Points missed: _____ Student's Name: _____

Total score: _____/100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2710 (Tarnoff) – Discrete Structures
TEST 3 for Fall Semester, 2004

Read this before starting!

- This test is closed book and closed notes
- You may ***NOT*** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

QUESTIONS BEGIN HERE!

For problems 1 though 3, let A = {1, 2, 3, 4, 5} and B = {a, b, c}. Determine whether the each of the relations R from A to B in these problems is a function. (2 points each)

1. R = {(1, a), (2, b), (1, b), (5, a), (4, b)} ☐ Function ☑ Not a function

Since 1 is paired with a and b, it doesn't have a relation with a unique value in B.

2. R = {(1, a), (2, b), (3, b)} ☑ Function ☐ Not a function

3. R = {(5, b), (4, c), (3, a), (2, a), (1, b)} ☑ Function ☐ Not a function

For problems 4 through 6, determine the range of the function given the domain A. In other words, if $f(a) = b$, then what is the set of all values of b that f produces from A? (2 points each)

4. a ∈ A = Real numbers; $f(a) = a^2$ Range of $f(a)$ = Positive Real values

5. a ∈ A = Positive integers; $f(a) = a \pmod 5$ Range of $f(a)$ = {0, 1, 2, 3, 4}

6. a ∈ A = Integers; $f(a) = 2 \cdot a + 1$ Range of $f(a)$ = Odd Integers

For problems 7 though 9, let the universal set U = {a, b, c, d, e, … x, y, z}. Given the subset A, determine the output of the given characteristic or membership function f_A. (1 point each)

7. A = {t, h, o, m, a, s} $f_A(b)$ = 0

8. A = {a, e, i, o, u} $f_A(y)$ = 0

9. A = {b, o, r, i, n, g } f_A(b) = __1_____

For problems 10 through 12, let *f* be the mod-20 function. Compute the output for each of the problems. (2 points each)

10. *f*(65) = __5_____ 11. *f*(32) = _12_____ 12. *f*(0) = __0_____

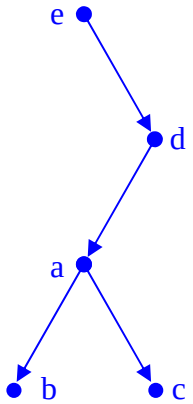
13. Assume that a hashing function ***h*** is used to store customer records to one of ***n*** linked lists. If each customer is assigned a unique 6-digit account number and the hashing function h is the mod 101 function, then how many linked lists will be needed? (2 points)

Since there are 101 possible outcomes from the mod 101 function, {0, 1, 2, 3, …, 99, 100}, then there are 101 lists uniquely identified by the hashing function h.

a.) 1 b.) 2^6 c.) $2^6 - 1$ d.) 99
e.) 100 f.) 101 g.) 10^6 h.) $10^6 - 1$

For problems 14 through 17, each relation ***R*** is defined on the set ***A***. In each case, determine if ***R*** is a ***rooted*** tree, and if it is, what is the root? If there is no root, leave that space blank. (3 points each)

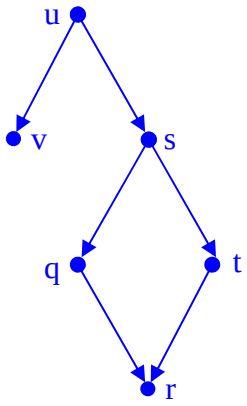
14. ***A*** = {a, b, c, d, e}
R = {(a, b), (a, c), (e, d), (d, a)}



☑ ***R*** is a rooted tree ☐ ***R*** is not a rooted tree
If ***R*** is a rooted tree, the root is: __e_____

15. ***A*** = {q, r, s, t, u, v}
R = {(t, r), (u, s), (u, v), (s, q), (q, r), (s, t)}

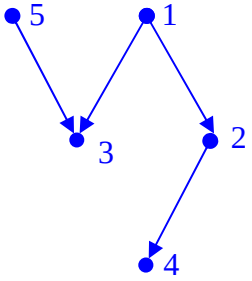
☐ ***R*** is a rooted tree ☑ ***R*** is not a rooted tree
If ***R*** is a rooted tree, the root is: _____



There's a cycle (s, t, r, q) which is not allowed in a tree.

16. $\boldsymbol{A} = \{1, 2, 3, 4, 5\}$
$\boldsymbol{R} = \{(1, 3), (1, 2), (5, 3), (2, 4)\}$

☐ $\boldsymbol{R}$ is a rooted tree ☑ $\boldsymbol{R}$ is not a rooted tree
If $\boldsymbol{R}$ is a rooted tree, the root is: _____

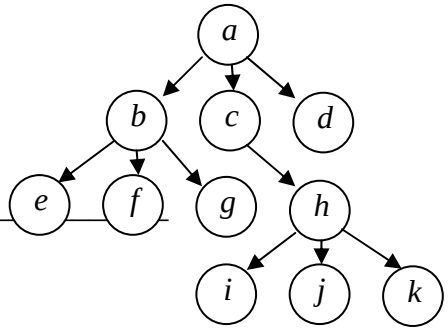


Tree has two roots, 5 and 1, which is not allowed with a rooted tree.

17. $\boldsymbol{A} = \{1, 2, 3, 4, 5\}$
$\boldsymbol{R} = \{(4, 5), (1, 2), (3, 4), (2, 3)\}$

☑ $\boldsymbol{R}$ is a rooted tree ☐ $\boldsymbol{R}$ is not a rooted tree
If $\boldsymbol{R}$ is a rooted tree, the root is: 1

For problems 18 through 24, use the rooted tree T shown in the figure to the right. (2 points each)



18. What is the height of *T*? 3

19. *T* is an *n*-tree. What is the value of *n*? 3

20. List all of the leaves of *T*. d, e, f, g, i, j, k

21. List all of the siblings of *c*. b, d

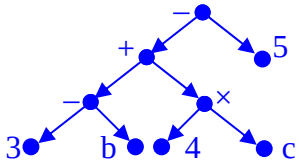
22. List all of the offspring of *c*. h

23. List all of the descendants of *c*. h, i, j, k

24. What is the minimum number of vertices that would need to be added to T for it to be a complete 4-tree? 6

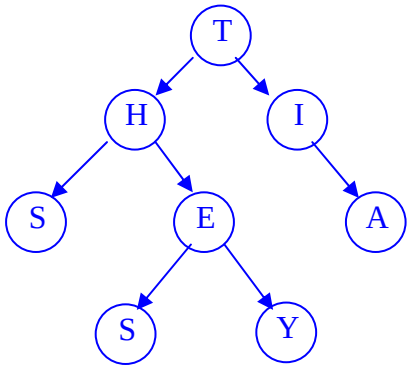
To make a 4-tree, each ***non-leaf*** vertex must have four offspring. The non-leaf vertices are: a, b, c, and h. Vertex a already has three offspring, so it only needs 1 more. Vertex b also already has three offspring, so it only needs 1 more bringing our total to 2. Vertex C only has one offspring, so it needs 3 more bringing our total to 5. Last of all, h has three offspring and needs one more. Therefore, our minimum number of additional vertices is 6.

25. Construct the tree of the algebraic expression
((3 – *b*) + (4 × *c*)) – 5. (5 points)

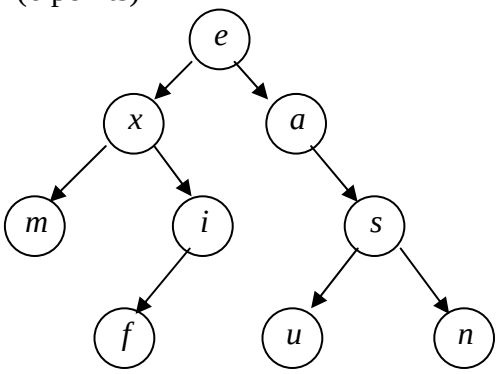


26. The following is the doubling linked list representation of a binary positional labeled tree. Construct the digraph of this tree with each vertex labeled as indicated. (6 points)

index	left	data	right
1	5		0
2	0	A	0
3	9	H	7
4	0	I	2
5	3	T	4
6	0	Y	0
7	8	E	6
8	0	S	0
9	0	S	0



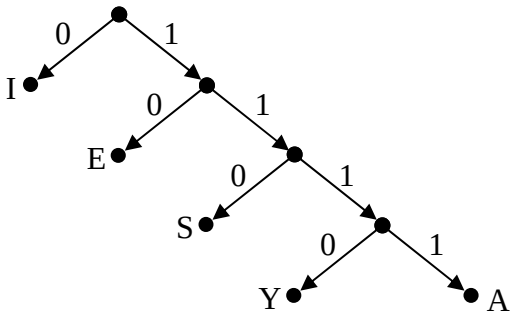
27. Fill in the LEFT and RIGHT arrays in the table to the left for the tree shown below. Note that I want you to put the root vertex starting at index 7. (6 points)



index	left	data	right
1	7		0
2	0	*m*	0
3	5	*i*	0
4	2	*x*	3
5	0	*f*	0
6	8	*s*	9
7	4	*e*	10
8	0	*u*	0
9	0	*n*	0
10	0	*a*	6

28. Use the Huffman code tree shown to the right to find the string of 0's and 1's that represents the word **SAY**. (4 points)

11011111110



29. Use the Huffman code tree shown to the right to decode the message **01101011111101110**. (4 points)

ISEASY

30. The expression shown below is written in Polish (prefix) notation. Evaluate it to the final integer result. Note that all of the numbers are single digit integers. (3 points)

+ – 3 2 × 6 – 4 2 = + 1 × 6 2 = + 1 12 = 13

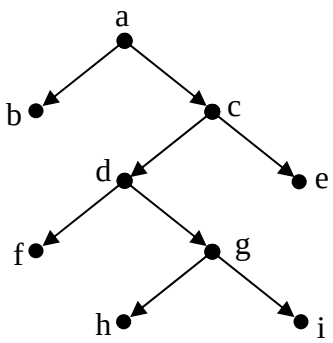
31. The expression shown below is written in reverse Polish (postfix) notation. Evaluate it to the final integer result. Note that all of the numbers are single digit integers. (3 points)

8 7 – 3 6 3 ÷ + × = 1 3 6 3 ÷ + × = 1 3 2 + × = 1 5 × = 5

32. True or False: Parentheses are not needed in order to successfully evaluate expressions derived in any of the following notations: Polish (prefix), inorder (infix), or reverse-Polish (postfix). (2 points)

True for all ***but*** inorder (infix). Therefore, overall statement is false.

33. List the vertices in the order that they are visited in a preorder search of the tree shown to the right. (3 points)

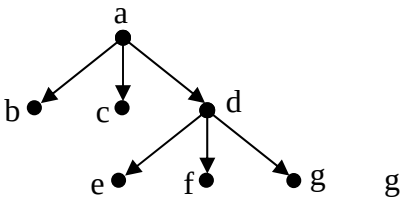


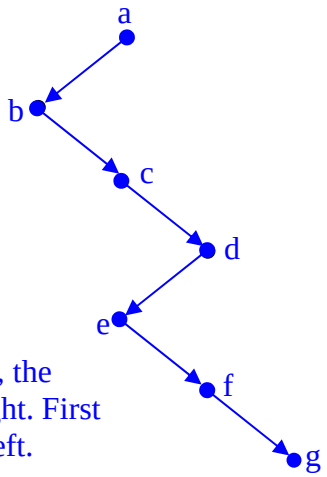
a b c d f g h i e

34. List the vertices in the order that they are visited in an inorder search of the same tree from problem 33. (3 points)

b a f d h g i c e

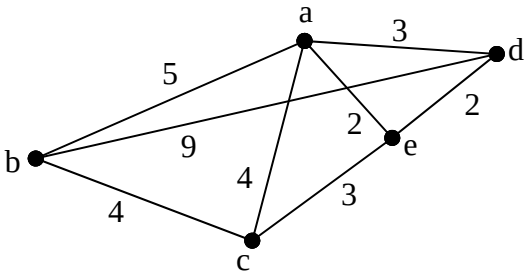
35. In the space to the right, convert the tree shown below to a binary positional tree. (4 points)

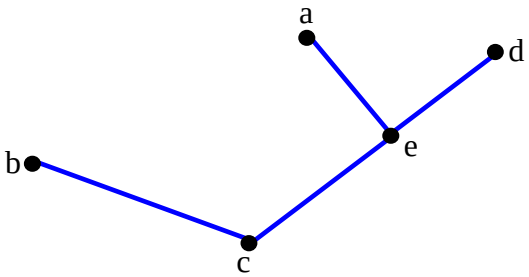




Using the process shown on page 269 of our textbook, the original tree is converted into the binary tree to the right. First sibling to right goes to right while first child goes to left.

36. Use any method you wish to determine the minimal spanning tree for the connected graph shown below and to the left. Draw the connections of the minimal spanning tree using the vertices shown to the right. (5 points)





37. True or False: There is more than one possible minimal spanning tree for the graph in problem 35. (2 points)

38. Make sure your name is on the front page. (1 point) Obvious, I hope.