %</u> *(If <u>any</u> of the following conditions are true)*
  A. Unsafe procedure(s) used at any point

    <u>file 03932</u>

Question 90

Draw the pattern of the magnetic field produced by electric current through a straight wire and through a wire coil:
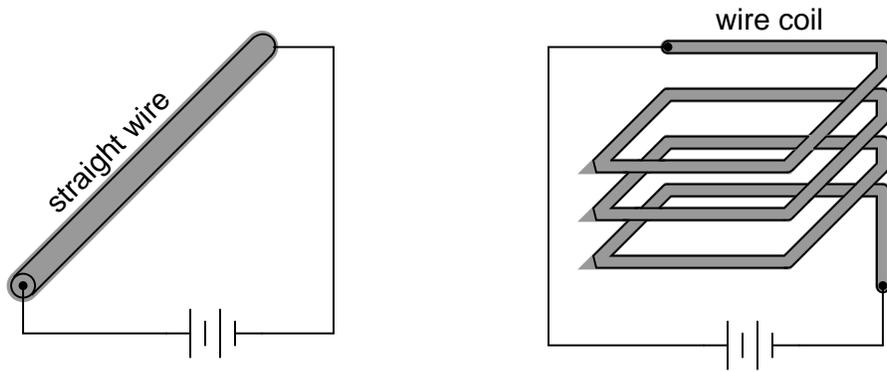
Explain your answer using either the *right-hand rule* (conventional flow) or the *left-hand rule* (electron flow).

file 00175

Question 91

$\boxed{\int f(x)\, dx \quad \textit{Calculus alert!}}$

If the number of turns of wire in an electromagnet coil is tripled, what happens to the magnitude of the magnetic flux (Φ) generated by it, assuming that none of the other variables change (current through the coil, reluctance of magnetic circuit, etc.)?

If the number of turns of wire in an inductor is tripled, what happens to the magnitude of the induced voltage for a given rate of magnetic flux change over time ($\frac{d\phi}{dt}$)?

If the number of turns of wire in an inductor is tripled, what happens to the magnitude of its inductance, measured in Henrys? Explain your answer.

file 00467

Question 92

Suppose you wished to build a component with no other purpose than to provide inductance in an electric circuit (an *inductor*). How might you design such a device to perform this function, and how could you maximize its inductance?

file 00210

Question 93

Magnetic fields, like all fields, have two fundamental measures: field *force* and field *flux*. In an inductor, which of these field quantities is directly related to current through the wire coil, and which is directly related to the amount of energy stored?

Based on this relationship, which magnetic field quantity changes when a bar of iron is brought closer to a wire coil, connected to a source of constant current?

Iron bar

Coil
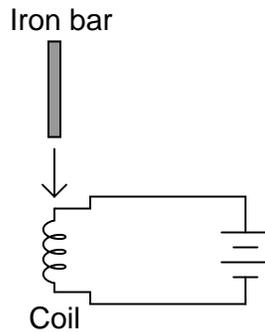
file 01137

Question 94

$\int f(x)\,dx$ Calculus alert!

Digital logic circuits, which comprise the inner workings of computers, are essentially nothing more than arrays of switches made from semiconductor components called *transistors*. As switches, these circuits have but two states: on and off, which represent the binary states of 1 and 0, respectively.

The faster these switch circuits are able to change state, the faster the computer can perform arithmetic and do all the other tasks computers do. To this end, computer engineers keep pushing the limits of transistor circuit design to achieve faster and faster switching rates.

This race for speed causes problems for the power supply circuitry of computers, though, because of the current "surges" (technically called *transients*) created in the conductors carrying power from the supply to the logic circuits. The faster these logic circuits change state, the greater the $\frac{di}{dt}$ rates-of-change exist in the conductors carrying current to power them. Significant voltage drops can occur along the length of these conductors due to their parasitic inductance:

L_{stray}

V_{drop}

V_{supply}

Logic gate circuit

L_{stray}

V_{drop}
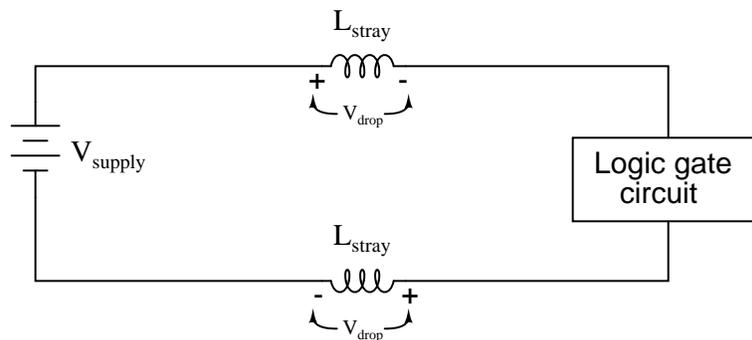
Suppose a logic gate circuit creates transient currents of 175 amps per nanosecond (175 A/ns) when switching from the "off" state to the "on" state. If the total inductance of the power supply conductors is 10 picohenrys (9.5 pH), and the power supply voltage is 5 volts DC, how much voltage remains at the power terminals of the logic gate during one of these "surges"?

file 00469

Question 95

One of the fundamental principles of calculus is a process called *integration*. This principle is important to understand because it is manifested in the behavior of inductance. Thankfully, there are more familiar physical systems which also manifest the process of integration, making it easier to comprehend.

If we introduce a constant flow of water into a cylindrical tank with water, the water level inside that tank will rise at a constant rate over time:
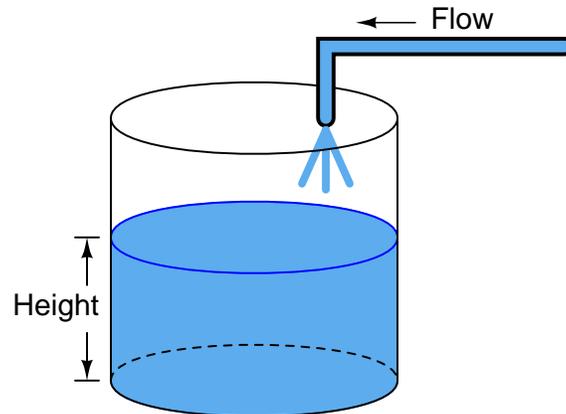
In calculus terms, we would say that the tank *integrates* water flow into water height. That is, one quantity (flow) dictates the rate-of-change over time of another quantity (height).

Like the water tank, electrical **inductance** also exhibits the phenomenon of integration with respect to time. Which electrical quantity (voltage or current) dictates the rate-of-change over time of which other quantity (voltage or current) in an inductance? Or, to re-phrase the question, which quantity (voltage or current), when maintained at a constant value, results in which other quantity (current or voltage) steadily ramping either up or down over time?

file 01382

Question 96

Electrical inductance has a close mechanical analogy: *inertia*. Explain what mechanical "inertia" is, and how the quantities of velocity and force applied to an object with mass are respectively analogous to current and voltage applied to an inductance.

file 01138

Question 97

What happens to the inductance of an inductor as its core becomes *saturated*? Does the inductance value increase, decrease, or remain the same? Explain your answer.

file 00473

Question 98

Suppose you wished to build a component with no other purpose than to provide capacitance in an electric circuit (a *capacitor*). How might you design such a device to perform this function, and how could you maximize its capacitance?

file 00192

What is a *Leyden Jar*, and how it its construction similar to the construction of all *capacitors*?
file 00186

Question 100

The storage of electric charge in a capacitor is often likened to the storage of water in a vessel:

Vessel holding water

Charged capacitor

+

-
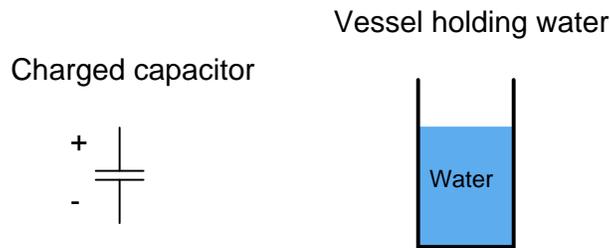
Water

Complete this analogy, relating the electrical quantities of charge (Q), voltage (E or V), and capacitance (C) to the quantities of water height, water volume, and vessel dimensions.
file 00188

Question 101

How does the rate of charge flow (current) into and out of a capacitor relate to the amount of voltage across its terminals? How does the rate of water flow into and out of a vessel relate to the amount of water stored in that vessel?

. . .

+

current

-

. . .

flow

Water
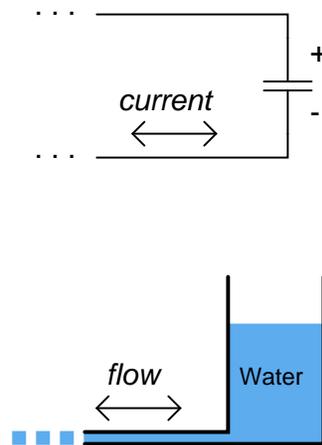
file 00193

Question 102

$\boxed{\int f(x)\, dx \quad \textit{Calculus alert!}}$

One of the fundamental principles of calculus is a process called *integration*. This principle is important to understand because it is manifested in the behavior of capacitance. Thankfully, there are more familiar physical systems which also manifest the process of integration, making it easier to comprehend.

If we introduce a constant flow of water into a cylindrical tank with water, the water level inside that tank will rise at a constant rate over time:
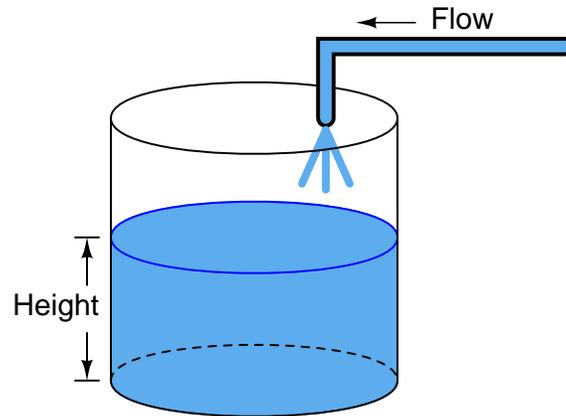
In calculus terms, we would say that the tank *integrates* water flow into water height. That is, one quantity (flow) dictates the rate-of-change over time of another quantity (height).

Like the water tank, electrical **capacitance** also exhibits the phenomenon of integration with respect to time. Which electrical quantity (voltage or current) dictates the rate-of-change over time of which other quantity (voltage or current) in a capacitance? Or, to re-phrase the question, which quantity (voltage or current), when maintained at a constant value, results in which other quantity (current or voltage) steadily ramping either up or down over time?

file 01383

Question 103

Electrical capacitance has a close mechanical analogy: *elasticity*. Explain what the term "elasticity" means for a mechanical spring, and how the quantities of velocity and force applied to a spring are respectively analogous to current and voltage applied to a capacitance.

file 01139

Question 104

Very large capacitors (typically in excess of 1 Farad!) are often used in the DC power wiring of high-power audio amplifier systems installed in automobiles. The capacitors are connected in parallel with the amplifier's DC power terminals, as close to the amplifier as possible, like this:
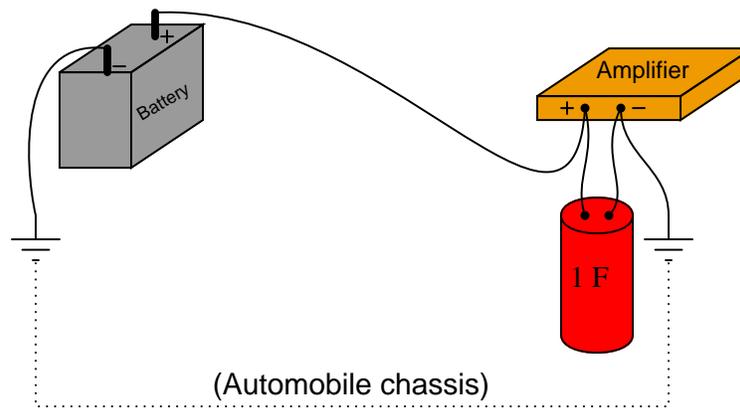
What is the purpose of having a capacitor connected in parallel with the amplifier's power terminals? What benefit does this give to the audio system, overall?

file 00206

Question 105

Capacitors and inductors alike have the ability to both *store* and *release* energy. This makes them more complicated than resistors, which merely dissipate energy. As a consequence, the relationship between direction of current and polarity of voltage is a bit more complex for capacitors and inductors than it is for power sources and resistors:
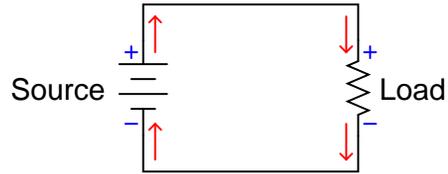
*Conventional flow notation used
in drawing current direction arrows*

Draw current arrows and voltage polarity marks (+ and - symbols) next to each of the following components, the left group representing a battery, capacitor, and inductor all acting as *energy sources* and the right group representing a resistor, capacitor, and inductor all acting as *energy loads*:
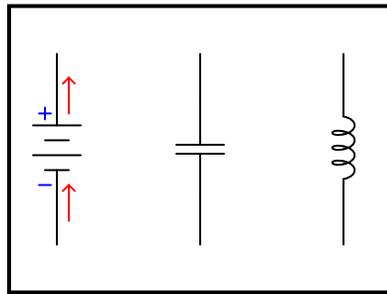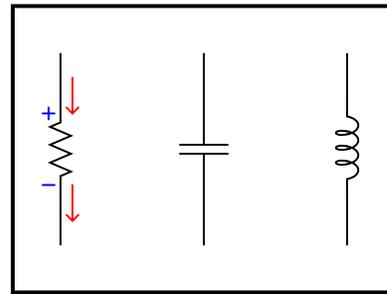
file 03725

Question 106

When the pushbutton is pressed, the relay immediately energizes and sends power to the electric horn. When the pushbutton is released, the horn remains on for a few moments before turning off, due to the capacitor's stored charge continuing to power the relay coil. So, the capacitor and relay form a *time-delay* control circuit for the horn:
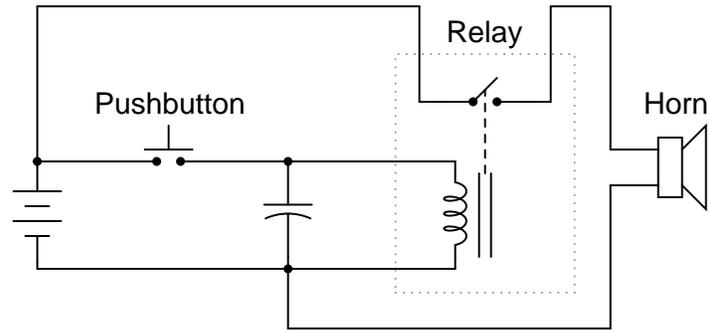
Suppose this circuit has functioned as designed for quite some time, then one day develops a problem. The horn sounds immediately when the pushbutton is pressed (as it should), bit it immediately silences instead of continuing to sound for a few more moments when the pushbutton is released. Based on this information, identify these things:

- <u>Two</u> components or wires in the circuit that you know must be in good working condition.

- <u>Two</u> components or wires in the circuit that could possibly be bad (and thus cause the off-delay action to fail).

  file 03441

Question 107

A 10 μF capacitor is charged to a voltage of 20 volts. How many coulombs of electric charge are stored in this capacitor?

file 00187

Question 108

An important parameter in capacitor performance is *ESR*. Define ESR, and explain what causes it.

file 01539

Question 109

Capacitors often have letter codes following the three-digit number codes. For example, here are some typical capacitor codes, complete with letters:

- 473K
- 102J
- 224M
- 331F

Determine the meaning of letters used on capacitor labels, what the respective numeric values are for all the available letters, and then finally what these four specific number/letter codes mean (shown above).

file 03651

60

Question 110

Suppose a capacitor is charged by a voltage source, and then switched to a resistor for discharging:
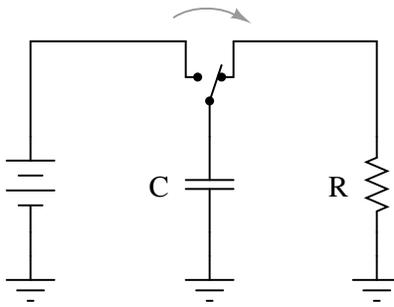
Would a larger capacitance value result in a slower discharge, or a faster discharge? How about a larger resistance value? You may find the "Ohm's Law" equation for capacitance helpful in answering both these questions:

$$i = C\frac{dv}{dt}$$

Now consider an inductor, "charged" by a current source and then switched to a resistor for discharging:
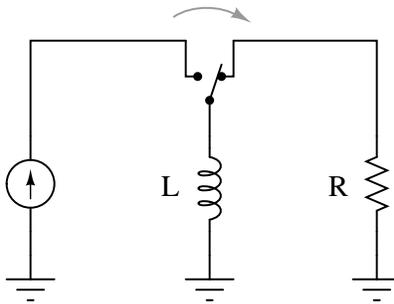
Would a larger inductance value result in a slower discharge, or a faster discharge? How about a larger resistance value? You may find the "Ohm's Law" equation for inductance helpful in answering both these questions:

$$v = L\frac{di}{dt}$$

file 01798

61

Question 111

While many students find it easy to understand how the value of R affects the time constant of a resistor-capacitor charging circuit (more R means slower charging; less R means faster charging), the opposite behavior of resistor-inductor circuits (less R means slower charging; more R means faster charging) seems incomprehensible:
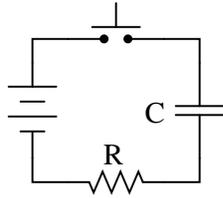
More R means slower charging
Less R means faster charging

Makes sense!
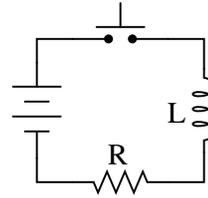
More R means faster charging
Less R means slower charging

Why???

Resistor-capacitor circuit charging behavior probably makes more sense to students because they realize resistance controls charging current, which in turn directly effects how quickly the capacitor's voltage may rise:

$$i = C\frac{dv}{dt}$$

Current (i) is proportional to the rate of change of voltage ($\frac{dv}{dt}$). Since current is inversely proportional to resistance in a circuit powered by a voltage source, so must be the capacitor charging rate.

One way to help make the inductor charging circuit more sensible is to replace the series voltage-source/resistor combination with its Norton equivalent, a parallel current-source/resistor combination:

Norton equivalent
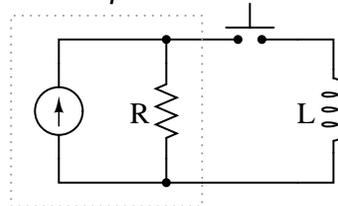
Re-analyze the circuit in this form, and try to explain why more resistance makes the inductor charging time faster and less resistance makes the inductor charging time slower.

file 03533

Question 112

The decay of a variable (either voltage or current) in a time-constant circuit (RC or LR) follows this mathematical expression:

$$e^{-\frac{t}{\tau}}$$

Where,

$e =$ Euler's constant (≈ 2.718281828)

$t =$ Time, in seconds

$\tau =$ Time constant of circuit, in seconds

Calculate the value of this expression as t increases, given a circuit time constant (τ) of 1 second. Express this value as a *percentage*:

$t = 1$ second

$t = 2$ seconds

$t = 3$ seconds

$t = 4$ seconds

$t = 5$ seconds

$t = 6$ seconds

$t = 7$ seconds

$t = 8$ seconds

$t = 9$ seconds

$t = 10$ seconds

Based on your calculations, how would you describe the change in the expression's value over time as t increases?

file 00442

Question 113

The decay of a variable (either voltage or current) in a time-constant circuit (RC or LR) follows this mathematical expression:

$$e^{-\frac{t}{\tau}}$$

Where,

e = Euler's constant (≈ 2.718281828)
t = Time, in seconds
τ = Time constant of circuit, in seconds

Calculate the value of this expression as t increases, given a circuit time constant (τ) of 2 seconds. Express this value as a *percentage*:

$t = 1$ second
$t = 2$ seconds
$t = 3$ seconds
$t = 4$ seconds
$t = 5$ seconds
$t = 6$ seconds
$t = 7$ seconds
$t = 8$ seconds
$t = 9$ seconds
$t = 10$ seconds

Also, express the percentage value of any *increasing* variables (either voltage or current) in an RC or LR charging circuit, for the same conditions (same times, same time constant).
file 00450

Question 114

Write a mathematical expression for calculating the percentage value of any *increasing* variables (either voltage or current) in an RC or LR time-constant circuit.

Hint: the formula for calculating the percentage of any *decreasing* variables in an RC or LC time-constant circuit is as follows:

$$e^{-\frac{t}{\tau}}$$

Where,

e = Euler's constant (≈ 2.718281828)
t = Time, in seconds
τ = Time constant of circuit, in seconds

Here, the value of the expression starts at 1 (100%) at time $= 0$ and approaches 0 (0%) as time approaches ∞. What I'm asking you to derive is an equation that does just the opposite: start with a value of 0 when time $= 0$ and approach a value of 1 as time approaches ∞.
file 00451

Question 115

Calculating variables in reactive circuits using time-constant formulae can be time consuming, due to all the keystrokes necessary on a calculator. Even worse is when a calculator is not available! You should be prepared to estimate circuit values without the benefit of a calculator to do the math, though, because a calculator may not always be available when you need one.

Note that Euler's constant (e) is approximately equal to 3. This is not a close approximation, but close enough for "rough" estimations. If we use a value of three instead of e's true value of $2.718281828\cdots$, we may greatly simplify the "decay" time constant formula:

$$\text{Percentage of change} \approx 3^{-\frac{t}{\tau}}$$

Suppose that a capacitive discharge circuit begins with a full-charge voltage of 10 volts. Calculate the capacitor's voltage at the following times as it discharges, assuming $\tau = 1$ second:

$t = 0$ seconds ; $E_C =$
$t = 1$ second ; $E_C =$
$t = 2$ seconds ; $E_C =$
$t = 3$ seconds ; $E_C =$
$t = 4$ seconds ; $E_C =$
$t = 5$ seconds ; $E_C =$

Without using a calculator, you should at least be able to calculate voltage values as *fractions* if not decimals!

file 01804

Question 116

Calculating variables in reactive circuits using time-constant formulae can be time consuming, due to all the keystrokes necessary on a calculator. Even worse is when a calculator is not available! You should be prepared to estimate circuit values without the benefit of a calculator to do the math, though, because a calculator may not always be available when you need one.

Note that Euler's constant (e) is approximately equal to 3. This is not a close approximation, but close enough for "rough" estimations. If we use a value of three instead of e's true value of $2.718281828\cdots$, we may greatly simplify the "increasing" time constant formula:

$$\text{Percentage of change} \approx 1 - 3^{-\frac{t}{\tau}}$$

Suppose that a capacitive charging circuit begins fully discharged (0 volts), and charges to an ultimate value of 10 volts. Calculate the capacitor's voltage at the following times as it discharges, assuming $\tau = 1$ second:

$t = 0$ seconds ; $E_C =$
$t = 1$ second ; $E_C =$
$t = 2$ seconds ; $E_C =$
$t = 3$ seconds ; $E_C =$
$t = 4$ seconds ; $E_C =$
$t = 5$ seconds ; $E_C =$

Without using a calculator, you should at least be able to calculate voltage values as *fractions* if not decimals!

file 01805

Question 117

Determine the number of time constants (τ) that 7.5 seconds is equal to in each of the following reactive circuits:

- RC circuit; R = 10 kΩ, C = 220 μF ; 7.5 sec =
- RC circuit; R = 33 kΩ, C = 470 μF ; 7.5 sec =
- RC circuit; R = 1.5 kΩ, C = 100 μF ; 7.5 sec =
- RC circuit; R = 790 Ω, C = 9240 nF ; 7.5 sec =
- RC circuit; R = 100 kΩ, C = 33 pF ; 7.5 sec =

- LR circuit; R = 100 Ω, L = 50 mH ; 7.5 sec =
- LR circuit; R = 45 Ω, L = 2.2 H ; 7.5 sec =
- LR circuit; R = 1 kΩ, L = 725 mH ; 7.5 sec =
- LR circuit; R = 4.7 kΩ, L = 325 mH ; 7.5 sec =
- LR circuit; R = 6.2 Ω, L = 25 H ; 7.5 sec =

file 01802

Question 118

Suppose we were measuring the interior temperature of an insulated box recently removed from a refrigerator, as it was being warmed by the ambient air around it:

(Ambient air warmer than inside of box)

Box
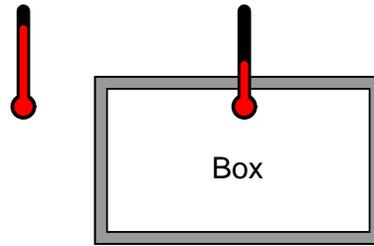
Graphing the box's temperature over time, we see a curve that looks something like this:

Box temp.
(degrees)

Time ⟶
(minutes)
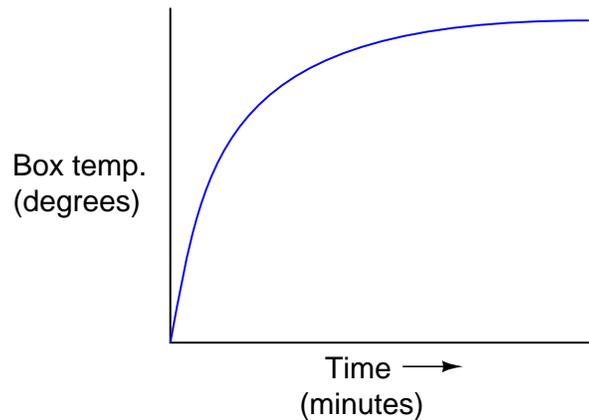
An engineer approaches you and says she wants you to build an electrical circuit that *models* this thermal system. What kind of circuit would you consider building for the engineer, to make a realistic electrical analogue of the box's temperature? Be as specific as you can in your answer.

file 01218

Question 119

When a circuit designer needs a circuit to provide a time delay, he or she almost always chooses an RC circuit instead of an LR circuit. Explain why this is.

file 01800

Question 120

At a party, you happen to notice a mathematician taking notes while looking over the food table where several pizzas are set. Walking up to her, you ask what she is doing. "I'm mathematically modeling the consumption of pizza," she tells you. Before you have the chance to ask another question, she sets her notepad down on the table and excuses herself to go use the bathroom.

Looking at the notepad, you see the following equation:

$$\text{Percentage} = \left(1 - e^{-\frac{t}{5.8}}\right) \times 100\%$$

Where,

t = Time in minutes since arrival of pizza.

The problem is, you don't know whether the equation she wrote describes the percentage of pizza eaten or the percentage of pizza remaining on the table. Explain how you would determine which percentage this equation describes. How, exactly, can you tell if this equation describes the amount of pizza already eaten or the amount of pizza that remains to be eaten?

file 03309

Question 121

The following expression is frequently used to calculate values of changing variables (voltage and current) in RC and LR timing circuits:

$$e^{-\frac{t}{\tau}} \qquad \text{or} \qquad \frac{1}{e^{\frac{t}{\tau}}}$$

If we evaluate this expression for a time of $t = 0$, we find that it is equal to 1 (100%). If we evaluate this expression for increasingly larger values of time ($t \to \infty$), we find that it approaches 0 (0%).

Based on this simple analysis, would you say that the expression $e^{-\frac{t}{\tau}}$ describes the percentage that a variable has changed from its initial value in a timing circuit, or the percentage that it has *left* to change before it reaches its final value? To frame this question in graphical terms . . .
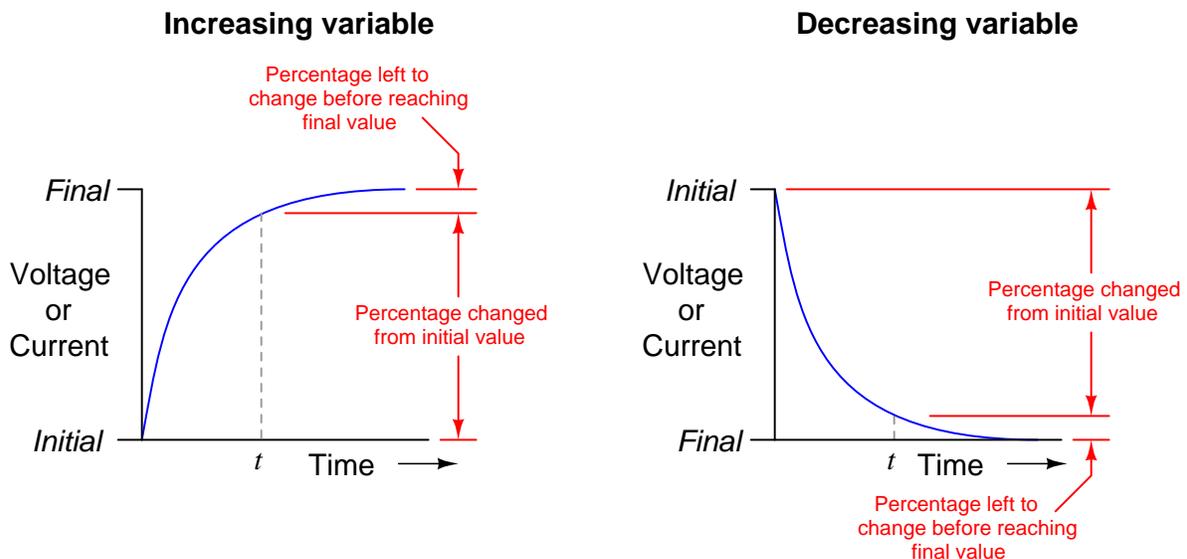
Which percentage does the expression $e^{-\frac{t}{\tau}}$ represent in each case? Explain your answer.

file 02946

Question 122

Electromechanical relays are extremely useful devices, but they have their idiosyncrasies. One of them is a consequence of the fact that the relay's coil acts as an inductor, storing energy in its magnetic field.

In the following circuit, a relay is used to switch power to a large electric motor, while being controlled by a light-duty pushbutton switch:
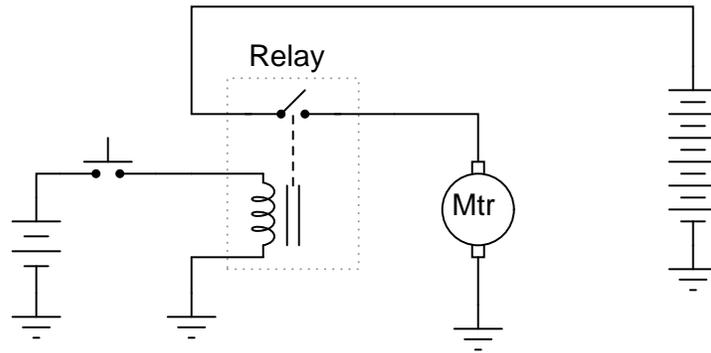
The problem here is every time the pushbutton switch is released, the contacts arc significantly. This happens because the inductor releases all of its stored energy as a high-voltage spark across the opening contacts. *Inductive kickback* is the phrase commonly used to describe this effect, and over time it will prematurely destroy the switch.

An electronics technician understands the nature of the problem and proposes a solution. By connecting a light bulb in parallel with the relay coil, the coil's energy now has a safer place to dissipate whenever the pushbutton switch contacts open:
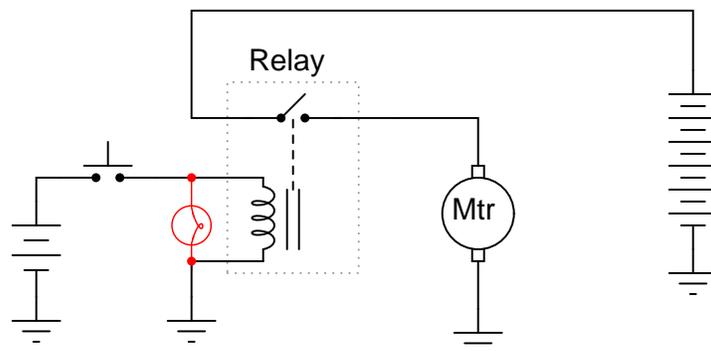
Instead of that stored energy manifesting itself as a high-voltage arc at the switch, it powers the light bulb for a brief time after the switch opens, dissipating in a non-destructive manner.

However, the addition of the light bulb introduces a new, unexpected problem to the circuit. Now, when the pushbutton switch is released, the relay delays for a fraction of a second before disengaging. This causes the motor to "overshoot" its position instead of stopping when it is supposed to.

Explain why this happens, with reference to the LR time constant of this circuit before and after the addition of the lamp.

<u>file 01801</u>

Question 123

Determine the capacitor voltage and capacitor current at the specified times (time $t = 0$ milliseconds being the exact moment the switch contacts close). Assume the capacitor begins in a fully discharged state:
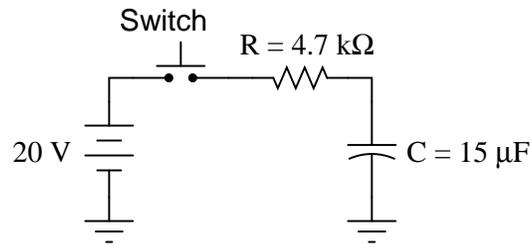
Time	V_C (volts)	I_C (mA)
0 ms		
30 ms		
60 ms		
90 ms		
120 ms		
150 ms		

file 03556

Question 124

Determine the inductor voltage and inductor current at the specified times (time $t = 0$ milliseconds being the exact moment the switch contacts close):
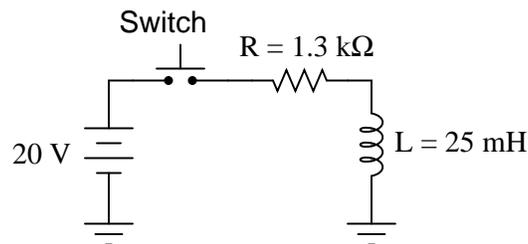
Time	V_L (volts)	I_L (mA)
0 μs		
10 μs		
20 μs		
30 μs		
40 μs		
50 μs		

file 03589

Question 125

Determine the inductor voltage and inductor current at the specified times (time $t = 0$ milliseconds being the exact moment the switch contacts close):
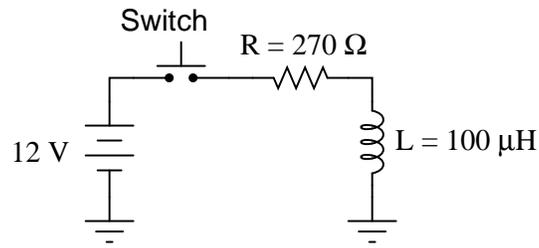
Time	V_L (volts)	I_L (mA)
0 ns		
250 ns		
500 ns		
750 ns		
1.00 μs		
1.25 μs		

file 03590

Question 126

Complete this table of values for inductor voltage and current. Consider time $= 0$ to be the precise moment the switch closes:
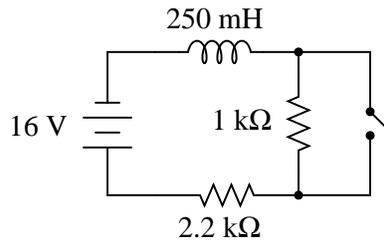
Time (μs)	V_L (V)	I_L (mA)
0		
50		
100		
150		
200		
250		
300		
350		
400		

file 01815

71

Question 127

Calculate the voltage across a 2.5 H inductor after "charging" through a series-connected resistor with 50 Ω of resistance for 75 milliseconds, powered by a 6 volt battery. Assume that the inductor has an internal resistance of 14 Ω.

Also, express this amount of time (75 milliseconds) in terms of how many *time constants* have elapsed.

Hint: it would be helpful in your analysis to draw a schematic diagram of this circuit showing the inductor's inductance and 14 ohms of resistance as two separate (idealized) components. This is a very common analysis technique in electrical engineering: to regard the parasitic characteristics of a component as a separate component in the same circuit.

file 00454

Question 128

Calculate the amount of time it takes for a 10 μF capacitor to discharge from 18 volts to 7 volts if its ultimate (final) voltage when fully discharged will be 0 volts, and it is discharging through a 22 kΩ resistor.

file 02941

Question 129

Determine the amount of time needed after switch closure for the capacitor voltage (V_C) to reach the specified levels:
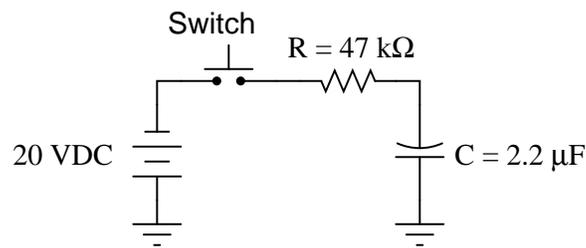
V_C	Time
0 volts	
-5 volts	
-10 volts	
-15 volts	
-19 volts	

Trace the direction of current in the circuit while the capacitor is charging, and be sure to denote whether you are using electron or conventional flow notation.

Note: the voltages are specified as negative quantities because they are negative with respect to (positive) ground in this particular circuit.

file 03118

Question 130

Determine the amount of time needed for the capacitor voltage (V_C) to fall to the specified levels after the switch is thrown to the "discharge" position, assuming it had first been charged to full battery voltage:

Switch

R = 190 kΩ

12 V

C = 17 µF
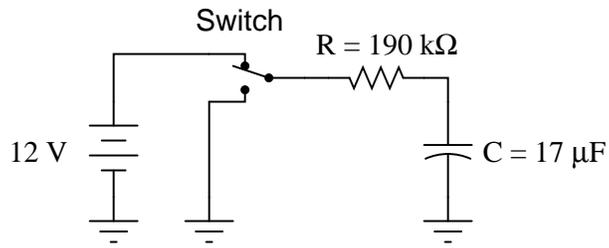
V_C	Time
10 volts	
8 volts	
6 volts	
4 volts	
2 volts	

Trace the direction of electron flow in the circuit, and also mark all voltage polarities.

file 02943

73

A submarine sonar system uses a "bank" of parallel-connected capacitors to store the electrical energy needed to send brief, powerful pulses of current to a transducer (a "speaker" of sorts). This generates powerful sound waves in the water, which are then used for echo-location. The capacitor bank relieves the electrical generators and power distribution wiring aboard the submarine from having to be rated for huge surge currents. The generator trickle-charges the capacitor bank, and then the capacitor bank quickly dumps its store of energy to the transducer when needed:
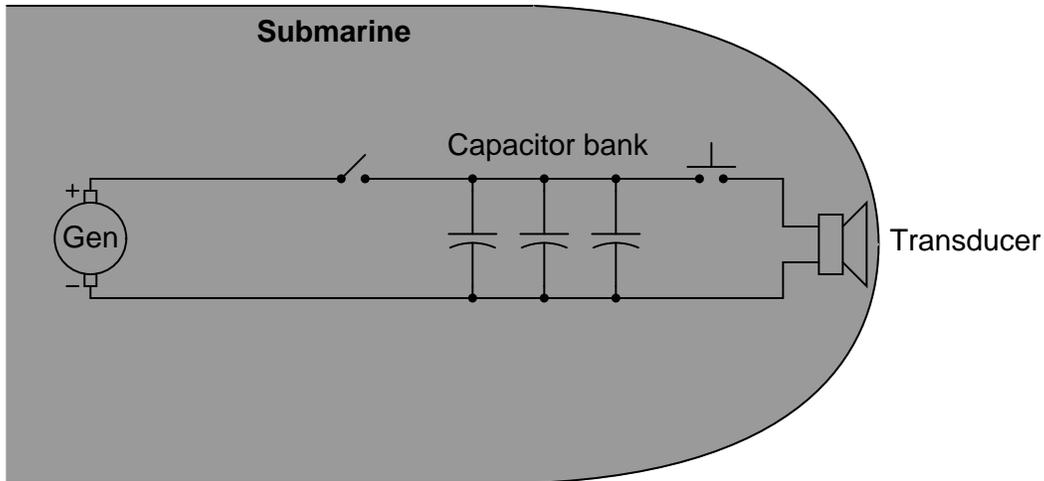
As you might well imagine, such a capacitor bank can be lethal, as the voltages involved are quite high and the surge current capacity is enormous. Even when the DC generator is disconnected (using the "toggle" disconnect switch shown in the schematic), the capacitors may hold their lethal charge for many days.

To help decreases the safety risk for technical personnel working on this system, a "discharge" switch is connected in parallel with the capacitor bank to automatically provide a path for discharge current whenever the generator disconnect switch is opened:
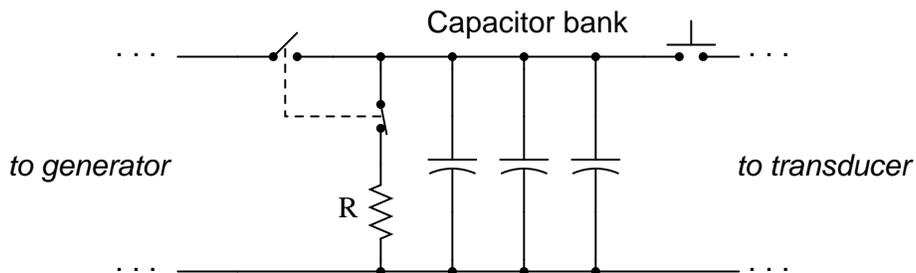
Suppose the capacitor bank consists of forty 1500 μF capacitors connected in parallel (I know the schematic only shows three, but . . .), and the discharge resistor is 10 kΩ in size. Calculate the amount of time it takes for the capacitor bank to discharge to 10 percent of its original voltage and the amount of time it takes to discharge to 1 percent of its original voltage once the disconnect switch opens and the discharge switch closes.

file 03119

74

Question 132

The model "555" integrated circuit is a very popular and useful "chip" used for timing purposes in electronic circuits. The basis for this circuit's timing function is a resistor-capacitor (RC) network:
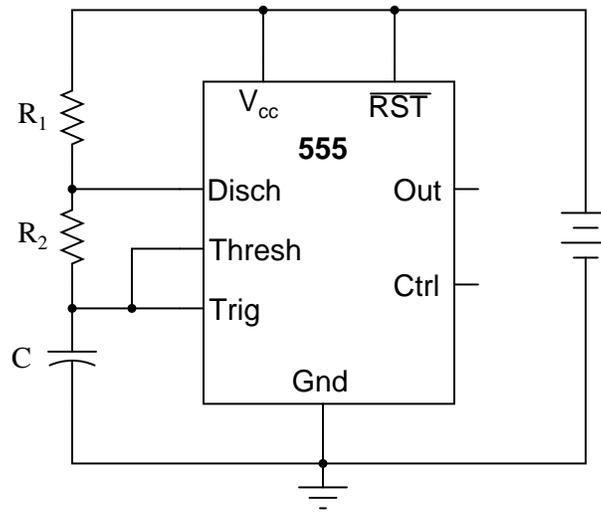
In this configuration, the "555" chip acts as an *oscillator*: switching back and forth between "high" (full voltage) and "low" (no voltage) output states. The time duration of one of these states is set by the charging action of the capacitor, through both resistors (R_1 and R_2 in series). The other state's time duration is set by the capacitor discharging through one resistor (R_2):

Capacitor charging through R1 and R2 (series)   *Capacitor discharging through R2 only*
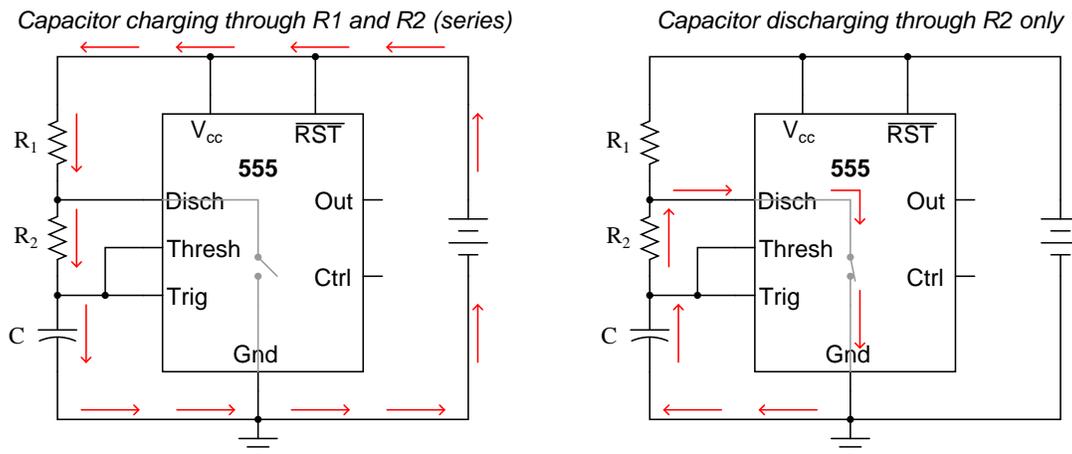
Note: all currents shown in the direction of conventional flow

Obviously, the charging time constant must be $\tau_{charge} = (R_1 + R_2)C$, while the discharging time constant is $\tau_{discharge} = R_2 C$. In each of the states, the capacitor is either charging or discharging 50% of the way between its starting and final values (by virtue of how the 555 chip operates), so we know the expression $e^{\frac{-t}{\tau}} = 0.5$, or 50 percent.[†]

[†] For those who must know why, the 555 timer in this configuration is designed to keep the capacitor voltage cycling between $\frac{1}{3}$ of the supply voltage and $\frac{2}{3}$ of the supply voltage. So, when the capacitor is

Develop two equations for predicting the "charge" time and "discharge" time of this 555 timer circuit, so that anyone designing such a circuit for specific time delays will know what resistor and capacitor values to use.

file 01807

Question 133

Calculate the rate of change of voltage $(\frac{dv}{dt})$ for the capacitor at the exact instant in time where the switch moves to the "charge" position. Assume that prior to this motion the switch had been left in the "discharge" position for some time:

1k5

Charge   Discharge

22 V

3μ3
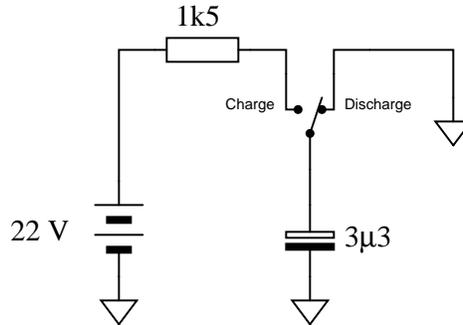
file 03558

Question 134

Calculate the rate of change of current $(\frac{di}{dt})$ for the inductor at the exact instant in time where the switch moves to the "charge" position.

470

Charge   Discharge

14 V

50m
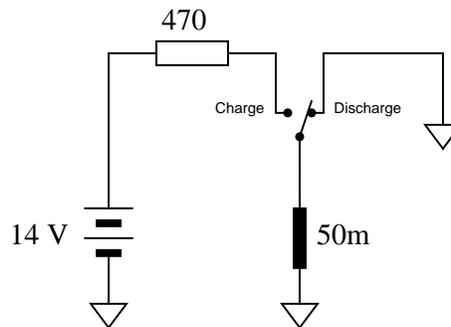
file 03559

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charging from $\frac{1}{3}V_{CC}$ to its (final) value of full supply voltage (V_{CC}), having this charge cycle interrupted at $\frac{2}{3}V_{CC}$ by the 555 chip constitutes charging to the half-way point, since $\frac{2}{3}$ of half-way between $\frac{1}{3}$ and 1. When discharging, the capacitor starts at $\frac{2}{3}V_{CC}$ and is interrupted at $\frac{1}{3}V_{CC}$, which again constitutes 50% of the way from where it started to where it was (ultimately) headed.

Question 135

An electrical testing laboratory has designed and built a high-current "surge" power supply for testing the effects of large electrical currents on certain types of components. The basic idea is that a bank of capacitors are charged to a high voltage by a DC power source (through a current-limiting resistor to protect the power source from overcurrent damage), then quickly discharged through a switch to the test load:
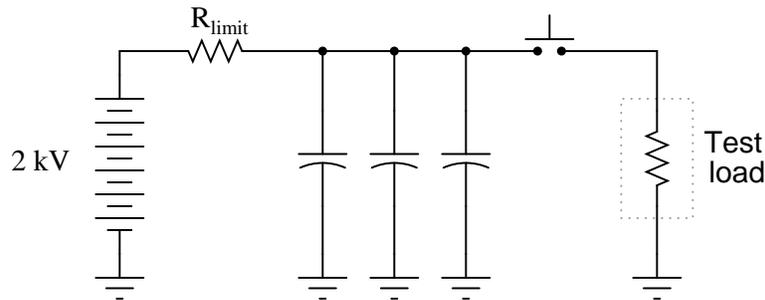
Due to the magnitude of the transient currents generated by the discharging capacitors, a mechanical switch (as suggested by the pushbutton symbol) is impractical. Instead, an *air gap switch* is constructed from two metal balls with a needle-point "ionizer" electrode between them. The ionizer electrode connects to a simple inductor/switch circuit that generates a high-voltage pulse sufficient to create a spark. The ionized air created in the spark provides a low-resistance path through the air which the 2000 volts from the capacitor bank can now traverse, thus completing the circuit for surge current between the capacitors and the test load:
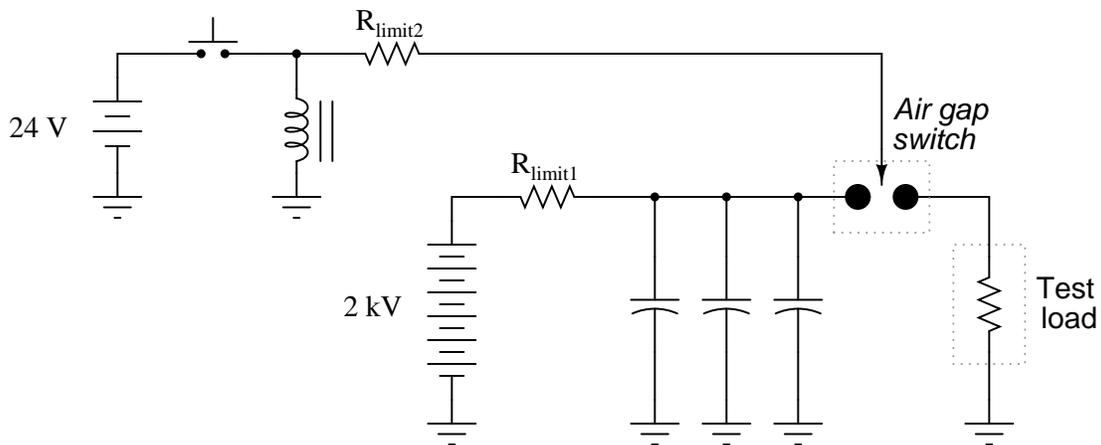
Suppose this surge power supply circuit failed to work after several months of good service. Nothing happens any more when the pushbutton switch is pressed and released. No spark is heard or seen in the air gap, and the test load receives no surge of current.

A voltmeter placed (carefully!) in parallel with the capacitor bank indicates a full charge voltage of 2035 volts, which is within normal parameters. Based on this information, identify these things:

- <u>Two</u> components in the circuit that you know must be in good working condition.

- <u>Two</u> components in the circuit that could possibly be bad, and the mode of their failure (either open or shorted).

  <u>file 03442</u>

Question 136

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Inductors store energy in the form of a magnetic field. We may calculate the energy stored in an inductance by integrating the product of inductor voltage and inductor current ($P = IV$) over time, since we know that power is the rate at which work (W) is done, and the amount of work done to an inductor taking it from zero current to some non-zero amount of current constitutes energy stored (U):

$$P = \frac{dW}{dt}$$

$$dW = P\,dt$$

$$U = W = \int P\,dt$$

Find a way to substitute inductance (L) and current (I) into the integrand so you may integrate to find an equation describing the amount of energy stored in an inductor for any given inductance and current values.

file 03547

Question 137

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Capacitors store energy in the form of an electric field. We may calculate the energy stored in a capacitance by integrating the product of capacitor voltage and capacitor current ($P = IV$) over time, since we know that power is the rate at which work (W) is done, and the amount of work done to a capacitor taking it from zero voltage to some non-zero amount of voltage constitutes energy stored (U):

$$P = \frac{dW}{dt}$$

$$dW = P\,dt$$

$$U = W = \int P\,dt$$

Find a way to substitute capacitance (C) and voltage (V) into the integrand so you may integrate to find an equation describing the amount of energy stored in a capacitor for any given capacitance and voltage values.

file 03548

Question 138

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Differential equations may be used to model the charging behavior of an RC circuit. Take, for instance, this simple RC circuit:
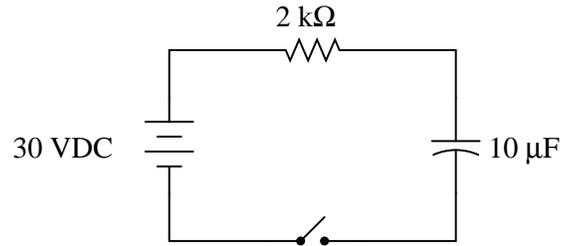
We may develop a loop equation based on Kirchhoff's Voltage Law, knowing that the voltage of the power source is constant (30 volts), and that the voltage drops across the capacitor and resistor are $V_C = \frac{Q}{C}$ and $V_R = IR$, respectively:

$$30 - IR - \frac{Q}{C} = 0$$

To turn this into a true differential equation, we must express one of the variables as the derivative of the other. In this case, it makes sense to define I as the time-derivative of Q:

$$30 - \frac{dQ}{dt}R - \frac{Q}{C} = 0$$

Show that the specific solution to this differential equation, assuming an initial condition of $Q = 0$ at $t = 0$, is as follows:

$$Q = 0.0003(1 - e^{-50t})$$

Also, show this solution in a form where it solves for capacitor voltage (V_C) instead of capacitor charge (Q).

file 01510

79

Question 139

Differential equations may be used to model the charging behavior of an L/R circuit. Take, for instance, this simple L/R circuit:
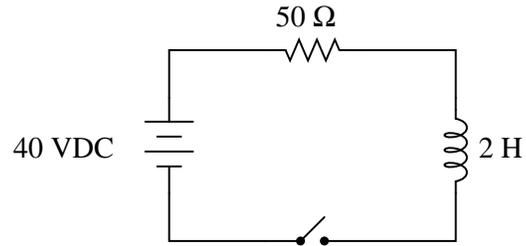
We may develop a loop equation based on Kirchhoff's Voltage Law, knowing that the voltage of the power source is constant (40 volts), and that the voltage drops across the inductor and resistor are $V_L = L\frac{dI}{dt}$ and $V_R = IR$, respectively:

$$40 - IR - L\frac{dI}{dt} = 0$$

Show that the specific solution to this differential equation, assuming an initial condition of $I = 0$ at $t = 0$, is as follows:

$$I = 0.8(1 - e^{-25t})$$

file 01511

Answer 1

A changing current through a coil produces a voltage drop that opposes the direction of change.

Answer 2

"Inductance" is the capacity of a conductor to store energy in the form of a magnetic field, resulting from an applied current. You may also find a definition of "inductance" stated in terms of opposition to change in applied current over time.

Inductance is caused by the establishment of a magnetic field around a conductor.

Answer 3

Lower-case variables represent *instantaneous* values, as opposed to average values. The expression $\frac{di}{dt}$ represents the *instantaneous rate of change of current over time.*

Follow-up question: manipulate this equation to solve for the other two variables ($\frac{di}{dt} = \cdots$; $L = \cdots$).

Answer 4

When the switch closes, current will steadily increase at a linear rate over time:
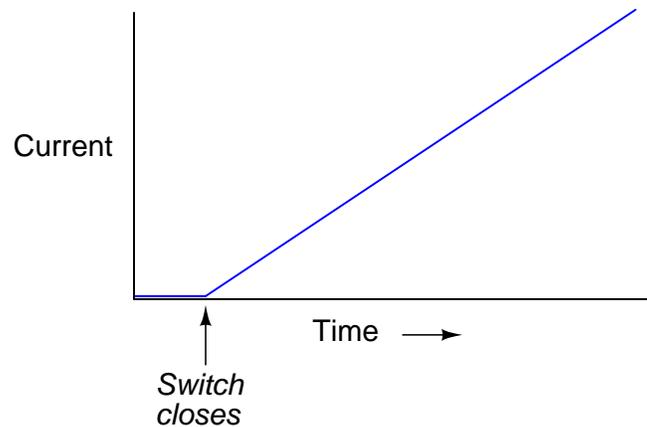
Challenge question: real wire coils contain electrical resistance (unless they're made of superconducting wire, of course), and we know how voltage equilibrium occurs in resistive circuits: the current converges at a value necessary for the resistance to drop an equal amount of voltage as the source. Describe, then, what the current does in a circuit with a *real* wire coil, not a superconducting wire coil.

Answer 5

Approximately 2360 turns of wire for the air core, and approximately 37 turns of wire for the iron core.

New inductance equation:

$$L = \frac{\pi N^2 r^2 \mu}{l}$$

Answer 6

Here I show the answer in two different forms: current shown as *electron flow* (left) and current shown as *conventional flow* (right).

Electron flow   **Conventional flow**
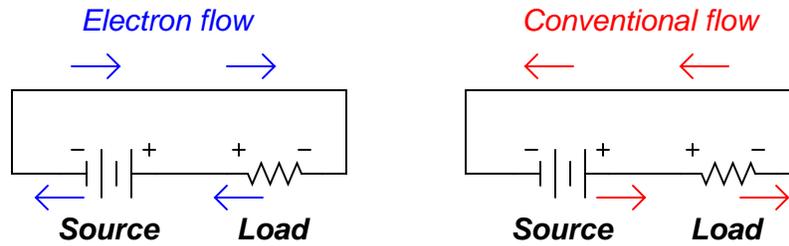
Whichever notation you choose to follow in your analysis of circuits, the understanding should be the same: the reason voltage polarities across the resistor and battery differ despite the same direction of current through both is the flow of power. The battery acts as a *source*, while the resistor acts as a *load*.

Answer 7

As the applied current increases, the inductor acts as a load, accumulating additional energy from the current source. Acting as a load, the voltage dropped by the inductor will be in the same polarity as across a resistor.

Current *increasing*
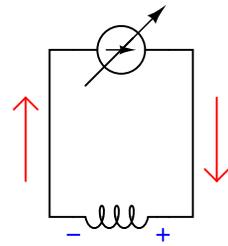
*Inductor as a **load***

As the applied current decreases, the inductor acts as a source, releasing accumulated energy to the rest of the circuit, as though it were a current source itself of superior current. Acting as a source, the voltage dropped by the inductor will be in the same polarity as across a battery, powering a load.

Current *decreasing*
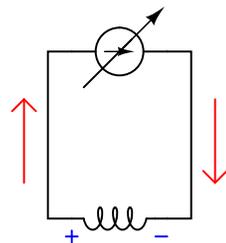
*Inductor as a **source***

Answer 8

Inductors oppose changes in **current**, reacting to such changes by producing a **voltage**.

Answer 9

I won't directly describe how a bifilar winding is made, but I'll give you a hint. Compare the inductance of a straight piece of wire, versus one that is folded in half:
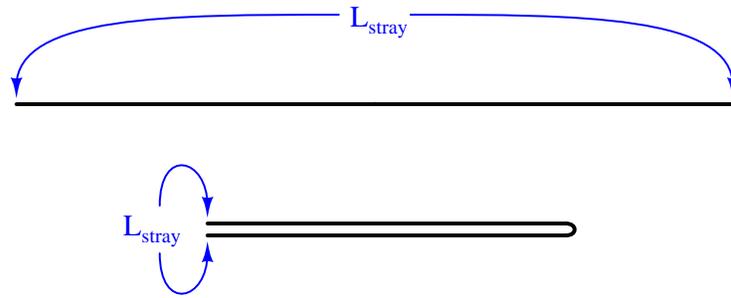
Now, how could a non-inductive *coil* of wire be made using the same principle?

Answer 10

Inductance naturally exists along any conductor. The longer the conductor, the more inductance, all other factors being equal.

Answer 11

There are a couple of different ways to explain how an electromagnet coil can generate a much greater voltage than what it is energized from (the battery). One way is to explain the origin of the high voltage using Faraday's Law of electromagnetic induction ($e = N\frac{d\phi}{dt}$, or $e = L\frac{di}{dt}$). Another way is to explain how it is the nature of an inductor to oppose any *change* in current over time. I'll leave it to you to figure out the exact words to say!

Answer 12

This inductor will drop 22.5 volts.

Answer 13

In real life, through, the inductor would drop more than 6 volts, due to winding resistance.

Follow-up question: research the typical winding resistance of a 4 henry inductor.

Answer 14

Series connection: 45 volts total. Parallel connection: 11.25 volts total.

Follow-up question: what do these figures indicate about the nature of series-connected and parallel connected inductors? In other words, what single inductor value is equivalent to two series-connected 5 H inductors, and what single inductor value is equivalent to two parallel-connected 5 H inductors?

Answer 15

25 mH

Answer 16

$L_{total} = 783.26$ mH

Note: the following formula is not the only possible one for describing total inductance in this circuit!

$$L_{total} = L_1 + \frac{L_2 L_3}{L_2 + L_3}$$

Answer 17

2.145 H

Answer 18

At the moment that a short-circuit fault occurs, the sudden increase in current constitutes a very large $\frac{di}{dt}$ value, which the inductor momentarily "opposes" by dropping voltage.

Follow-up question: why use an inductor to limit the short-circuit fault current? Why not use a resistor instead?

Answer 19

- Red, Grn, Brn, Gld = 250 μH, +/- 5%
- Wht, Org, Red, Sil = 9300 μH, +/- 10%
- Grn, Gry, Blk = 58 μH, +/- 20%
- Vio, Blu, Org, Gld = 76 mH, +/- 5%

Answer 20

A "healthy" inductor should register as a very low resistance between its terminals. If the inductor has an iron core, there should be infinite resistance (no continuity) between either winding terminal and the core.

Follow-up question: what do you suppose is the most likely failure "mode" of an inductor, open or shorted? Explain your answer.

Answer 21
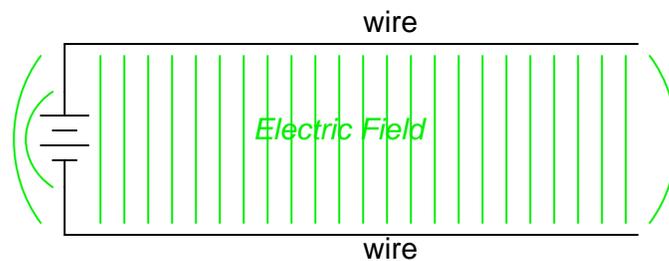
Follow-up question: explain how electric flux lines differ in geometry from magnetic flux lines.

Answer 22

"Static cling," where articles of clothing are attracted to one another after being dried in a machine, is an example of an electric field strong enough to produce tangible, physical attraction over a distance. Another, similar effect is that of peoples' hair standing on end prior to a lightning strike.

In both cases, what condition causes such a strong electric field to develop?

Answer 23

"Capacitance" is the capacity of two separated conductors to store energy in the form of an electric field, resulting from an applied voltage. You may also find a definition of "capacitance" stated in terms of opposition to change in applied voltage over time.

Capacitance is caused by the establishment of an electric field between two conductors.

Answer 24

If you calculated a distance in the order of 2 million meters (2×10^6 meters), you made a common mistake! The proper answer is 17.71 micro-meters (17.71×10^{-6} meters), or 0.01771 millimeters.

Answer 25

It is the nature of capacitance to store electrical charges, manifested in the form of static voltage. Testing for the presence of a stored charge between the two conductors of a cable would be one way to prove the existence of capacitance within the cable. I'll leave the details of testing for a stored electrical charge to you!

Answer 26

Field force is a direct function of applied voltage, and field flux is a direct function of stored charge.

If a sheet of glass is inserted between two metal plates connected to a constant voltage source, the electric field force between the plates will remain unchanged, while the electric field flux will increase (and along with it, the amount of charge stored on the plates).

Follow-up question: explain how the variable of *electric permittivity* is relevant to the described situation.

Answer 27

All I'll reveal here is that charge is directly proportional to both voltage *and* capacitance. This equation is an easy one to find on your own, by researching through various electronics textbooks!

Follow-up question: calculate the amount of charge stored in a 330 μF capacitor charged with a voltage of 12 volts.

Answer 28

As the applied voltage increases, the capacitor acts as a load, accumulating additional energy from the voltage source. Acting as a load, the current going "through" the capacitor will be in the same direction as through a resistor.

Voltage *increasing*

Currents shown using conventional flow notation
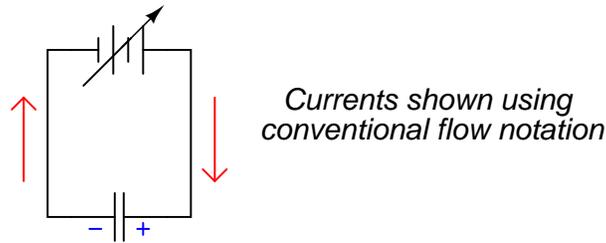
*Capacitor as a **load***

As the applied voltage decreases, the capacitor acts as a source, releasing accumulated energy to the rest of the circuit, as though it were a voltage source itself of superior voltage. Acting as a source, the current going "through" the capacitor will be in the same direction as through a battery, powering a load.

Voltage *decreasing*

Currents shown using conventional flow notation
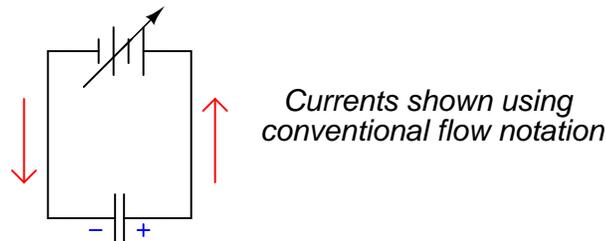
*Capacitor as a **source***

Answer 29

Lower-case variables represent *instantaneous* values, as opposed to average values. The expression $\frac{de}{dt}$, which may also be written as $\frac{dv}{dt}$, represents the *instantaneous rate of change of voltage over time*.

Follow-up question: manipulate this equation to solve for the other two variables ($\frac{de}{dt} = \cdots$; $C = \cdots$).

Answer 30

Capacitors oppose changes in **voltage**, reacting to such changes by producing a **current**.

Answer 31

Mica, electrolytic, mylar, paper, air, and ceramic are some of the more popular styles.

Answer 32

"Electrolytic" capacitors use a very thin layer of metal oxide as the dielectric. The manufacturing process involves applying a DC voltage to the new capacitor to create the oxide layer. This "polarizes" it for life, preventing its use with reverse polarity.

Follow-up question: identify the schematic symbol for any type of polarized capacitor, electrolytics included.

Answer 33

The capacitor's *working voltage* is 100 volts DC. Failure of the dielectric material will result from exceeding this voltage rating. In electrolytic capacitors especially, the failure can be violent!

Answer 34

Capacitors have the ability to store dangerous voltage and charge levels even when external energy sources have been disconnected.

Answer 35

This capacitor will have a constant current of 94 milliamps (mA) going "through" it. The word "through" is placed in quotation marks because capacitors have no continuity.

Answer 36

Series connection: 47 milliamps (mA) total. Parallel connection: 188 milliamps (mA) total.

Follow-up question: what do these figures indicate about the nature of series-connected and parallel connected capacitors? In other words, what single capacitor value is equivalent to two series-connected 470 μF capacitors, and what single capacitor value is equivalent to two parallel-connected 470 μF capacitors?

Answer 37

16.5 μF

Answer 38

132.998 pF

Follow-up question: suppose one of the terminal block's hold-down screws were to come loose on one of the leads for the middle capacitor, making a bad (open) connection. What effect would this have on the total capacitance?

0.33 µF
1000 V
(dipped)

.33
1 kV

1.0 µF
50 V
(dipped)

1.0
50 V

4700 MFD
36 V

4700 µF
36 V
(tubular)

10,000 pF
25 V
(molded)

103
25 V

47 pF
100 V
(dipped)

47
100 V

4.7 µF
16 V
(dipped)

4.7µ
16 V
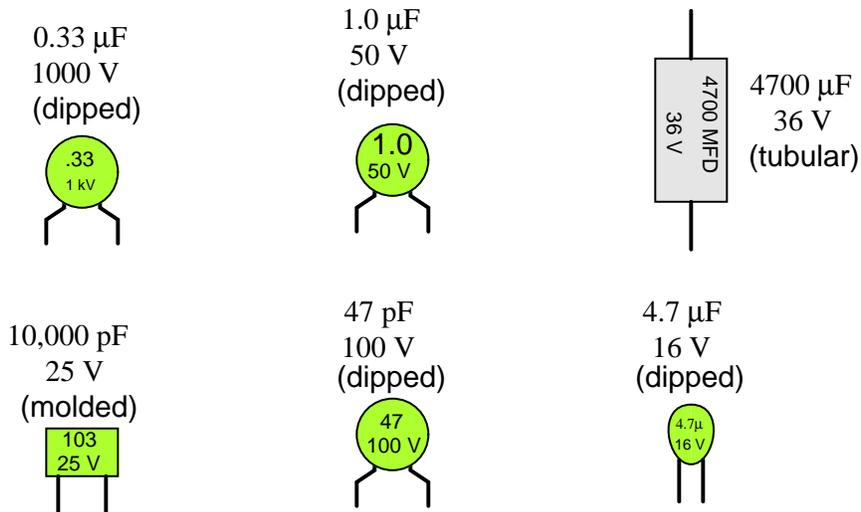
Answer 40

A "healthy" capacitor should register as an open circuit between its terminals, after the initial charging period.

Follow-up question: what do you suppose is the most likely failure "mode" of a capacitor, open or shorted? Explain your answer.

Answer 41

The capacitor's voltage will increase asymptotically over time when the switch is closed, with an ultimate voltage equal to the voltage of the source (battery).

Answer 42

The inductor's current will increase asymptotically over time when the switch is closed, with an ultimate current equal to the voltage of the source (battery) divided by the total resistance of the circuit.

Answer 43

E_C

I_C

Time ⟶

Switch
closes
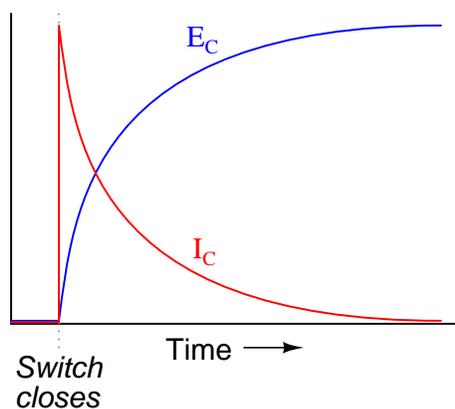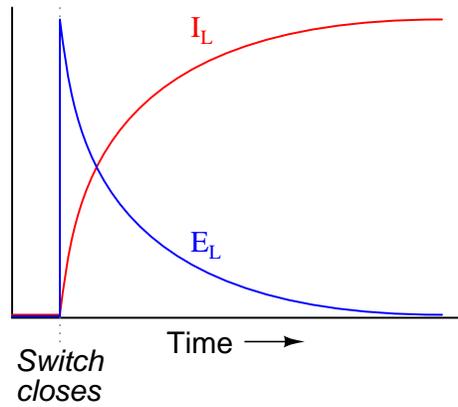
The percentage is 63.2% for each time constant.
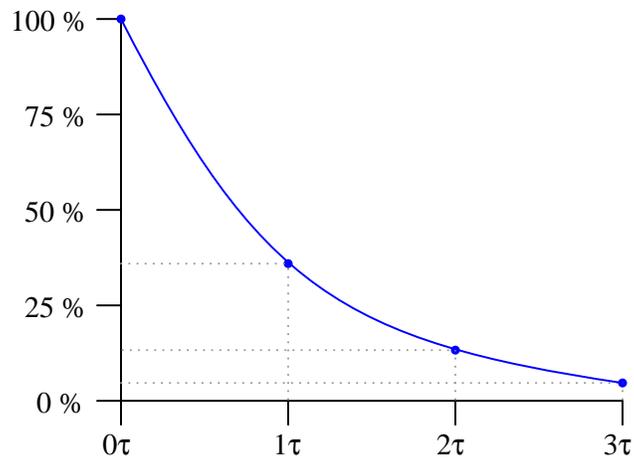
I will answer this question with another question: what value of R in each of these circuits will result in the greatest power dissipation at the load, large R or small R?
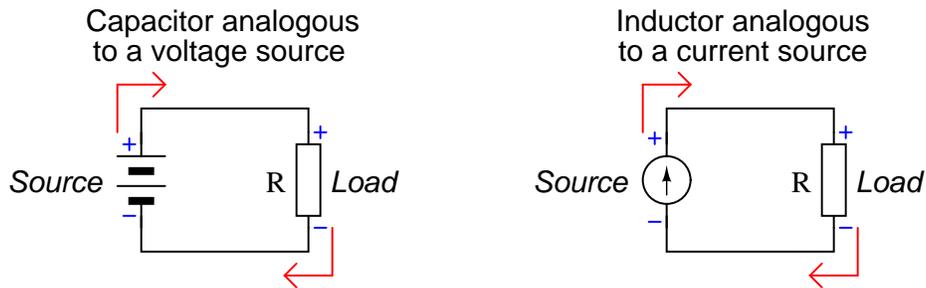
Large R values slow down RC circuits and speed up LR circuits, while small R values speed up RC circuits and slow down LR circuits.

Answer 47

Org, Blk, Org, Org, Vio

Answer 48

Org, Vio, Grn, Brn, Blu

Answer 49

If you said, "five time constants' worth" (5τ), you might not be thinking deeply enough! In actuality, the voltage and current in such a circuit *never* finally reach stable values, because their approach is asymptotic.

However, after 5 time constants' worth of time, the variables in an RC or LR circuit will have settled to within 0.6% of their final values, which is good enough for most people to call "final."

Answer 50

Yellow, Black, Green, Silver (assuming 5 time constants' worth of time: just less than 1% charge). Yellow, Orange, Green, Silver for a discharge down to 1% in 15 seconds.

Answer 51
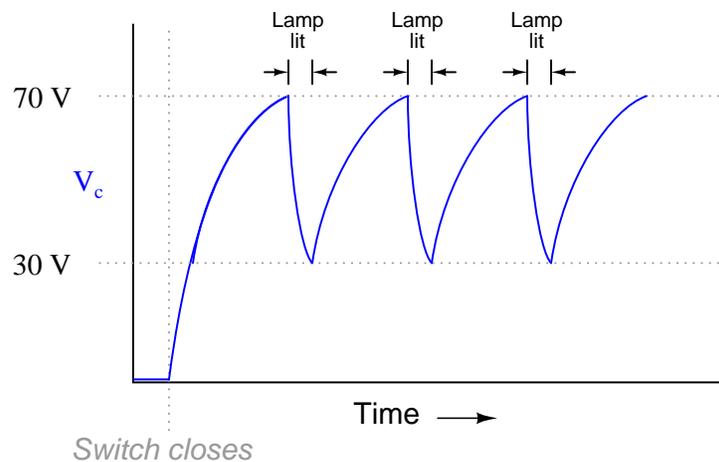
Follow-up question: assuming a source voltage of 100 volts, a resistor value of 27 kΩ, and a capacitor value of 22 μF, calculate the amount of time it takes for the capacitor to charge from 30 volts to 70 volts (assuming the neon bulb draws negligible current during the charging phase).

Answer 52
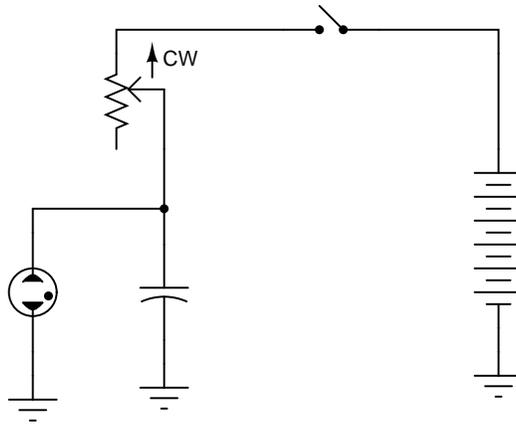
Answer 53

The following circuit does not meet all the design criteria, but it gives you an idea of where to begin in your design:
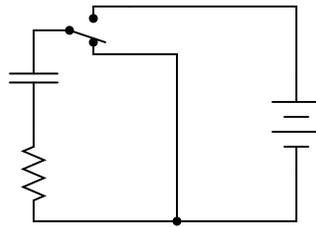
Answer 54

Wire #1 needs to be closest to the projectile source (the gun), while wire #2 needs to be further downrange.

Answer 55

$$e^{-x} = \frac{1}{e^x}$$

Answer 56

This equation models the percentage of pizza *remaining on the table* at time t, not how much has already been eaten.

Answer 57

Here is a hint: set x to zero and evaluate each equation.

Answer 58
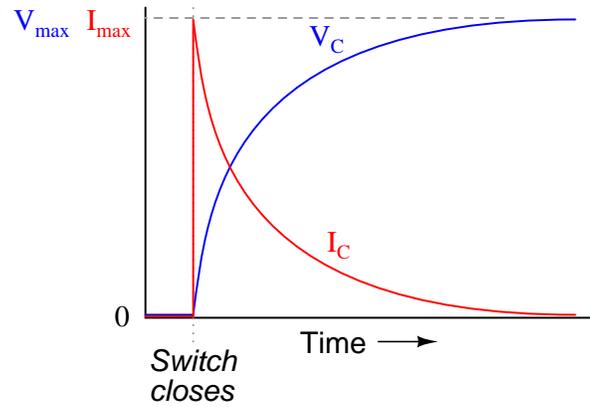
$$I_C = I_{max}\left(e^{-\frac{t}{\tau}}\right)$$

$$V_C = V_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Answer 59

$V_C = \underline{1.9915\ \text{volts}}$ @ $t = 3$ seconds

$I_C = \underline{995.74\ \mu\text{A}}$ @ $t = 3$ seconds
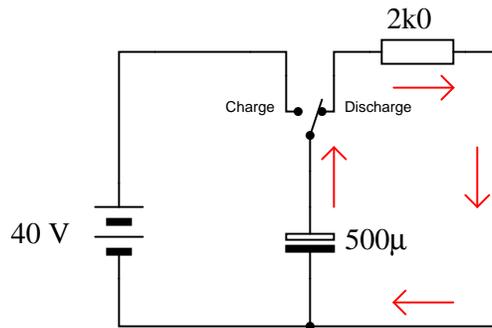
Arrows pointing in the direction
of conventional flow

Answer 60

$V_C = \underline{\text{-13.08 volts}}$ @ $t = 45$ ms

$I_C = \underline{236.6\ \mu\text{A}}$ @ $t = 45$ ms

Follow-up question: show the directions of charge and discharge current in this circuit.

Answer 61

Before the switch closes:

V_C = zero
V_R = zero
V_{switch} = maximum
I = zero

At the instant of switch closure:

V_C = zero
V_R = maximum
V_{switch} = zero
I = maximum

Long after the switch has closed:

V_C = maximum
V_R = zero
V_{switch} = zero
I = zero

Follow-up question: which of these variables remained the same immediately before and immediately after switch closure? Explain why.

Answer 62

Before the switch closes:

V_L = zero
V_R = zero
V_{switch} = maximum
I = zero

At the instant of switch closure:

V_L = maximum
V_R = zero
V_{switch} = zero
I = zero

Long after the switch has closed:

V_L = zero
V_R = maximum
V_{switch} = zero
I = maximum

Follow-up question: which of these variables remained the same immediately before and immediately after switch closure? Explain why.

Answer 63

$I_{switch-left}$ = 2 mA
$V_{switch-right}$ = 182 V

Follow-up question: suppose this circuit were built and tested, and it was found that the voltage developed across the inductor at the moment the switch moved to the right-hand position far exceeded 182 volts. Identify some possible problems in the circuit which could account for this excessive voltage.

Answer 64

I'll let you discuss this question with your classmates and instructor!

Answer 65

- 1 time constant (τ) after connecting the resistor: $V_C = 36.79$ volts
- 2 time constants (2τ) after connecting the resistor: $V_C = 13.53$ volts
- 3 time constants (3τ) after connecting the resistor: $V_C = 4.979$ volts
- 4 time constants (4τ) after connecting the resistor: $V_C = 1.832$ volts
- 5 time constants (5τ) after connecting the resistor: $V_C = 0.6738$ volts

Follow-up question: write an equation solving for these voltages at the specified times.

Answer 66

$E_C = 3.537$ volts @ $t = 9$ seconds.

9 s = 1.915 time constants (1.915τ)

Answer 67

$I_L = 109.11$ mA @ $t = 6$ milliseconds

6 ms = 2.4 time constants (2.4τ)

Answer 68

Time	V_C (volts)
0 ms	0
30 ms	12.29
60 ms	19.71
90 ms	24.19
120 ms	26.89
150 ms	28.52

Answer 69

94
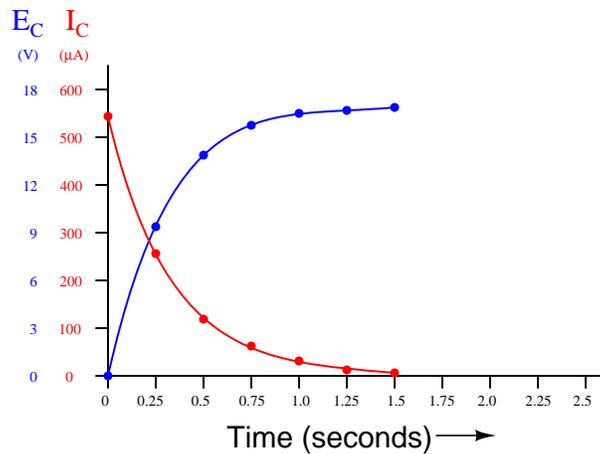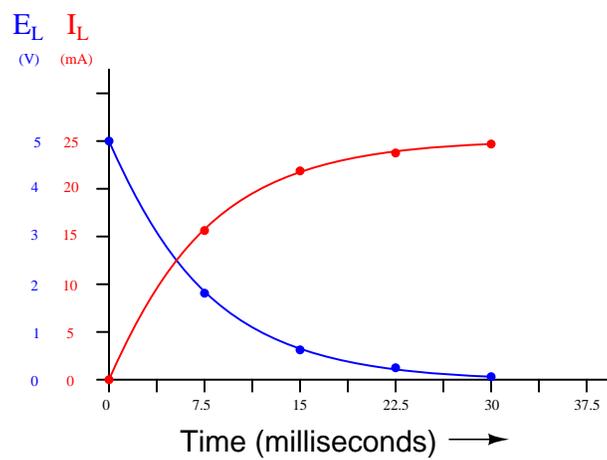
Answer 70

Answer 71

Showing all the necessary steps:

$$x = e^{-\frac{t}{\tau}}$$

$$\ln x = \ln \left(e^{-\frac{t}{\tau}} \right)$$

$$\ln x = -\frac{t}{\tau}$$

$$t = -\tau \ln x$$

Answer 72

0.591 seconds

Answer 73
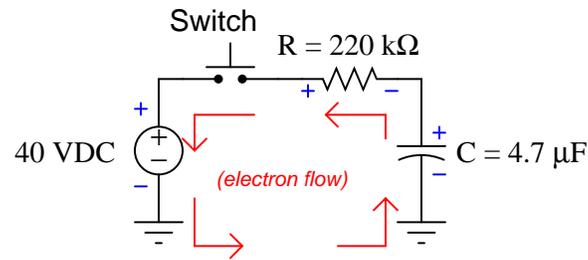
V_C	Time
0 volts	0 ms
10 volts	297.5 ms
20 volts	716.7 ms
30 volts	1.433 s
40 volts	> 5 s

Answer 74

$t_{delay} = 1.437$ seconds

Answer 75

I recommend the following circuit for testing the capacitor:
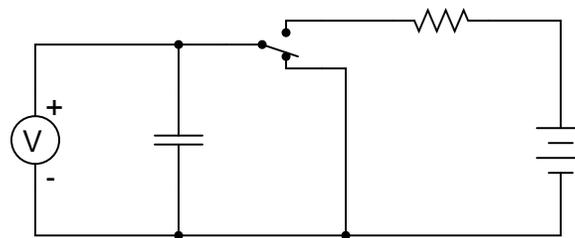
The equation is yours to develop – I will not reveal it here. However, this does not mean there is no way to verify the accuracy of your equation, once you write it. Explain how it would be possible to prove the accuracy of your algebra, once you have written the equation.

Answer 76

Figure 1:

$V_{C(initial)} = 0$ V (given) $V_{C(final)} = 15$ V
$I_{C(initial)} = 1.5$ mA $I_{C(final)} = 0$ mA
$V_{R(initial)} = 15$ V $V_{R(final)} = 0$ V
$I_{R(initial)} = 1.5$ mA $I_{R(final)} = 0$ mA

Figure 2:

$V_{C(initial)} = 25$ V (given) $V_{C(final)} = 0$ V
$I_{C(initial)} = 25$ mA $I_{C(final)} = 0$ mA
$V_{R(initial)} = 25$ V $V_{R(final)} = 0$ V
$I_{R(initial)} = 25$ mA $I_{R(final)} = 0$ mA

96

Figure 3:

$V_{C(initial)} = 4$ V $V_{C(final)} = 7$ V
$I_{C(initial)} = 81$ μA $I_{C(final)} = 0$ μA
$V_{R(initial)} = 3$ V $V_{R(final)} = 0$ V
$I_{R(initial)} = 81$ μA $I_{R(final)} = 0$ μA

Figure 4:

$V_{C(initial)} = 10$ V $V_{C(final)} = 12$ V
$I_{C(initial)} = 606$ μA $I_{C(final)} = 0$ μA
$V_{R(initial)} = 2$ V $V_{R(final)} = 0$ V
$I_{R(initial)} = 606$ μA $I_{R(final)} = 0$ μA

Follow-up question: explain why the inductor value (in Henrys) is irrelevant in determining "initial" and "final" values of voltage and current.

Answer 77

Figure 1:

$V_{L(initial)} = 15$ V $V_{L(final)} = 0$ V
$I_{L(initial)} = 0$ mA $I_{L(final)} = 1.5$ mA
$V_{R(initial)} = 0$ V $V_{R(final)} = 15$ V
$I_{R(initial)} = 0$ mA $I_{R(final)} = 1.5$ mA

Figure 2:

$V_{L(initial)} = 50.4$ V $V_{L(final)} = 0$ V
$I_{L(initial)} = 3.38$ mA $I_{L(final)} = 825$ μA
$V_{R(initial)} = 74.4$ V $V_{R(final)} = 18.1$ V
$I_{R(initial)} = 3.38$ mA $I_{R(final)} = 825$ μA

Figure 3:

$V_{L(initial)} = 370$ V $V_{L(final)} = 0$ V
$I_{L(initial)} = 10$ mA $I_{L(final)} = 0$ mA
$V_{R(initial)} = 370$ V $V_{R(final)} = 0$ V
$I_{R(initial)} = 10$ mA $I_{R(final)} = 0$ mA

Figure 4:

$V_{L(initial)} = 6$ V $V_{L(final)} = 0$ V
$I_{L(initial)} = 638$ μA $I_{L(final)} = 1.91$ mA
$V_{R(initial)} = 3$ V $V_{R(final)} = 9$ V
$I_{R(initial)} = 638$ μA $I_{R(final)} = 1.91$ mA

Follow-up question: explain why the inductor value (in Henrys) is irrelevant in determining "initial" and "final" values of voltage and current.

Answer 78

The term $V(t)$ uses symbolism common to calculus and pre-calculus, pronounced "V of t," meaning "voltage at time t". It means that the variable V changes as a *function* of time t.

$(V_f - V_0)$ represents the amount of *change* that the voltage would go through, from the start of the charge/discharge cycle until eternity. Note that the sign (positive or negative) of this term is very important!

$\left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right)$ is the fractional value (between 0 and 1, inclusive) expressing how far the voltage has changed from its initial value to its final value.

Follow-up question: why is it important to add the final V_0 term to the expression? Why not leave the formula at $V(t) = (V_f - V_0)\left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right)$?

Answer 79

$E_C = 3.974$ V @ 3 seconds

Follow-up question: identify at least one failure in this circuit which would cause the capacitor to remain completely discharged no matter what position the switch was in.

Answer 80

$E_{switch} = 40.91$ V @ $t = 0$ seconds
$E_{switch} = 9.531$ V @ $t = 15$ milliseconds

Follow-up question: predict all voltage drops in this circuit in the event that the inductor fails open (broken wire inside).

Answer 81

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 82

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 83

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 84

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 85

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 86

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 87

Use circuit simulation software to verify your predicted and measured parameter values.

Answer 88

I do not provide a grading rubric here, but elsewhere.

Answer 89

Be sure to document all steps taken and conclusions made in your troubleshooting!

Answer 90
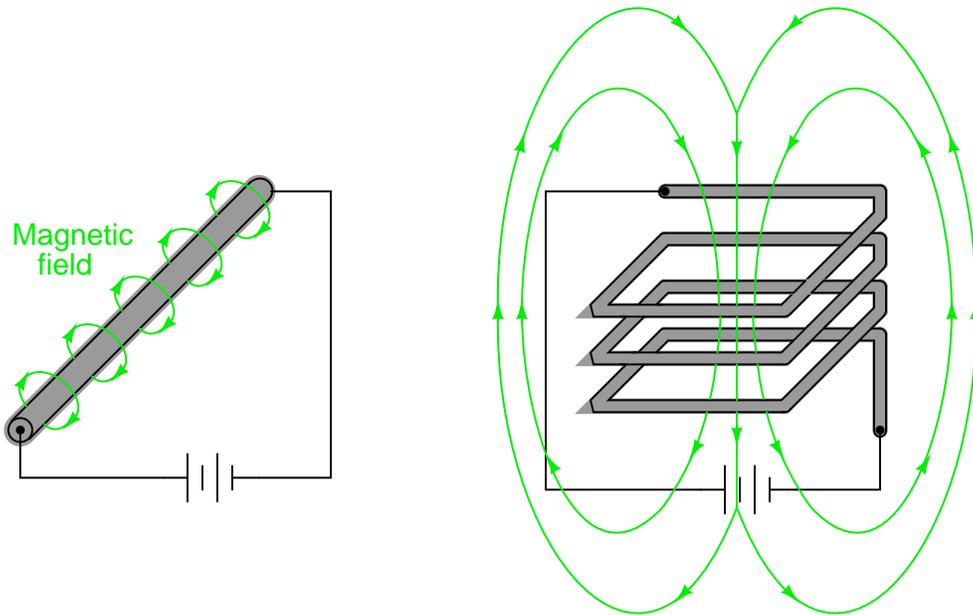
Answer 91

If N triples, then Φ triples, all other factors being equal.

If $\frac{d\phi}{dt}$ triples, then e triples, all other factors being equal.

If N triples, then L increases by a factor of *nine*, all other factors being equal.

Answer 92

I'll let you determine how an inductor is constructed, from your own research.

To increase inductance:

- Increase number of "turns" in coil
- Increase diameter of coil
- Decrease length of coil
- Increase permeability of core material

Answer 93

Field force is a direct function of coil current, and field flux is a direct function of stored energy.

If an iron bar is brought closer to a wire coil connected to a constant current source, the magnetic field force generated by the coil will remain unchanged, while the magnetic field flux will increase (and along with it, the amount of energy stored in the magnetic field).

Answer 94

Voltage remaining at logic gate terminals during current transient = 3.338 V

Answer 95

In an inductance, current is the time-integral of voltage. That is, the applied voltage across the inductor dictates the rate-of-change of current through the inductor over time.

Challenge question: can you think of a way we could exploit the similarity of inductive voltage/current integration to *simulate* the behavior of a water tank's filling, or any other physical process described by the same mathematical relationship?

Answer 96

As an object is subjected to a constant, unbalanced force, its velocity changes at a linear rate:

$$F = m\frac{dv}{dt}$$

Where,
F = Net force applied to object
m = Mass of object
v = Velocity of object
t = Time

In a similar manner, a pure inductance experiencing a constant voltage will exhibit a constant rate of current change over time:

$$e = L\frac{di}{dt}$$

Answer 97

As the core of an inductor becomes saturated with magnetic flux, there will be less change in flux for a given change in current (the derivative $\frac{d\phi}{di}$ will be less):
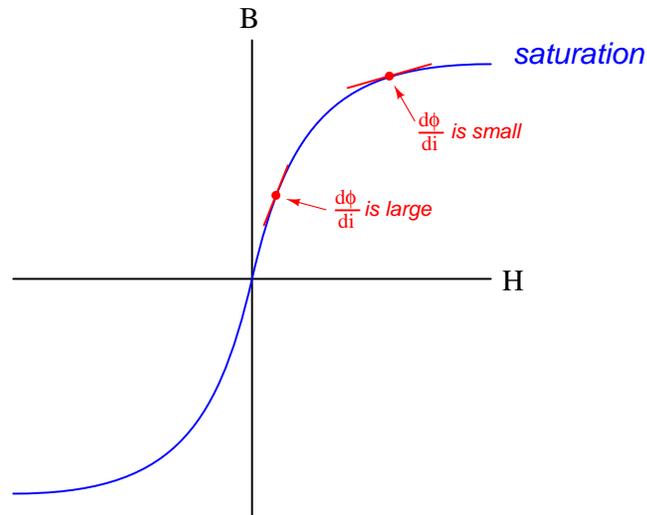
This causes the inductance to *decrease*.

Answer 98

I'll let you determine how a capacitor is constructed, from your own research.

To increase capacitance:

- Increase plate area
- Decrease spacing between plates
- Increase permittivity of dielectric

Answer 99

A "Leyden Jar" is a device used by early experimenters of static electricity to store electrical charges. It is made from a glass jar, lined inside and outside with metal foil. The glass insulates the two layers of metal foil from each other, and permits the storage of electric charge, manifested as a voltage between the two foil layers.

All capacitors share a common design feature of Leyden jars: the separation of two conductive plates by an insulating medium.

Answer 100

Electrical charge ≡ Water volume
Voltage ≡ Height of water column in vessel
Capacitance ≡ Area of vessel, measured on a cross-section with a horizontal plane

Answer 101

Rather than simply give you an answer here, I'll let you figure this out for yourself. Think very carefully about the water-in-a-vessel analogy when answering this question! Fill a glass with water, if necessary, to gain an intuitive understanding of these quantities.

Answer 102

In a capacitance, voltage is the time-integral of current. That is, the applied current "through" the capacitor dictates the rate-of-change of voltage across the capacitor over time.

Challenge question: can you think of a way we could exploit the similarity of capacitive voltage/current integration to *simulate* the behavior of a water tank's filling, or any other physical process described by the same mathematical relationship?

Answer 103

As a spring is compressed at a constant velocity, the amount of reaction force it generates increases at a linear rate:

$$v = \frac{1}{k}\frac{dF}{dt}$$

Where,

v = Velocity of spring compression
k = Spring "stiffness" constant
F = Reaction force generated by the spring's compression
t = Time

In a similar manner, a pure capacitance experiencing a constant current will exhibit a constant rate of voltage change over time:

$$i = C\frac{de}{dt}$$

Answer 104

The capacitor minimizes voltage transients seen at the amplifier's power terminals due to voltage drops along the power cables (from the battery) during transient current pulses, such as those encountered when amplifying heavy bass "beats" at high volume.

Incidentally, this same technique is used in computer circuitry to stabilize the power supply voltage powering digital logic circuits, which draw current from the supply in rapid "surges" as they switch between their "on" and "off" states. In this application, the capacitors are known as *decoupling* capacitors.

Answer 105
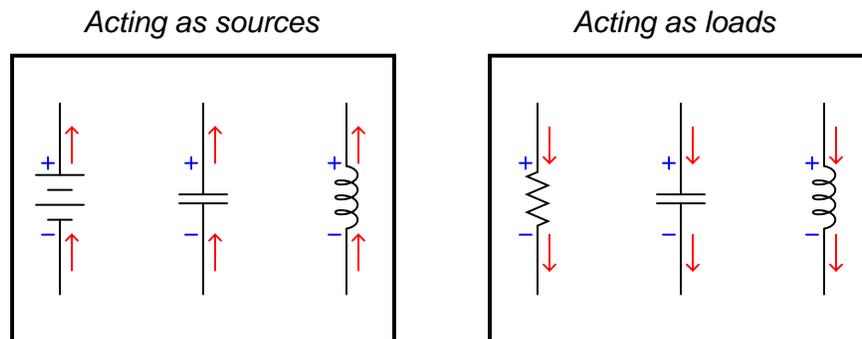
Follow-up question: what form does the stored energy take inside each type of reactive component? In other words, *how* does an inductor store energy, and *how* does a capacitor store energy?

Answer 106

Obviously the horn, relay, switch, and battery are all working normally. The capacitor could be bad (failed open), as well as the wiring/connections between the capacitor and the coil.

Answer 107

200 μC of charge.

Answer 108

"ESR" stands for *Equivalent Series Resistance*, usually being a consequence of the capacitor's dielectric substance.

Answer 109

Letter codes are used to designate *tolerance*, just like the last color band on most resistors. I will let you research the letter code equivalencies on your own! Same for the specific values of the four capacitor labels shown.

Answer 110

For the RC circuit:
- Larger capacitance = slower discharge
- Larger resistance = slower discharge

For the LR circuit:
- Larger inductance = slower discharge
- Larger resistance = faster discharge

Answer 111

The answer relates to this equation (of course!):

$$v = L\frac{di}{dt}$$

The inductor's "charge" is a direct function of current, the change of which (rate of charge) is directly proportional to the amount of voltage impressed across the inductor. In a Norton equivalent circuit, the output voltage is directly proportional to the internal resistance. So, a greater resistance results in a faster charging time.

Answer 112

$t = 1$ second ; $e^{-\frac{t}{\tau}} = 36.788\%$
$t = 2$ seconds ; $e^{-\frac{t}{\tau}} = 13.534\%$
$t = 3$ seconds ; $e^{-\frac{t}{\tau}} = 4.979\%$
$t = 4$ seconds ; $e^{-\frac{t}{\tau}} = 1.832\%$
$t = 5$ seconds ; $e^{-\frac{t}{\tau}} = 0.6738\%$
$t = 6$ seconds ; $e^{-\frac{t}{\tau}} = 0.2479\%$
$t = 7$ seconds ; $e^{-\frac{t}{\tau}} = 0.09119\%$
$t = 8$ seconds ; $e^{-\frac{t}{\tau}} = 0.03355\%$
$t = 9$ seconds ; $e^{-\frac{t}{\tau}} = 0.01234\%$
$t = 10$ seconds ; $e^{-\frac{t}{\tau}} = 0.004540\%$

Answer 113

$t = 1$ second ; $e^{-\frac{t}{\tau}} = 60.65\%$; increasing variable $= 39.35\%$
$t = 2$ seconds ; $e^{-\frac{t}{\tau}} = 36.79\%$; increasing variable $= 63.21\%$
$t = 3$ seconds ; $e^{-\frac{t}{\tau}} = 22.31\%$; increasing variable $= 77.69\%$
$t = 4$ seconds ; $e^{-\frac{t}{\tau}} = 13.53\%$; increasing variable $= 86.47\%$
$t = 5$ seconds ; $e^{-\frac{t}{\tau}} = 8.208\%$; increasing variable $= 91.79\%$
$t = 6$ seconds ; $e^{-\frac{t}{\tau}} = 4.979\%$; increasing variable $= 95.02\%$
$t = 7$ seconds ; $e^{-\frac{t}{\tau}} = 3.020\%$; increasing variable $= 96.98\%$
$t = 8$ seconds ; $e^{-\frac{t}{\tau}} = 1.832\%$; increasing variable $= 98.17\%$
$t = 9$ seconds ; $e^{-\frac{t}{\tau}} = 1.111\%$; increasing variable $= 98.89\%$
$t = 10$ seconds ; $e^{-\frac{t}{\tau}} = 0.6738\%$; increasing variable $= 99.33\%$

Answer 114

$$(1 - e^{-\frac{t}{\tau}})(100\%)$$

Answer 115

$t = 0$ seconds ; $E_C = 10$ V

$t = 1$ second ; $E_C = \frac{10}{3}$ V $= 3.33$ V

$t = 2$ seconds ; $E_C = \frac{10}{9}$ V $= 1.11$ V

$t = 3$ seconds ; $E_C = \frac{10}{27}$ V $= 0.370$ V

$t = 4$ seconds ; $E_C = \frac{10}{81}$ V $= 0.123$ V

$t = 5$ seconds ; $E_C = \frac{10}{243}$ V $= 0.0412$ V

Follow-up question: without using a calculator to check, determine whether these voltages are *over*-estimates or *under*-estimates.

Answer 116

$t = 0$ seconds ; $E_C = 0$ V
$t = 1$ second ; $E_C = \frac{20}{3}$ V $= 6.67$ V
$t = 2$ seconds ; $E_C = \frac{80}{9}$ V $= 8.89$ V
$t = 3$ seconds ; $E_C = \frac{260}{27}$ V $= 9.63$ V
$t = 4$ seconds ; $E_C = \frac{800}{81}$ V $= 9.88$ V
$t = 5$ seconds ; $E_C = \frac{2420}{243}$ V $= 9.96$ V

Follow-up question: without using a calculator to check, determine whether these voltages are *over*-estimates or *under*-estimates.

Answer 117

- RC circuit; R = 10 kΩ, C = 220 μF ; 7.5 sec = 3.41 τ
- RC circuit; R = 33 kΩ, C = 470 μF ; 7.5 sec = 0.484 τ
- RC circuit; R = 1.5 kΩ, C = 100 μF ; 7.5 sec = 50.0 τ
- RC circuit; R = 790 Ω, C = 9240 nF ; 7.5 sec = 1027 τ
- RC circuit; R = 100 kΩ, C = 33 pF ; 7.5 sec = 2,272,727 τ

- LR circuit; R = 100 Ω, L = 50 mH ; 7.5 sec = 15,000 τ
- LR circuit; R = 45 Ω, L = 2.2 H ; 7.5 sec = 153.4 τ
- LR circuit; R = 1 kΩ, L = 725 mH ; 7.5 sec = 10,345 τ
- LR circuit; R = 4.7 kΩ, L = 325 mH ; 7.5 sec = 108,462 τ
- LR circuit; R = 6.2 Ω, L = 25 H ; 7.5 sec = 1.86 τ

Answer 118

Usually, voltage is the electrical variable used to represent the physical quantity being modeled (in this case, temperature), so we need a circuit where voltage starts to increase rapidly, then gradually approaches a maximum value over time.

I won't show you a schematic diagram for the correct circuit, but I will give you several hints:

- The circuit uses a capacitor and a resistor.
- It is sometimes referred to as a *passive integrator* circuit.
- Engineers sometimes refer to it as a *first-order lag network*.
- You've seen this circuit before, just not in the context of modeling a physical process!

Answer 119

Capacitors are generally cheaper and easier to work with than inductors for making time delay circuits.

Answer 120

This equation models the percentage of pizza *eaten* at time t, not how much remains on the table.

Answer 121

Whether the variable in question is increasing or decreasing over time, the expression $e^{-\frac{t}{\tau}}$ describes the percentage that a variable has left to change before it reaches its final value.

Follow-up question: what could you add to or modify about the expression to make it describe the percentage that a variable has already changed from its initial value? In other words, alter the expression so that it is equal to 0% at $t = 0$ and approaches 100% as t grows larger ($t \to \infty$).

Answer 122

Adding the light bulb to this circuit *increased* the circuit's time constant, causing the coil's inductance to discharge at a slower rate.

Answer 123

Time	V_C (volts)	I_C (mA)
0 ms	0	4.255
30 ms	6.932	2.781
60 ms	11.46	1.817
90 ms	14.42	1.187
120 ms	16.35	0.7757
150 ms	17.62	0.5069

Answer 124

Time	V_L (volts)	I_L (mA)
0 μs	20	0
10 μs	11.89	6.238
20 μs	7.069	9.947
30 μs	4.203	12.15
40 μs	2.499	13.46
50 μs	1.485	14.24

Answer 125

Time	V_L (volts)	I_L (mA)
0 ns	12	0
250 ns	6.110	21.82
500 ns	3.111	32.92
750 ns	1.584	38.58
1.00 μs	0.8065	41.46
1.25 μs	0.4106	42.93

Answer 126

Time (μs)	V_L (V)	I_L (mA)
0	5.00	5.00
50	3.22	5.81
100	2.07	6.33
150	1.34	6.67
200	0.860	6.88
250	0.554	7.02
300	0.357	7.11
350	0.230	7.17
400	0.148	7.21

Answer 127

Equivalent schematic:
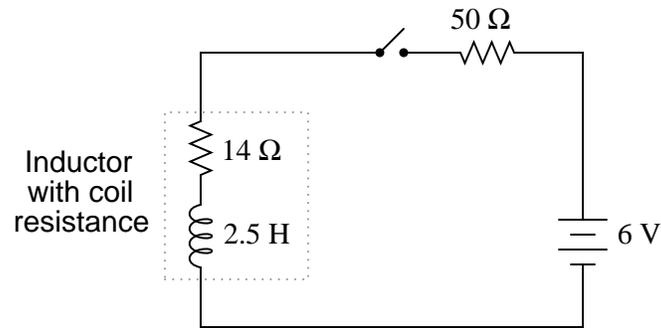
$E_L = 2.00$ V @ $t = 75$ milliseconds
75 ms = 1.92 time constants (1.92τ)

Answer 128

0.208 seconds

Answer 129

V_C	Time
0 volts	0 ms
-5 volts	29.75 ms
-10 volts	71.67 ms
-15 volts	143.3 ms
-19 volts	309.8 ms

While the capacitor is charging, electron flow moves clockwise and conventional flow moves counterclockwise.

Answer 130
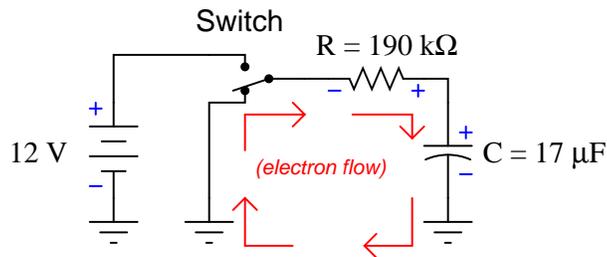
V_C	Time
10 volts	588.9 ms
8 volts	1.31 s
6 volts	2.24 s
4 volts	3.55 s
2 volts	5.79 s

Answer 131

Time to reach $10\% \approx 23$ minutes

Time to reach $1\% \approx 46$ minutes

Follow-up question: without using the time constant formula again, calculate how long it will take to discharge to 0.1% of its original voltage. How about 0.01%?

Answer 132

$$t_{charge} = -\ln 0.5 (R_1 + R_2) C$$

$$t_{discharge} = -\ln 0.5 R_2 C$$

Answer 133

$\frac{dv}{dt} = 4.44$ kV/s or 4.44 V/ms

Answer 134

$\frac{di}{dt} = 280$ A/s or 0.28 A/ms

Follow-up question: does the resistor value have any effect on this initial $\frac{di}{dt}$? Explain why or why not.

Answer 135

The 2 kV DC power source must be functioning properly, as is its current-limiting resistor (R_{limit1}). Possible failed components include:

- R_{limit2} failed open
- Inductor failed open or shorted
- Pushbutton switch failed open or shorted
- 24 VDC supply failed
- Air gap switch "ionizing" needle tip burnt or otherwise worn

Follow-up question: explain how one would *safely* continue diagnostic measurements in a circuit such as this where there is much potential for electric shock and other hazards.

Answer 136

$$U = \frac{1}{2} L I^2$$

Answer 137

$$U = \frac{1}{2} C V^2$$

$$30 - \frac{dQ}{dt}R - \frac{Q}{C} = 0$$

$$30 - \frac{Q}{C} = \frac{dQ}{dt}R$$

$$\frac{30 - \frac{Q}{C}}{R} = \frac{dQ}{dt}$$

$$\frac{\frac{30C}{C} - \frac{Q}{C}}{R} = \frac{dQ}{dt}$$

$$\frac{30C - Q}{RC} = \frac{dQ}{dt}$$

$$\frac{dt}{RC} = \frac{dQ}{30C - Q}$$

$$\int \frac{1}{RC}\, dt = \int \frac{1}{30C - Q}\, dQ$$

Substitution: $u = 30C - Q$; $\dfrac{du}{dQ} = -1$; $dQ = -du$

$$\frac{1}{RC} \int dt = -\int \frac{1}{u}\, du$$

$$\frac{t}{RC} + K_1 = -|\ln u|$$

$$-\frac{t}{RC} - K_1 = |\ln u|$$

$$e^{-\frac{t}{RC} - K_1} = |u|$$

$$K_2 e^{-\frac{t}{RC}} = u$$

$$K_2 e^{-\frac{t}{RC}} = 30C - Q$$

General solution: $Q = 30C - K_2 e^{-\frac{t}{RC}}$

Given the initial condition that the charge stored in the capacitor is zero ($Q = 0$) at time zero ($t = 0$), the constant of integration must be equal to $30C$ in our specific solution:

$$Q = 30C - 30Ce^{-\frac{t}{RC}}$$

$$Q = 30C(1 - e^{-\frac{t}{RC}})$$

Substituting the given component values into this specific solution gives us the final equation:

$$Q = 0.0003(1 - e^{-50t})$$

Showing a final equation in terms of capacitor voltage instead of capacitor charge:

$$\frac{Q}{C} = \frac{30C}{C}(1 - e^{-\frac{t}{RC}})$$

$$V_C = 30(1 - e^{-50t})$$

Answer 139

$$40 - IR - L\frac{dI}{dt} = 0$$

$$40 - IR = L\frac{dI}{dt}$$

$$\frac{40 - IR}{L} = \frac{dI}{dt}$$

$$\frac{dt}{L} = \frac{dI}{40 - IR}$$

$$\int \frac{1}{L}\, dt = \int \frac{1}{40 - IR}\, dI$$

Substitution: $u = 40 - IR$; $\frac{du}{dI} = -R$; $dI = -\frac{1}{R}du$

$$\frac{1}{L}\int dt = -\frac{1}{R}\int \frac{1}{u}\, du$$

$$\frac{t}{L} + K_1 = -\frac{1}{R}|\ln u|$$

$$-\frac{tR}{L} + K_2 = |\ln u|$$

$$e^{-\frac{tR}{L}+K_2} = |u|$$

$$K_3 e^{-\frac{tR}{L}} = u$$

$$K_3 e^{-\frac{tR}{L}} = 40 - IR$$

$$IR = 40 - K_3 e^{-\frac{tR}{L}}$$

$$I = \frac{40}{R} - K_4 e^{-\frac{tR}{L}}$$

Given the initial condition of zero current ($I = 0$) at time zero ($t = 0$), the constant of integration must be equal to $\frac{40}{R}$ in our specific solution:

$$I = \frac{40}{R} - \frac{40}{R}e^{-\frac{tR}{L}}$$

$$I = \frac{40}{R}(1 - e^{-\frac{tR}{L}})$$

Substituting the given component values into this specific solution gives us the final equation:

$$I = 0.8(1 - e^{-25t})$$

Notes

Notes 1

Self-induction is not a difficult concept to grasp if one already possesses a good understanding of electromagnetism, electromagnetic induction, and Lenz's Law. Some students may struggle understanding self-induction, because it is probably the first application they've seen where these three phenomena inter-relate simultaneously.

Notes 2

Ask students what unit of measurement inductance is expressed in. Also, ask them if they think the inductance of any given conductor changes with the applied current or stored energy, or if inductance is a quantity independent of particular electrical conditions.

Notes 3

I have found that the topics of capacitance and inductance are excellent contexts in which to introduce fundamental principles of calculus to students. The time you spend discussing this question and questions like it will vary according to your students' mathematical abilities.

Even if your students are not ready to explore calculus, it is still a good idea to discuss how the relationship between current and voltage for an inductance involves *time*. This is a radical departure from the time-independent nature of resistors, and of Ohm's Law!

Notes 4

Students who do not yet understand the concept of inductance may be inclined to suggest that the current in this circuit will be infinite, following Ohm's Law ($I = E/R$). One of the purposes of this question is to reveal such misunderstandings, so that they may be corrected.

This circuit provides an excellent example of the calculus principle *integration*, where the application of a steady voltage across the inductor results in a steadily *increasing* current. Whether or not you should touch on this subject depends on the mathematical aptitude of your students.

Notes 5

This problem is first and foremost an algebraic manipulation exercise: solving for N given the values of the other variables. Students should be able to research the value of μ_0 quite easily, being a well-defined physical constant.

Note that in this equation, the Greek letter "mu" (μ) is not a metric prefix, but rather an actual variable! This confuses many students, who are used to interpreting μ as the metric prefix "micro" ($\frac{1}{1,000,000}$).

Note also how the re-written equation puts pi (π) ahead of all the variables in the numerator of the fraction. This is not absolutely necessary, but it is conventional to write constants before variables. Do not be surprised if some students ask about this, as their answers probably looked like this:

$$L = \frac{N^2 \pi r^2 \mu}{l}$$

Notes 6

This type of distinction is very important in the study of physics as well, where one must determine whether a mechanical system is *doing work* or whether *work is being done on it*. A clear understanding of the relationship between voltage polarity and current direction for sources and loads is very important for students to have before they study reactive devices such as inductors and capacitors!

Notes 7

Relating the polarity of voltage across an inductor to a change of applied current over time is a complex concept for many students. Since it involves rates of change over time, it is an excellent opportunity to introduce calculus concepts ($\frac{d}{dt}$).

Vitally important to students' conceptual understanding of an inductor exposed to increasing or decreasing currents is the distinction between an electrical energy *source* versus a *load*. Students need to think "battery" and "resistor," respectively when determining the relationship between direction of current and voltage drop. The complicated aspect of inductors (and capacitors!) is that they may switch character in an instant, from being a source of energy to being a load, and visa-versa. The relationship is not fixed as it is for resistors, which are always energy *loads*.

Notes 8

Emphasize to your students that inductance is an essentially *reactive* property, opposing change in current over time. It is not steady current that inductors react to, only changing current.

Notes 9

This technique is very useful in reducing or eliminating parasitic inductance. Typically, parasitic inductance is not a problem unless very high rates of current change are involved, such as in high-frequency AC circuits (radio, high-speed digital logic, etc.). In such applications, knowing how to control stray inductance is very important to proper circuit operation.

Notes 10

In high-frequency AC circuits, such as computer circuits where pulses of voltage oscillate at millions of cycles per second, even short lengths of wire or traces on a circuit board may present substantial trouble by virtue of their stray inductance. Some of this parasitic inductance may be reduced by proper circuit board assembly, some of it by a re-design of component layout on the circuit board.

According to an article in IEEE Spectrum magazine ("*Putting Passives In Their Place*", July 2003, Volume 40, Number 7, page 29), the transient currents created by fast-switching logic circuits can be as high as 500 amps/ns, which is a $\frac{di}{dt}$ rate of 500 *billion* amps per second!! At these levels, even a few picohenrys of parasitic inductance along component leads and circuit board traces will result in significant voltage drops.

Notes 11

One way to help understand how an inductor could produce such large voltages is to consider it as a *temporary current source*, which will output as much voltage as necessary in an effort to maintain constant current. Just as ideal current sources are dangerous to open-circuit, current-carrying inductors are likewise capable of generating tremendous transient voltages.

Although there was no real safety hazard with my experiment, there potentially could have been, provided different circumstances. Discuss with your students what would have been necessary to create an actual safety hazard.

Notes 12

Don't give your students the equation with which to perform this calculation! Let them find it on their own. The $\frac{di}{dt}$ notation may be foreign to students lacking a strong mathematical background, but don't let this be an obstacle to learning! Rather, use this as a way to introduce those students to the concept of *rates of change*, and to the calculus concept of the *derivative*.

Notes 13

Ahhh, the controversy of conventional versus electron flow. The existence of two contradicting conventions for denoting direction of electric current irritates me to no end, especially when the one upon which almost all electronic device symbolism is based on is actually incorrect with regard to charge flow through metallic conductors (the majority case in electric circuits)! Your students will surely encounter both "conventional" and "electron" flow in their careers, so be sure to introduce them to *both* conventions.

Discuss with your students the consequences of winding resistance in real inductors. Is it significant? Work together with your students to calculate how much extra voltage would be dropped across the inductor, based on their research on the typical winding resistance of a 4 henry inductor (ask them where they obtained the information!), given a $\frac{di}{dt}$ rate of 1.5 amps per second.

Ask your students if they think it might be possible to create an inductor with no "stray" resistance at all to interfere with perfect, theoretical inductor behavior. What would be required to make the "perfect" inductor?

Notes 14

If your students are having difficulty answering the follow-up question in the Answer, ask them to compare these voltage figures (45 V and 11.25 V) against the voltage that would be dropped by just one of the 5 H inductors under the same condition (an applied current changing at a rate of 4.5 amps per second).

It is, of course, important that students know how series-connected and parallel connected inductors behave. However, this is typically a process of rote memorization for students rather than true understanding. With this question, the goal is to have students come to a realization of inductor connections based on their understanding of series and parallel voltages and currents.

Notes 15

Ask your students this question: given capacitors and inductors, which of these component types behave more like resistors, in terms of their series versus parallel equivalent values?

Notes 16

It is most helpful to first draw a schematic diagram for this inductor network before trying to perform any inductance calculations, in order that a clear understanding of the series/parallel connections be established.

Notes 17

Inductors tend to be even more scarce than resistors in regard to the availability of different values, so knowing how to combine different inductors together to achieve a desired total inductance is a practical skill.

It should be noted to your students that inductors may interact with one another if placed too close, due to the magnetic field of one inductor "linking" with the coil of another inductor. This is called *mutual inductance*, and if present alters the otherwise straightforward series-parallel inductance relationships that one would expect to be analogous to series-parallel resistances.

Notes 18

Power system reactors are usually installed in substations, where they appear as coils of wire (no iron core) a few feet in diameter, usually located near circuit breakers.

Notes 19

Students will note a lot of similarities between the inductor color code and the resistor color code, but they should also note one significant difference!

Notes 20

Have your students actually test a few inductors with their ohmmeters in class. Unlike capacitor checking with an ohmmeter, there is never a substantial "charging" period! If your students have already learned about capacitor checking with an ohmmeter, ask them to explain why there is no "charging" action indicated by the ohmmeter when connected to an inductor.

If there were an observable "charging" time exhibited by an inductor when measured by an ohmmeter, what would it appear as, in terms of the ohmmeter's indication?

Normally, I don't give away answers to follow-up questions in the "Notes" section, but here I feel it may be necessary. Studies have shown that inductors have about an equal chance of failing open as they do failing shorted. Of course, this will vary with the specific design and application of the inductor, but there is no mode of failure clearly more probable than the other.

Notes 21

Students may note that electric lines of flux do not follow the same paths that magnetic lines of flux would. Whereas magnetic lines of flux are always circular, electric lines of flux always terminate between points.

Note to your students the relevance of this fact in shielding: unlike magnetic shields which must *divert* the inevitable paths of magnetic flux lines, electric shields are able to *terminate* electric flux lines.

Notes 22

Electric field force is also used in some precision voltage measuring instruments ("electrostatic" meter movements), as well as the more common *electroscope*. If you happen to have either an electrostatic meter movement or an electroscope available in your classroom, use it to demonstrate the physical effects of electric fields.

Notes 23

Ask students what unit of measurement capacitance is expressed in. Also, ask them if they think the capacitance of any given conductor pair changes with the applied voltage or stored energy, or if capacitance is a quantity independent of particular electrical conditions.

Notes 24

This problem is first and foremost an algebraic manipulation exercise. Then, it is merely a matter of solving for d given the proper values. Finding ϵ could be difficult, though, and this is by design: I want students to learn the significance of *absolute* permittivity!

Notes 25

The purpose of this question is to make students think critically and creatively about capacitance. There is more than one way to test for capacitance in a cable, so do not limit students to one method only!

Notes 26

The concept of a *field* is quite abstract. Electric fields in particular are abstract because they cannot be tangibly perceived, at least not outside of dangerous voltage levels. Magnetic fields, which everyone should be familiar with from playing with magnets, may serve as an illustration of fields in general, but it is very important for students of electricity and electronics to understand that electric and magnetic fields are two different entities, albeit closely related (by Maxwell's Laws).

Notes 27

This is one of those equations usually discussed somewhere in the first few months of basic electronics education, and promptly forgotten by most. It can be very useful, however, when dealing with charge pumps and other switched-capacitor circuitry.

Notes 28

Relating the direction of current in a capacitor to a change of applied voltage over time is a complex concept for many students. Since it involves rates of change over time, it is an excellent opportunity to introduce calculus concepts ($\frac{d}{dt}$).

Vitally important to students' conceptual understanding of a capacitor exposed to increasing or decreasing voltages is the distinction between an electrical energy *source* versus a *load*. Students need to think "battery" and "resistor," respectively when determining the relationship between direction of current and voltage drop. The complicated aspect of capacitors (and inductors!) is that they may switch character in an instant, from being a source of energy to being a load, and visa-versa. The relationship is not fixed as it is for resistors, which are always energy *loads*.

Notes 29

I have found that the topics of capacitance and inductance are excellent contexts in which to introduce fundamental principles of calculus to students. The time you spend discussing this question and questions like it will vary according to your students' mathematical abilities.

Even if your students are not ready to explore calculus, it is still a good idea to discuss how the relationship between current and voltage for a capacitance involves *time*. This is a radical departure from the time-independent nature of resistors, and of Ohm's Law!

Notes 30

Emphasize to your students that capacitance is an essentially *reactive* property, opposing change in voltage over time. It is not steady voltage that capacitors react to, only changing voltage.

Notes 31

There is much to be researched on different capacitor types and characteristics! Encourage your students to research both textbooks and capacitor manufacturer literature for more information.

Notes 32

There are many features of electrolytic capacitors unique to that type, not the least of which being their means of manufacture. Since these capacitors are used so often for low-voltage electronic applications, it is well worth the students' time to study them well and know their idiosyncrasies.

Notes 33

As with resistors and resistance ratings, there is more to the rating of a capacitor than merely capacitance! Discuss with your students the importance of safety when working with capacitors, not only from shock hazard but also from explosions (resulting from excessive voltage).

Notes 34

An interesting follow-up question to pose would be: how do we safely discharge a capacitor charged with dangerous levels of voltage?

Notes 35

Don't give your students the equation with which to perform this calculation! Let them find it on their own. The $\frac{dv}{dt}$ notation may be foreign to students lacking a strong mathematical background, but don't let this be an obstacle to learning! Rather, use this as a way to introduce those students to the concept of *rates of change*, and to the calculus concept of the *derivative*.

Notes 36

If your students are having difficulty answering the follow-up question in the Answer, ask them to compare these current figures (47 mA and 188 mA) against the current that would go through just one of the 470 μF capacitors under the same condition (an applied voltage changing at a rate of 200 volts per second).

It is, of course, important that students know how series-connected and parallel connected capacitors behave. However, this is typically a process of rote memorization for students rather than true understanding. With this question, the goal is to have students come to a realization of capacitor connections based on their understanding of series and parallel voltages and currents.

Notes 37

Capacitors often confuse new students of electronics because their values do not add up the same as resistors. It is important in answering this question that your students understand *why* series capacitances combine as they do. There is more than one way to explain this phenomenon – explain in terms of capacitor dimensions, or in terms of voltage drop and charge storage.

Notes 38

It is most helpful to first draw a schematic diagram for this capacitor network before trying to perform any capacitance calculations, in order that a clear understanding of the series/parallel connections be established.

Notes 39

Ask your students how they can tell whether a capacitor's value is given in *micro*farads or *pico*farads. There is a way, even if the metric prefix is not printed on the capacitor!

Notes 40

Have your students actually test a few capacitors with their ohmmeters in class. For large capacitor values, the charging time may be substantial! Students need to be aware of this, and the effect it has on the ohmmeter's indication.

Although an ohmmeter check of a capacitor is not a comprehensive analysis, it is certainly better than nothing, and will detect the more common faults.

Notes 41

A qualitative analysis of this circuit's behavior may be performed without using any calculus, or even algebra. Ask students to explain what happens to current in the circuit as the capacitor's voltage begins to increase, and then what effect that has on the rate of voltage rise, and so on, graphing the results for all too see.

Hint: the rate of voltage rise across a capacitor is in direct proportion to the quantity of current going "through" the capacitor.

Notes 42

A qualitative analysis of this circuit's behavior may be performed without using any calculus, or even algebra. Ask students to explain what happens to the voltage across the inductor as the circuit current beings to increase, and what effect that has on the rate of current rise, and so on, graphing the results for all too see.

Hint: the rate of current rise through an inductor is in direct proportion to the quantity of voltage dropped across the inductor.

Notes 43

Have your students explain why the voltage and current curves are shaped as they are.

Notes 44

Have your students explain why the voltage and current curves are shaped as they are.

Notes 45

I like to use the example of radioactive decay to introduce time constants, because it seems most people have at least heard of something called "half-life," even if they don't know exactly what it is.

Incidentally, the choice to measure decay in either "half lives" or "time constants" is arbitrary. The curve for radioactive decay is the exact same curve as that of an RC or LR discharge process, and is characterized by the same differential equation:

$$\frac{dQ}{dt} = -kQ$$

Where,

Q = Decaying variable (grams of substance, volts, amps, whatever)
k = Relative decay rate
t = Time

By solving this separable differential equation, we naturally arrive at an equation expressing Q in terms of an exponential function of e:

$$Q = Q_0 e^{-kt}$$

Thus, it makes more sense to work with units of "time constants" based on e than with "half lives," although admittedly "half-life" is a concept that makes more intuitive sense. Incidentally, 1 half-life is equal to 0.693 time constants, and 1 time constant is equal to 1.443 half-lives.

Notes 46

The leap I expect students to make here is to think about the exchange of energy for both capacitors and inductors as they deliver power to a resistive load. Which ever values of R make the respective sources work the hardest (dissipate the most power at the load) will be the values that make the reactive components discharge quickest.

Notes 47

In order for students to answer this question, they must research the RC time constant equation and review the 5-band resistor color code.

Notes 48

In order for students to answer this question, they must research the LR time constant equation and review the 5-band resistor color code.

Notes 49

The stock answer of "5 time constants" as the amount of time elapsed between the transient event and the "final" settling of voltage and current values is widespread, but largely misunderstood. I've encountered more than a few graduates of electronics programs who actually believe there is something special about the number 5, as though everything grinds to a halt at exactly 5 time constants worth of time after the switch closes.

In reality, the rule of thumb of "5 time constants" as a settling time in RC and LR circuits is an approximation only. Somewhere I recall reading an old textbook that specified *ten* time constants as the time required for all the variables to reach their final values. Another old book declared *seven* time constants. I think we're getting impatient as the years roll on!

Notes 50

In order to answer this question, students must not only be able to calculate time constants for a simple RC circuit, but they must also remember the resistor color code so as to choose the right size based on color. A very practical problem, and important for safety reasons too!

Notes 51

What we have here is a very simple strobe light circuit. This circuit may be constructed in the classroom with minimal safety hazard if the DC voltage source is a hand-crank generator instead of a battery bank or line-powered supply. I've demonstrated this in my own classroom before, using a hand-crank "Megger" (high-range, high-voltage ohmmeter) as the power source.

Notes 52

Ask your students to explain *why* the potentiometer has the speed-changing effect it does on the circuit's flash rate. Would there be any other way to change this circuit's flash rate, without using a potentiometer?

Notes 53

This is an interesting question, as the students are challenged with the design of a circuit, and that the example circuit shown in the answer does not completely fulfill the specified criteria. In order for a student to derive a circuit from the one given in the answer, they must first assess that circuit's operation, to determine what it can and cannot do.

Note that it will not suffice to merely replace the fixed-value resistor with a variable resistor, as this would not provide *independent* adjustment of charging and discharging time constants.

Notes 54

This is an interesting question because it requires the students to reason through the function of a practical measuring circuit. Not only must students grasp the charge/discharge behavior of a capacitor, but they also must relate it to the practical purpose of the intervalometer, recognizing the importance of voltmeter characteristics as well. Expect substantial discussion on this question.

Notes 55

I am usually not a fan of discussing calculator keystroke sequences to college-level students, but I have resorted to this at the college where I teach because so many of my students have no idea of how to evaluate exponential expressions! Of course, the actual keystrokes one must push to evaluate this expression depend on the brand of calculator used.

Notes 56

While some may wonder what this question has to do with electronics, it is an exercise in qualitative analysis. This skill is very important for students to master if they are to be able to distinguish between the equations $e^{-\frac{t}{\tau}}$ and $1 - e^{-\frac{t}{\tau}}$, both used in time-constant circuit analysis.

The actual procedure for determining the nature of the equation is simple: consider what happens as t begins at 0 and as it increases to some arbitrary positive value. Some students may rely on their calculators, performing actual calculations to see whether the percentage increases or decreases with increasing t. Encourage them to analyze the equation qualitatively rather than quantitatively, though. They should be able to tell which way the percentage changes with time without having to consider a single numerical value!

Notes 57

It is very important for students to understand what this expression means and how it works, lest they rely solely on memorization to use it in their calculations. As I always tell my students, rote memorization *will* fail you! If a student does not comprehend why the expression works as it does, they will be helpless to retain it as an effective "tool" for performing calculations in the future.

Notes 58

Have your students explain why the voltage and current curves are shaped as they are.

Notes 59

Here, students must choose which equation(s) to use for the calculation, calculate the time constant for the circuit, and put all the variables in the right place to obtain the correct answers. Discuss all these steps with your students, allowing them to explain how they approached the question.

If anyone asks, let them know that the capacitor symbol shown represents a polarized (electrolytic) capacitor.

Notes 60

Here, students must choose which equation(s) to use for the calculation, calculate the time constant for the circuit, and put all the variables in the right place to obtain the correct answers. Discuss all these steps with your students, allowing them to explain how they approached the question.

If anyone asks, let them know that the capacitor symbol shown represents a polarized (electrolytic) capacitor.

Notes 61

The purpose of this question is to preview the concept of "initial" and "final" values in RC circuits, before they learn to use the "universal time constant formula."

Notes 62

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Notes 63

The main purpose of this question is to get students thinking in terms of "initial" and "final" values for LR circuits, and how one might calculate them. It is largely a conceptual question, with just a bit of calculation necessary.

One practical application of this circuit is for "stepping up" DC voltage. The circuit topology shown in the question is that of an *inverting* converter circuit. This form of DC-DC converter circuit has the ability to step voltage up *or* down, depending on the duty cycle of the switch's oscillation.

Notes 64

This is advice I always give my students, after seeing so many students get themselves into trouble by blindly plugging numbers into equations. *Think* before you act, is the motto here!

Actually, this general advice applies to most all physics-type problems: identify what it is you're trying to solve and what you have to work with before jumping into calculations.

Notes 65

Although students should be able to look up approximate answers to this question from almost any beginning electronics textbook, the point here is to get them to relate the question to an actual formula so they may calculate this on their own.

Notes 66

Here, students must choose which equation to use for the calculation, calculate the time constant for the circuit, and put all the variables in the right place to obtain the correct answer. Discuss all these steps with your students, allowing them to explain how they approached the question.

Notes 67

Here, students must choose which equation to use for the calculation, calculate the time constant for the circuit, and put all the variables in the right place to obtain the correct answer. Discuss all these steps with your students, allowing them to explain how they approached the question.

Notes 68

Be sure to have your students share their problem-solving techniques (how they determined which equation to use, etc.) in class.

Notes 69

I intentionally left the graph unscaled in the problem, so that students might determine their own scales to plot the points in. The scaling shown in the answer is obviously not ideal, as the graphs have reached their terminal values (for all practical purposes) well before the horizontal axis is complete.

Notes 70

I intentionally left the graph unscaled in the problem, so that students might determine their own scales to plot the points in. The scaling shown in the answer is obviously not ideal, as the graphs have reached their terminal values (for all practical purposes) well before the horizontal axis is complete.

In my experience, most American high school graduates are extremely weak in logarithms. Apparently this is not taught very well at the high school level, which is a shame because logarithms are a powerful mathematical tool. You may find it necessary to explain to your students what a logarithm is, and exactly why it "un-does" the exponent.

When forced to give a quick presentation on logarithms, I usually start with a generic definition:

$$\text{Given: } b^a = c$$

$$\text{Logarithm defined: } \log_b c = a$$

Verbally defined, the logarithm function asks us to find the power (a) of the base (b) that will yield c.

Next, I introduce the common logarithm. This, of course, is a logarithm with a base of 10. A few quick calculator exercises help students grasp what the common logarithm function is all about:

$$\log 10 =$$

$$\log 100 =$$

$$\log 1000 =$$

$$\log 10000 =$$

$$\log 100000 =$$

$$\log \frac{1}{10} =$$

$$\log \frac{1}{100} =$$

$$\log \frac{1}{1000} =$$

After this, I introduce the *natural logarithm*: a logarithm with a base of e (Euler's constant):

$$\text{Natural logarithm defined: } \ln x = \log_e x$$

Have your students do this simple calculation on their calculators, and explain the result:

$$\ln 2.71828 =$$

Next comes an exercise to help them understand how logarithms can "undo" exponentiation. Have your students calculate the following values:

$$e^2 =$$

$$e^3 =$$

$$e^4 =$$

Now, have them take the natural logarithms of each of those answers. They will find that they arrive at the original exponent values (2, 3, and 4, respectively). Write this relationship on the board as such for your students to view:

$$\ln e^2 = 2$$

$$\ln e^3 = 3$$

$$\ln e^4 = 4$$

Ask your students to express this relationship in general form, using the variable x for the power instead of an actual number:

$$\ln e^x = x$$

It should now be apparent that the natural logarithm function has the ability to "undo" a power of e. Now it should be clear to your students why the given sequence of algebraic manipulations in the answer for this question is true.

Notes 72

In order for students to solve this problem, they must algebraically manipulate the "normal" time-constant formula to solve for time instead of solving for voltage.

Notes 73

Some students may write 5.17 seconds as the time required to charge to 40 volts (5 time constants' worth of time). If so, remind them that the "standard" of 5τ is arbitrary, and that theoretically the capacitor *never* actually reaches full charge.

Notes 74

In order for students to solve this problem, they must algebraically manipulate the "normal" time-constant formula to solve for time instead of solving for voltage.

Notes 75

In developing equations, students often feel "abandoned" if the instructor does not provide an answer for them. How will they ever know if their equation is correct? If the phenomenon the equation seeks to predict is well-understood, though, this is not a problem.

Notes 76

Once students grasp the concept of *initial* and *final* values in time-constant circuits, they may calculate any variable at any point in time for any RC or LR circuit (not for RLC circuits, though, as these require the solution of a second-order differential equation!). All they need is the universal time-constant equation:

$$x = x_{initial} + (x_{final} - x_{initial})\left(1 - e^{\frac{-t}{\tau}}\right)$$

(x, of course, represents either voltage or current, depending on what is being calculated.)

One common mistake new students often commit is to consider "initial" values as those values of voltage and current existing in the circuit *before* the switch is thrown. However, "initial" refers to those values *at the very first instant the switch moves to its new position*, not before.

Notes 77

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Notes 78

This so-called "universal time-constant formula" is my own (Tony R. Kuphaldt's) invention. A product of frustration from trying to teach students to calculate variables in RC and LR time-constant circuits using one formula for decay and another one for increasing values, this equation works for *all* conditions. Vitally important to this formula's accuracy, however, is that the student correctly assesses the initial and final values. This is the biggest problem I see students having when they go to calculate voltages or currents in time-constant circuits.

In my Lessons In Electric Circuits textbook, I introduce this formula in a slightly different form:

$$\Delta V = (V_f - V_0) \left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right)$$

This explains the purpose of my follow-up question: to challenge students who might simply read the book's version of the formula and not consider the difference between it and what is presented here!

Notes 79

This problem is unique in that the capacitor does not discharge all the way to 0 volts when the switch is moved to the lower position. Instead, it discharges down to a (final) value of 3 volts. Solving for the answer requires that students be a bit creative with the common time-constant equations ($e^{-\frac{t}{\tau}}$ and $1 - e^{-\frac{t}{\tau}}$).

The follow-up question is simply an exercise in troubleshooting theory.

Notes 80

There is quite a lot to calculate in order to reach the solutions in this question. There is more than one valid way to approach it, as well. An important fact to note: the voltage across the switch contacts, in both examples, is greater than the battery voltage! Just as capacitive time-constant circuits can generate currents in excess of what their power sources can supply, inductive time-constant circuits can generate voltages in excess of what their power sources can supply.

The follow-up question is simply an exercise in troubleshooting theory.

Notes 81

You will need an inductance meter in your lab to do this exercise. If you don't have one, you should get one right away!

Notes 82

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Notes 83

Many modern digital multimeters come equipped with capacitance measurement built-in. If your students do not have these meters, you will either need to provide one for them to use, or provide an LCR meter. If you don't have either one of these instruments, you should get one right away!

Notes 84

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Notes 85

I recommend choosing resistor and capacitor values that yield time constants in the range that may be accurately tracked with a stopwatch. I also recommend using resistor values significantly less than the voltmeter's input impedance, so that voltmeter loading does not significantly contribute to the decay rate.

Good time values to use (t_1, t_2, t_3) would be in the range of 5, 10, and 15 seconds, respectively.

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Notes 86

Two very important "given" parameters are the relay coil resistance (R_{coil}) and the relay dropout voltage ($V_{dropout}$). These are best determined experimentally.

Many students fail to grasp the purpose of this exercise until it is explained. The idea here is to predict *when* the relay will "drop out" after the switch is opened. This means solving for t in the time-constant (decay) equation given the initial capacitor voltage, time constant (τ), and the capacitor voltage at time t. Because this involves the use of logarithms, students may be perplexed until given assistance.

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Notes 87

Use a variable-voltage, regulated power supply to supply any amount of DC voltage below 30 volts. Specify standard resistor values, all between 1 kΩ and 100 kΩ (1k5, 2k2, 2k7, 3k3, 4k7, 5k1, 6k8, 8k2, 10k, 22k, 33k, 39k 47k, 68k, 82k, etc.).

I have used this circuit as both a "quick" lab exercise and a troubleshooting exercise, using values of 10 kΩ for R1, R2, and R3; 15 kΩ for R(load1); 22 kΩ for R(load2); and 6 volts for the power supply. Of course, these component values are not critical, but they do provide easy-to measure voltages and currents without incurring excessive impedances that would cause significant voltmeter loading problems.

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Notes 88

The idea of a troubleshooting log is three-fold. First, it gets students in the habit of documenting their troubleshooting procedure and thought process. This is a valuable habit to get into, as it translates to more efficient (and easier-followed) troubleshooting on the job. Second, it provides a way to document student steps for the assessment process, making your job as an instructor easier. Third, it reinforces the notion that each and every measurement or action should be followed by reflection (conclusion), making the troubleshooting process more efficient.

Notes 89

The purpose of this assessment rubric is to act as a sort of "contract" between you (the instructor) and your student. This way, the expectations are all clearly known in advance, which goes a long way toward disarming problems later when it is time to grade.

Notes 90

In your students' research, they will encounter a "right-hand rule" as well as a "left-hand rule" for relating electric current with magnetic field directions. The distinction between the two rules depends on whether the text uses "conventional flow" notation or "electron flow" notation to denote the movement of electrical charge through the conductors. Sadly, this is another one of those concepts in electricity that has been made unnecessarily confusing by the prevalence of two "standard" notions for electric current.

Notes 91

This question presents an interesting problem in qualitative mathematics. It is closely related to the "chain rule" in calculus, where one function $y = f(x)$ is embedded within another function $z = f(y)$, such that $\frac{dz}{dy}\frac{dy}{dx} = \frac{dz}{dx}$. The purpose of this exercise is for students to gain a conceptual grasp of why inductance does not vary linearly with changes in N.

Of course, students can obtain the same (third) answer just by looking at the inductance formula (in terms of N, μ, A, and l), without all the conceptual work. It would be good, in fact, if a student happens to derive the same answer by inspection of this formula, just to add variety to the discussion. But the real purpose of this question, again, is a conceptual understanding of that formula.

Notes 92

These factors are important to understand for comprehending the function of variable inductors. Be sure to bring up the subject of variable inductors in your discussion with students.

Notes 93

The concept of a *field* is quite abstract, but at least magnetic fields are something within most peoples' realm of experience. This question is good for helping students distinguish between field force and field flux, in terms they should understand (constant current through a coil, versus the attractive force produced by a magnetic field flux).

Notes 94

Students will likely marvel at the $\frac{di}{dt}$ rate of 175 amps per nanosecond, which equates to 175 *billion* amps per second. Not only is this figure realistic, though, it is also low by some estimates (see <u>IEEE Spectrum</u> magazine, July 2003, Volume 40, Number 7, in the article "*Putting Passives In Their Place*"). Some of your students may be very skeptical of this figure, not willing to believe that "a computer power supply is capable of outputting 175 billion amps?!"

This last statement represents a very common error students commit, and it is based on a fundamental misunderstanding of $\frac{di}{dt}$. "175 billion amps per second" is not the same thing as "175 billion amps". The latter is an absolute measure, while the former is a *rate of change over time*. It is the difference between saying "1500 miles per hour" and "1500 miles". Just because a bullet travels at 1500 miles per hour does not mean it will travel 1500 miles! And just because a power supply is incapable of outputting 175 billion amps does not mean it cannot output a current that *changes* at a rate of 175 billion amps per second!

Notes 95

The concept of integration doesn't have to be overwhelmingly complex. Electrical phenomena such as capacitance and inductance may serve as excellent contexts in which students may explore and comprehend the abstract principles of calculus. The amount of time you choose to devote to a discussion of this question will depend on how mathematically adept your students are.

Notes 96

Explain to your students how the similarities between inertia and inductance are so close, that inductors can be used to electrically model mechanical inertia.

Notes 97

Ask your students to identify what condition(s) might lead to a condition of saturation. And, even if the extreme ends of the $B - H$ curve are avoided, what does the nonlinear shape of the $B - H$ plot indicate about the linearity of an inductor? The inductance formula ($e = L\frac{di}{dt}$) assumes perfect linearity, but is this really true for an inductor whose core exhibits this kind of magnetic flux/force relationship?

Notes 98

These factors affecting capacitance are very hypothetical when dealing with fixed-value capacitors. After all, few people will ever have to *design* or *construct* a capacitor. However, these factors are very practical and important to understand when dealing with stray capacitance between conductors, where conductor layout and placement are well within the control of those building an electrical system!

These factors are also important to understand for comprehending the function of variable capacitors. Be sure to bring up the subject of variable capacitors in your discussion with students.

Notes 99

Encourage your students to find a picture of a Leyden Jar, or even to build their own. One can't help but notice the functional equivalence between a capacitor and a jar: storing charge versus storing substance!

A jar is not the only object which may be transformed into a capacitor. Aluminum foil and paper sheets may also be used to make a rudimentary capacitor. Have your students experiment with building their own capacitors, especially if they have access to a capacitance meter which may be used to compare the capacitance of different designs.

Notes 100

Many students find this a helpful analogy of capacitor action. But it helps even more if students work together to *build* the analogy, and to truly understand it.

Perform some "thought experiments" with vessels of different size, relating the outcomes to charge storage in capacitors of different size.

Notes 101

The existence of such an appropriate analogy for capacitor action makes an explanation unnecessary, even if the concept takes a bit of thought to comprehend at first. It is important that students clearly distinguish the quantities of *current*, *voltage*, and *charge* in a capacitor circuit just as they clearly distinguish the quantities of *liquid height*, *flow rate*, and *liquid volume* in a hydraulic system.

Notes 102

The concept of integration doesn't have to be overwhelmingly complex. Electrical phenomena such as capacitance and inductance may serve as excellent contexts in which students may explore and comprehend the abstract principles of calculus. The amount of time you choose to devote to a discussion of this question will depend on how mathematically adept your students are.

Hopefully, the challenge question will stir your students' imaginations, as they realize the usefulness of electrical components as *analogues* for other types of physical systems.

Notes 103

Note to your students that spring stiffness (k) and capacitance (C) are inversely proportional to one another in this analogy.

Explain to your students how the similarities between inertia and capacitance are so close, that capacitors can be used to electrically model mechanical springs!

Notes 104

Audio system engineering usually inspires interest among music-loving students, especially young students who crave maximum audio power in their automobiles' sound systems! This question is designed to provoke interest as much as it is intended to explore capacitor function.

With regard to "decoupling" capacitors, your students will likely have to use capacitors in this manner when they progress to building semiconductor circuits. If you have a printed circuit board from a computer (a "motherboard") available to show your students, it would be a good example of decoupling capacitors in use.

Notes 105

Although the answer may seem a bit too easy – nay, even obvious – the point here is to get students to correlate the behavior of capacitors and inductors in terms of components they already understand very well. Then, they may correctly associate direction of current with polarity of voltage drop according to the direction of energy flow (source versus load) the reactive component is subjected to.

Notes 106

The purpose of this troubleshooting question is to get students to think in terms of fault elimination: deciding what things *cannot* be bad in order to better isolate what might be bad.

Notes 107

Don't give your students the equation with which to perform this calculation! Let them find it on their own.

Notes 108

Discuss with your students why ESR matters, especially in decoupling applications where capacitors are expected to handle large $\frac{dv}{dt}$ transients.

Notes 109

The capacitor tolerance codes are easy enough for students to research on their own. For your own reference, though:

- D = ± 0.5%
- F = ± 1%
- G = ± 2%
- H = ± 3%
- J = ± 5%
- K = ± 10%
- M = ± 20%
- P = +100%, -0%
- Z = +80%, -20%

Same for the four capacitor labels given in the question:

- 473K = 47 nF ± 10%
- 102J = 1 nF ± 5%
- 224M = 0.22 μF ± 20%
- 331F = 330 pF ± 1%

Notes 110

Students usually want to just use the $\tau = RC$ and $\tau = \frac{L}{R}$ formulae to answer questions like this, but unfortunately this does not lend itself to a firm conceptual understanding of time-constant circuit behavior.

If students need hints on how to answer the capacitance and inductance questions, ask them what the fundamental definitions of "capacitance" and "inductance" are (the ability to store energy . . .). Then ask them what takes longer to discharge (given the same power, or rate of energy release per unit time), a large reservoir of energy or a small reservoir of energy.

If students need hints on how to answer the resistance questions, ask them what each type of reactive component resists change in (voltage for capacitors and current for inductors). Then ask them what condition(s) are necessary to cause the most rapid change in those variables (high current for capacitors and high voltage for inductors). This is most evident by inspection of the differential equations $i = C\frac{dv}{dt}$ and $v = L\frac{di}{dt}$.

Notes 111

My answer could stand to be a little more detailed, but the main idea here is to have students piece together their own argument explaining why inductor charging rate is directly proportional to resistance. Of course, this is yet another example of the usefulness of Thévenin/Norton equivalent circuits. Sometimes a simple source conversion from one form to the other is all that is needed to achieve a conceptual breakthrough!

Notes 112

The purpose of this question is for students to learn the significance of the expression $e^{-\frac{t}{\tau}}$ by "playing" with the numbers. The negative exponent may confuse some students, so be sure to discuss its significance with all students, so that all understand what it means.

Another concept for students to grasp in this question is that of an *asymptotic* function: a function that approaches a final value in incrementally smaller intervals.

Notes 113

Do not simply tell your students how to calculate the values of the increasing variable. Based on their qualitative knowledge of time-constant circuit curves and their ability to evaluate the downward (decay) expression, they should be able to figure out how to calculate the increasing variable's value over time as well.

Some students will insist that you give them an equation to do this. They want to be told what to do, rather than solve the problem on their own based on an observation of pattern. It is very important that students of any science learn to recognize patterns in data, and that they learn to fit that data into a mathematical equation. If nothing else, these figures given in the answer for both decreasing and increasing variables should be plain enough.

Notes 114

Being able to derive an equation from numerical data is a complex, but highly useful, skill in all the sciences. Sure, your students will be able to find this mathematical expression in virtually any basic electronics textbook, but the point of this question is to *derive* this expression from an examination of data (and, of course, an examination of the other time-constant expression: $e^{-\frac{t}{\tau}}$).

Be sure to challenge your students to do this, by asking how they obtained the answer to this question. Do not "settle" for students simply telling you what the equation is – ask them to explain their problem-solving techniques, being sure that all students have contributed their insights.

Notes 115

Calculating the voltage for the first few time constants' worth of time should be easy without a calculator. I strongly encourage your students to develop their estimation skills, so that they may solve problems without being dependent upon a calculator. Too many students depend heavily on calculators – some are even dependent on specific brands or models of calculators!

Equally important as being able to estimate is knowing whether or not your estimations are over or under the exact values. This is especially true when estimating quantities relevant to safety and/or reliability!

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Equally important as being able to estimate is knowing whether or not your estimations are over or under the exact values. This is especially true when estimating quantities relevant to safety and/or reliability!

Notes 117

An interesting thing to note here is the span of time constant values available from common capacitor/inductor/resistor sizes. As students should notice, the capacitor-resistor combinations (all very practical values, I might add) create both longer and shorter time constant values than the inductor-resistor combinations, and that is even including the 25 Henry - 6.2 Ohm combination, which would be difficult (read: expensive) to achieve in real life.

Notes 118

I like to show this question to beginning students because it shows them an aspect of electrical circuits that they probably never thought to consider: that the behavior of a circuit might mimic the behavior of some other type of physical system or process, and that this principle might be exploited as a modeling tool for engineers. Before the advent of inexpensive digital computers, the *analog computer* was the simulation tool of choice among engineers of all persuasions. Using resistors, capacitors, and amplifier circuits, these marvelous machines modeled all kinds of physical systems, as electrical constructs of mathematical equations.

But even without knowing anything about calculus, amplifier circuits, or any advanced electronic concepts, beginning students should be able to understand the principle at work in this question.

Notes 119

The answer given here is purposely minimal. You should ask your students to give responses more thoughtful than this! Ask them why capacitors are cheaper than inductors. Ask them to explain what is meant by "easier to work with," in technical terms.

Notes 120

While some may wonder what this question has to do with electronics, it is an exercise in qualitative analysis. This skill is very important for students to master if they are to be able to distinguish between the equations $e^{-\frac{t}{\tau}}$ and $1 - e^{-\frac{t}{\tau}}$, both used in time-constant circuit analysis.

The actual procedure for determining the nature of the equation is simple: consider what happens as t begins at 0 and as it increases to some arbitrary positive value. Some students may rely on their calculators, performing actual calculations to see whether the percentage increases or decreases with increasing t. Encourage them to analyze the equation qualitatively rather than quantitatively, though. They should be able to tell which way the percentage changes with time without having to consider a single numerical value!

Notes 121

It is very important for students to understand what this expression means and how it works, lest they rely solely on memorization to use it in their calculations. As I always tell my students, rote memorization *will* fail you! If a student does not comprehend why the expression works as it does, they will be helpless to retain it as an effective "tool" for performing calculations in the future.

A good way to suggest students approach a problem such as this is to imagine t increasing in value. As t grows larger, what happens to the expression's overall value? Then, compare which of the two percentages (percentage traversed, or percentage remaining) follow the same trend. One not need touch a calculator to figure this out!

Notes 122

This answer is rather minimal, if not obvious. Of course, if the relay takes longer to de-energize, and we were told this has something to do with time constants, then the time constant of the circuit must have increased. What is not so obvious is *why* τ increased. Discuss this with your students, and see what conclusions they reached.

Notes 123

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Notes 124

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This circuit demands careful pre-analysis of the initial and final values. If students experience difficulty calculating the voltage and current figures here, it is probably due to incorrect initial and final values for voltage and/or current.

Although I have revealed a problem-solving technique in this question, it does not show the students exactly how to separate the inductor's 2.5 henrys of inductance and 14 ohms of resistance into two components, nor does it give away the answer. Discuss the analytical technique of drawing idealized components ("lumped parameters") as a problem-solving technique, and encourage students to use it whenever they are faced with analyzing a component exhibiting parasitic characteristics.

An excellent example of this technique is in "modeling" transformers. Transformers exhibit much more than just mutual inductance. They also exhibit self-inductance, leakage inductance, capacitance, resistance, and hysteretic losses. A comprehensive model for a transformer is a very complex thing, and it appears on a schematic to be a whole network of components connected together:

Equivalent circuit for a transformer
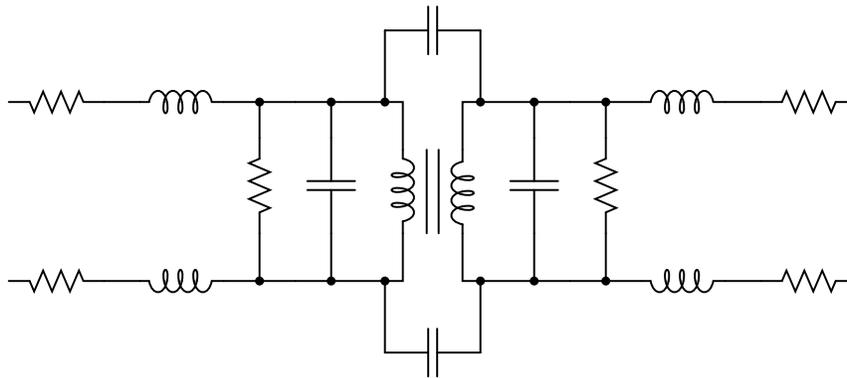
Each of these components is regarded as ideal (i.e., pure: possessing no parasitic characteristics), but together they "model" the behavior of a real transformer in terms readily applicable to existing mathematical techniques.

In order for students to solve this problem, they must algebraically manipulate the "normal" time-constant formula to solve for time instead of solving for voltage.

Ask your students to show how they algebraically solved the standard time constant equation for t using logarithms.

Ask your students to explain how they set up each calculation.

Notes 131

The follow-up question illustrates an important mathematical principle regarding logarithmic decay functions: for every passing of a fixed time interval, the system decays by the same *factor*. This is most clearly (and popularly) seen in the concept of *half-life* for radioactive substances, but it is also seen here for RC (or LR) circuits.

Notes 132

Although it may seem premature to introduce the 555 timer chip when students are just finishing their study of DC, I wanted to provide a practical application of RC circuits, and also of algebra in generating useful equations. If you deem this question too advanced for your student group, by all means skip it.

Incidentally, I simplified the diagram where I show the capacitor discharging: there is actually another current at work here. Since it wasn't relevant to the problem, I omitted it. However, some students may be adept enough to catch the omission, so I show it here:

Capacitor discharging through R2 only
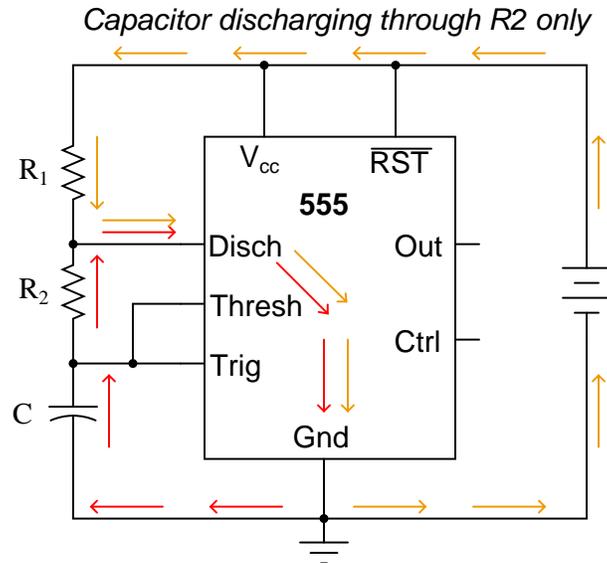
Note that this second current (through the battery) does not go anywhere near the capacitor, and so is irrelevant to the discharge cycle time.

Notes 133

Some students may think that a rate of change of 4.44 *kilo*volts per second harbors shock hazard, because, well, 4.44 *thousand* volts is a lot of voltage! Remind them that this is a *rate of change* and not an actual voltage figure. This number simply tells us how fast the voltage is changing, not how far it will rise to. It is the difference between saying that a car travels at 75 miles per hour and that a car will travel 75 miles.

Notes 134

Some students may think that a rate of change of 280 amps per second might burn up the wiring, because 280 amps seems like a lot of current. Remind them that this is a *rate of change* and not an actual current figure. This number simply tells us how fast the current is changing, not how far it will rise to. It is the difference between saying that a car travels at 75 miles per hour and that a car will travel 75 miles.

Notes 135

In addition to introducing the concept of an air gap switch and the notion of a "surge" power supply, this question challenges students to envision practical problems and their respective diagnostic techniques.

It goes without saying that a circuit such as this is *very* dangerous, and its construction should not be attempted by anyone lacking a thorough understanding of its relevant hazards. Having said that, I will admit to having built one myself, simply to test the validity of an air gap switch for high-voltage, high-current, transient switching. Yes, the concept does work!

Notes 136

The integration required to obtain the answer is commonly found in calculus-based physics textbooks, and is an easy (power rule) integration.

Notes 137

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Notes 138

RC time constant circuits are an excellent example of how to apply simple differential equations. In this case, we see that the differential equation is first-order, with separable variables, making it comparatively easy to solve.

It should also be evident to students that *any* initial condition of capacitor charge may be set into the general solution (by changing the value of the constant).

Notes 139

L/R time constant circuits are an excellent example of how to apply simple differential equations. In this case, we see that the differential equation is first-order, with separable variables, making it comparatively easy to solve.

It should also be evident to students that *any* initial condition for current may be set into the general solution (by changing the value of the constant).