

Points missed: \_\_\_\_\_

Student's Name: \_\_\_\_\_

Total score: \_\_\_\_\_/100 points

East Tennessee State University – Department of Computer and Information Sciences  
CSCI 1900 (Tarnoff) – Math for Computer Science  
TEST 1 for Summer Term, 2005

**Read this before starting!**

- This test is closed book and closed notes
- You may **NOT** use a calculator
- All answers must have a box drawn around them. This is to remove any ambiguity during grading. Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- ***If not otherwise identified, every integer is assumed to be represented using base 10.***
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

"Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarizing, the changing or falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

**Algebraic properties of sets:**

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cup A = A$
- $\overline{A \cap A} = A$
- $\overline{(\overline{A})} = A$
- $\overline{A} \cup A = U$
- $\overline{A} \cap A = \emptyset$
- $\overline{\emptyset} = U$
- $\overline{U} = \emptyset$
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$
- $\overline{A \cap B} = \overline{A} \cup \overline{B}$
- $A \cup U = U$
- $A \cap U = A$
- $A \cup \emptyset = A$  or  $A \cup \{ \} = A$
- $A \cap \emptyset = \emptyset$  or  $A \cap \{ \} = \{ \}$

**Addition principle of cardinality:**

- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

**Properties of characteristic functions:**

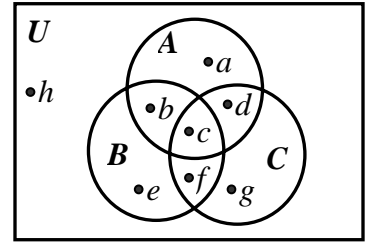
- $f_{A \cap B} = f_A f_B$ ; that is  $f_{A \cap B}(x) = f_A(x) f_B(x)$  for all  $x$ .
- $f_{A \cup B} = f_A + f_B - f_A f_B$ ; that is  $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) f_B(x)$  for all  $x$ .
- $f_{A \oplus B} = f_A + f_B - 2f_A f_B$ ; that is  $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2f_A(x) f_B(x)$  for all  $x$ .

**Properties of integers:**

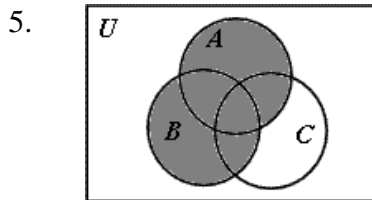
- If  $n$  and  $m$  are integers and  $n > 0$ , we can write  $m = qn + r$  for integers  $q$  and  $r$  with  $0 \leq r < n$ .
- Properties of divisibility:
  - If  $a \mid b$  and  $a \mid c$ , then  $a \mid (b + c)$       If  $a \mid b$  or  $a \mid c$ , then  $a \mid bc$
  - If  $a \mid b$  and  $a \mid c$ , where  $b > c$ , then  $a \mid (b - c)$       If  $a \mid b$  and  $b \mid c$ , then  $a \mid c$
- Every positive integer  $n > 1$  can be written uniquely as  $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} p_4^{k_4} \dots p_s^{k_s}$  where  $p_1 < p_2 < p_3 < p_4 < \dots < p_s$  are distinct primes that divide  $n$  and the  $k$ 's are positive integers giving the number of times each prime occurs as a factor of  $n$ .

Problems 1 through 4 refer to the Venn Diagram shown to the right. (2 points each)

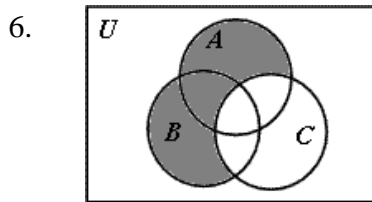
1. True or False:  $d \in A \cap C$
2. True or False:  $f \in A \cap B \cap C$
3. True or False:  $g \in \overline{(A \cup C)}$
4. True or False:  $f \in C - (A - B)$



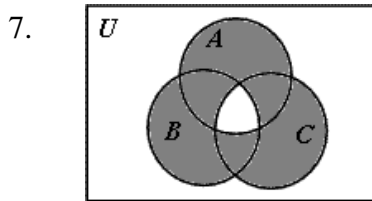
For problems 5, 6, and 7, use unions, intersections, and complements of the sets  $A$ ,  $B$ , and  $C$ , to write an expression to describe the set represented by the shaded area in the given Venn Diagram. (3 points each)



Answer for 5: \_\_\_\_\_



Answer for 6: \_\_\_\_\_



Answer for 7: \_\_\_\_\_

8. If  $A = \{a, e, i, o, u\}$  and  $B = \{a, b, c, d, e, f\}$ , compute the set represented by  $A \cap B$ . (2 points)

$A \cap B =$

9. If  $A = \{a, e, i, o, u\}$  and  $B = \{a, b, c, d, e, f\}$ , compute the set represented by  $A \oplus B$ . (2 points)

$A \oplus B =$

10. If  $A = \{a \mid a \text{ is an integer divisible by 2 or 3}\}$  and  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , compute the set represented by  $\overline{A}$ . (3 points)

$\overline{A} =$

11. If set  $A$  has 10 elements, set  $B$  has 8 elements, and they have 5 elements in common, how many elements are a member of  $A - B$ ? (3 points)

12. If set  $A$  has 10 elements, set  $B$  has 8 elements, and they have 5 elements in common, how many elements are a member of  $A \cap B$ ? (3 points)

13. Give the corresponding set to the sequence *thisiseasy*. (3 points)

14. Write a **recursive** formula for the n-th term of the sequence 2, -2, 2, -2, 2, -2, 2, -2, ... (3 points)

For problems 15 and 16, determine if the sequence defined by the formula is explicit or recursive. In addition, identify the value of the 3<sup>rd</sup> element in the sequence. (2 points each)

15.  $a_1 = 0; a_n = a_{n-1} + 5$                        explicit                       recursive                       $a_3 = \underline{\hspace{2cm}}$

16.  $b_n = 5(n - 1)$                                explicit                               recursive                               $b_3 = \underline{\hspace{2cm}}$

For problems 17, 18, and 19, tell whether or not the string on the left belongs to the regular set corresponding to the regular expression on the right. (2 points each)

17. string: 010101010101              regular expression:  $0(0 \vee 1)^*1$                Belongs               Doesn't belong

18. string:  $\Lambda$  (empty string)              regular expression:  $(0 \vee 1 \vee 2)^*$                Belongs               Doesn't belong

19. string: 1111100                      regular expression:  $1^*0^*1^*$                        Belongs                       Doesn't belong

20. Write three elements that are members of the regular set corresponding to the regular expression  $a(b^*)b$ . (3 points)

21. Give the regular expression corresponding to the regular set {c, ac, aac, aaac, aaaac, aaaaaac, aaaaaaac, ...}. (3 points)

22. True or False: 40394787484731 is divisible by 5. (2 points)

23. True or False: 234555432 is divisible by 3. (2 points)

24. True or False: 45239456603 is divisible by 11. (2 points)

25. True or False:  $2342324_5$  is a valid base 5 number. (2 points)

26. Write the expansion in base 5 of  $254_{10}$ . (4 points)

27. Convert the base 5 number  $2211_5$  to base 10. Just write out the formula with the correct values; do not worry about crunching the numbers for a final calculation. (4 points)

28. How many possible answers/results are there from the mod-24 function? (2 points)

29. What is the result of  $f(33)$  if  $f$  is the mod-4 function, i.e., calculate  $33 \bmod 4$ . (2 points)

30. If  $m = 65$  and  $n = 6$ , determine the values of  $q$  and  $r$  that satisfy the expression  $m = q \cdot n + r$  such that  $0 \leq r < n$ . (2 points)

$q =$  \_\_\_\_\_  $r =$  \_\_\_\_\_

31. Write the integer  $4400_{10}$  as a product of powers of primes. (4 points)

32. Use any method you wish to find the greatest common divisor of 180 and 126. (4 points)

33. Use any method you wish to find the least common multiple of 180 and 126. (4 points)

34. True or False: The matrix  $A$  has the same dimensions as  $A^T$ , i.e.,  $A^T$  has the same number of rows as  $A$  and the same number of columns as  $A$ . (2 points)
35. True or False: If  $A$ ,  $B$ , and  $C$  are matrices, then  $(A + B)C = AC + BC$ . (2 points)

For problems 36 through 39, use the matrices  $A$ ,  $B$ , and  $C$  shown below.

$$A = \begin{bmatrix} 2 & 1 & 8 \\ 3 & 2 & 9 \\ 1 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}$$

36. True or False: It is possible to compute  $B^T + C$ . (2 points)
37. True or False: The result of  $BC$  is matrix with 3 rows and 3 columns. (2 points)
38. True or False: It is possible to compute  $CA + B^T$ . (2 points)
39. Calculate  $AB$ . Show all work. (4 points)