Points missed: _____ Student's Name: ___________________________________

Total score: _____ /100 points

East Tennessee State University – Department of Computer and Information Sciences
CSCI 2150 (Tarnoff) – Computer Organization
TEST 1 for Spring Semester, 2005

Read this before starting!

- The total possible score for this test is 100 points.
- This test is ***closed book and closed notes***
- You may ***NOT*** use a calculator. Leave all numeric answers in the form of a formula.
- You may use one sheet of scrap paper that you must turn in with your test.
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer. Example:

$$\boxed{32F1_{16}}$$

- **1 point will be deducted** per answer for missing or incorrect units when required. **No** assumptions will be made for hexadecimal versus decimal, so you should always include the base in your answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

Basic Rules of Boolean Algebra:

1.	$A + 0 = A$	**7.**	$A \cdot A = A$
2.	$A + 1 = 1$	**8.**	$A \cdot \overline{A} = 0$
3.	$A \cdot 0 = 0$	**9.**	$\overline{\overline{A}} = A$
4.	$A \cdot 1 = A$	**10.**	$A + AB = A$
5.	$A + A = A$	**11.**	$A + \overline{A}B = A + B$
6.	$A + \overline{A} = 1$	**12.**	$(A + B)(A + C) = A + BC$

DeMorgan's Theorem: $\overline{(AB)} = (\overline{A} + \overline{B})$ $\qquad$ $\overline{(A + B)} = (\overline{A} \bullet \overline{B})$

Short-ish Answer (2 points each unless otherwise noted)

1. Calculate the period of a periodic pulse train with a frequency of 2 Gigahertz (2×10^9 Hertz). ***Be sure to include your units!***

$$\text{period} = \frac{1}{\text{frequency}} = \frac{1}{2\times10^9 \text{ Hertz}} = 0.5\times10^{-9} \text{ seconds} = 0.5 \text{ nanoseconds}$$

You could have left your answer as $1/(2\times10^9)$ seconds if you wanted to. I also needed to see the units of seconds to be sure that you knew what a period was measured with.

2. The AND operation in boolean algebra is analogous to what mathematical algebraic operation?

a.) addition b.) multiplicative inverse c.) negation
d.) multiplication e.) subtracting 1 f.) division

3. True or False: A signal with a 50% duty cycle has the same pulse width (duration of a logic one during a single cycle) regardless of the signal's frequency.

The duty cycle is the ratio of the duration of a logic one to the duration of the period. If the frequency changes, then the period changes. If the period changes, then the duration of the logic one must change. Therefore, the answer is false.

4. How many combinations of 1's and 0's can a 6-bit number (i.e., 6 binary variables) have?

a.) 2^{6-1} b.) $2^6 - 1$ c.) 2^{6+1} d.) 2^6 e.) $2^{6-1} - 1$ f.) None of the above

Each bit of a 6-bit value can take on 2 possible values. Therefore, the answer is $2\times2\times2\times2\times2\times2 = 2^6$.

5. For each of the following binary representations, give the decimal expression for the smallest/lowest value that can be represented using 9 bits. (2 points each)

a.) unsigned binary: Unsigned binary cannot take on any negative values, so the answer is $000000000_2 = 0_{10}$.

b.) 2's complement: The most negative 2's complement value is 100000000_2 which equals $-2^{(9-1)} = -2^8 = -256_{10}$

c.) signed magnitude: The most negative signed magnitude value is 111111111_2 which equals $-2^{(9-1)} - 1 = -2^8 - 1 = -255$

6. What is the **minimum** number of bits needed to represent 132_{10} in signed magnitude representation?

a.) 6 b.) 7 c.) 8 d.) 9 e.) 10 f.) None of the above

One of the easiest ways to do this is to convert 132_{10} to signed magnitude representation. Begin by determining the binary magnitude of 132_{10}.

$132 = 128 + 4 = 2^7 + 2^2$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	0	0	1	0	0

Since it takes 8 bits to represent the magnitude, and since an additional sign bit is needed for signed magnitude, then $\boxed{\textbf{9 bits}}$ are required.

7. True or False: The number 0010010001100010100011 is a valid BCD number.

Divided into nibbles, the number becomes 00 1001 0001 1000 1010 0011 (add two leading zeros to make the first nibble contain four bits.) The second to last nibble, 1010, is not a legal BCD value, and therefore, the number 0010010001100010100011 is not a BCD number.

8. True or False: The 8-bit value 10110110_2 represents the same value in both signed magnitude and 2's complement form.

The most significant bit in both signed magnitude and 2's complement representation is the sign bit. In 8-bits, this means that the value 10110110_2 is a negative value. Only positive values have the same values for both signed magnitude and 2's complement, so the answer is FALSE.

9. Write the complete truth table for a 2-input NOR gate.

The NOR gate is the inverted output of an OR gate, i.e., it outputs a 0 if any of the inputs equal 1.

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

The following two questions are based on the 8-bit binary addition shown below.

```
  01100011
 +00110110
  10011001
```

10. True or False: If the addition above is considered 8-bit 2's complement, an overflow has occurred.

In 2's complement addition, an overflow has occurred if the sign bits of the numbers being added are the same, but the result has a different sign bit. In the above example, two positive numbers are added resulting in a negative number. Therefore, an overflow has occurred.

11. True or False: If the addition above is considered 8-bit unsigned, an overflow has occurred.

In unsigned addition, an overflow has occurred if a carry is generated from the most significant bits. In the above example, no carry is generated, and therefore, **no** overflow has occurred.

12. How many positions and in what direction (left or right) must the number 00011000_2 be shifted in order to effectively divide it by 8? (3 points)

Number of positions to shift: ____3____ Direction to shift: ____right____

A division by 2 is accomplished by shifting all of the ones in a number to the right which reduces each integer power of 2 by one. Dividing by 8 is the same as dividing by 2 three times, therefore, a divide by 8 is realized with 3 shifts to the right.

13. Circle the function that would first be performed in the following expression.

$$A\cdot(B + F + (C\cdot D + E))$$

Just like in mathematical algebra, perform operations inside parenthesis first, and perform multiplication before addition. Therefore, the C·D term is performed first.

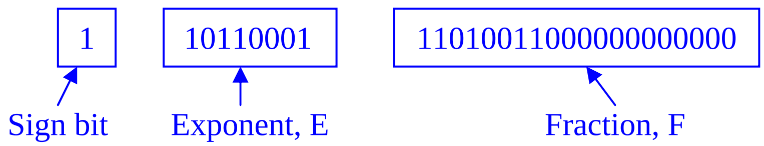
14. True or False: The two circuits below are equal.

DeMorgan's theorem tells us that an OR with an inverted output is the same as an AND with inverted inputs, i.e., to distribute an inverted output to the inputs of an OR gate, the OR must be changed to an AND. Therefore, the answer is TRUE.

Medium-ish Answer (4 points each)

15. Convert the floating-point number 11011000111010011000000000000000 to its binary exponential format, e.g., 1.1010110×2^{-12}, (which, by the way, is not even close to the right answer).

First, break the 32-bit number into its components.



A sign bit of 1 means that this will be a negative number.
The exponent, E, will be used to determine the power of two by which our mantissa will be multiplied. To use it, we must first convert it to a decimal integer using the unsigned method.

$$\begin{aligned}\text{Exponent, E} &= 10110001_2 \\ &= 2^7 + 2^5 + 2^4 + 2^0 \\ &= 128 + 32 + 16 + 1 \\ &= 177_{10}\end{aligned}$$

Substituting these components into our equation for floating point values gives us:

$$\begin{aligned}(\underline{+})1.F \times 2^{(E-127)} &= -1.11010011000000000000000 \times 2^{(177-127)} \\ &= -1.11010011 \times 2^{50}\end{aligned}$$

16. Convert 1100**.**0101 to decimal. (Leave your answer in expanded form.)

Remember that binary digits to the right of the point continue in descending order relative to the 2^0 position. Therefore, the powers of two are in order to the right of the point $2^{-1} = 0.5$, $2^{-2} = 0.25$, $2^{-3} = 0.125$, and $2^{-4} = 0.0625$.

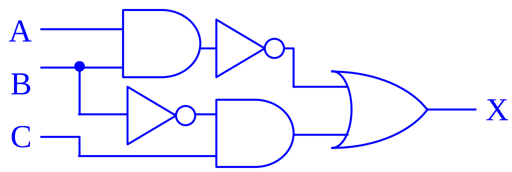
2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
1	1	0	0	0	1	0	1

Therefore, the answer is:

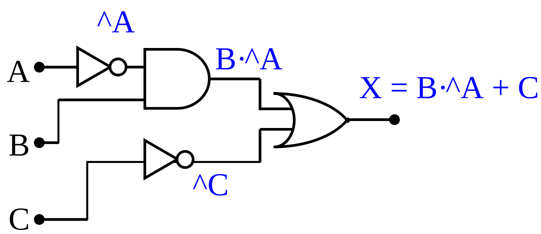
$$2^3 + 2^2 + 2^{-2} + 2^{-4} = 8 + 4 + 1/4 + 1/16 = 12 + 5/16 = 12.3125.$$

You could have left your answer in any of these forms in order to receive full credit.

17. Draw the circuit ***exactly*** as it is represented by the Boolean expression $\overline{A\bullet B} + C\bullet\overline{B}$.



18. In the space to the right, create the truth table for the circuit shown below.



A	B	C	^A	B·^A	^C	X = B·^A + ^C
0	0	0	1	0	1	1
0	0	1	1	0	0	0
0	1	0	1	1	1	1
0	1	1	1	1	0	1
1	0	0	0	0	1	1
1	0	1	0	0	0	0
1	1	0	0	0	1	1
1	1	1	0	0	0	0

19. Write the Boolean expression for the circuit shown in the previous problem. ***Do not simplify!***

$$X = \overline{A}\cdot B + \overline{C}$$

20. Convert 1101001010101101010010_2 to hexadecimal.

From our discussion of hexadecimal numbers, we saw how to make a table representing a conversion from binary nibble patterns to hexadecimal.

Binary	Hex	Binary	Hex
0000	0	1000	8
0001	1	1001	9
0010	2	1010	A
0011	3	1011	B
0100	4	1100	C
0101	5	1101	D
0110	6	1110	E
0111	7	1111	F

Begin our conversion by dividing up the binary pattern into nibbles. Remember to always start from the right in case leading zeros are needed.

0011 0100 1010 1011 0101 0010

Using the table, we find the hex digit that represents the binary nibble pattern: **$34AB52_{16}$**.

21. List two benefits of a digital circuit that uses fewer gates.

In class we discussed a number of benefits of a circuit with fewer gates. They are faster, cheaper, more reliable, take up less circuit board space, run cooler, and use less power (which is basically the same thing as "run cooler").

22. Use any method you wish to prove $A + \overline{A} \bullet B = A + B$.

The easiest way is to prove it using a truth table:

A	B	$\overline{A}$	$\overline{A}\cdot B$	$A + \overline{A}\cdot B$	A + B
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

There were also some proofs that I accepted such as the one shown below:

$$A + \overline{A}\cdot B = A + A\cdot B + \overline{A}\cdot B \qquad \text{Rule 10}$$

$$= A + B(A + \overline{A}) \qquad \text{Distributive Law}$$

$$= A + B\cdot 1 \qquad \text{Rule 6}$$

$$= A + B \qquad \text{Rule 4}$$

23. If an 8-bit binary number is used to represent an analog value in the range from –10 to 145, what is the accuracy of the system? In other words, if the binary number is incremented by one, how much change in the analog range is represented? (Leave your answer in the form of a fraction.)

A single increment = range/number of increments

$$= (145 - (-10))/(2^8 - 1)$$

$$= 155/255$$

$$= 0.60784$$

You could have left your answer in any of the last three forms.

24. Use DeMorgan's Theorem to distribute the inverse of the expression $\overline{A + B + C \bullet D}$ to the individual terms. ***Do not simplify!***

$\overline{A + B + C\cdot D}$ — First, distribute inverse across OR

$\overline{A}\cdot\overline{B}\cdot(\overline{C\cdot D})$ — Next, distribute inverse across C·D

$\overline{A}\cdot\overline{B}\cdot(\overline{C} + \overline{D})$ — This gives us the final answer.

Longer Answers (Points vary per problem)

25. Mark each boolean expression as ***true*** or ***false*** depending on whether the right and left sides of the equal sign are equivalent. Show all of your work to receive partial credit for incorrect answers. (3 points each)

a.) $A + \overline{(A + B)} + B = A$ Answer: **FALSE**

$A + \overline{A}\cdot\overline{B} + B$	DeMorgan's Theorem
$A + \overline{B} + B$	$A + \overline{A}\cdot B = A + B$
$A + 1$	Anything or'ed with inverse equals 1
1	Anything or'ed with 1 equals 1

b.) $AB(A + \overline{B}) = AB$ Answer: **TRUE**

$A\cdot B\cdot A + A\cdot B\cdot\overline{B}$	Distribution Law
$A\cdot A\cdot B + A\cdot B\cdot\overline{B}$	Commutative Law
$A\cdot B + A\cdot B\cdot\overline{B}$	Anything and'ed with itself equals itself
$A\cdot B + A\cdot 0$	Anything and'ed with inverse equals 0
$A\cdot B + 0$	Anything and'ed with 0 equals 0
$A\cdot B$	Anything or'ed with 0 equals itself

c.) $A + \overline{B} + C + \overline{(AB)} = 1$ Answer: **TRUE**

$A + \overline{B} + C + \overline{A} + \overline{B}$	DeMorgan's Theorem
$A + \overline{A} + \overline{B} + \overline{B} + C$	Commutative Law
$1 + B + \overline{B} + C$	Anything or'ed with its inverse equals 1
1	Anything or'ed with 1 equals 1

26. Fill in the blank cells of the table below with the correct numeric format. ***For cells representing binary values, only 8-bit values are allowed!*** If a value for a cell is invalid or cannot be represented in that format, write "X". Use your scrap paper to do your work. (2 points per cell)

Decimal	2's complement binary	Signed magnitude binary	Unsigned binary
106	**01101010**	**01101010**	**01101010**
–128	**10000000**	**X**	**X**
–72	**10111000**	**11001000**	**X**

In the top row, all three binary representations are the same for positive numbers, and since the MSB of the 2's complement value is 0, it is a positive value. Converting to decimal gives us $2^6 + 2^5 + 2^3 + 2^1 = 64 + 32 + 8 + 2 = 106$.

In the second row, we can automatically put an X in the last column as unsigned binary has no negative values. We can also put an X in the column for signed magnitude since -127 is the moste negative value that can be representing in this method with 8 bits. Two's complement can, however, represent -128. It is the value $128_{10} = 10000000_2$ with the bit's flipped giving us 10000000_2.

In the third row, we can once again put an X in the last column as unsigned binary has no negative values. To convert from signed mag to 2's complement, we first need to figure out what the binary representation of the positive value is. 11001000 in signed mag uses the MSB to represent the sign. Changing it to a zero gives us the positive value 01001000. The work below shows what happens after the two's complement is taken of it (the bit flippy thingy) with the values in red being copied straight down and the values in blue inverted.

0	1	0	0	1	0	0	0
1	0	1	1	1	0	0	0

To convert to decimal, go back to the original unsigned value, 01001000_2, and convert it to the positive decimal value.

$$01001000_2 = 2^6 + 2^3 = 64 + 8 = 72$$

Since the original value was negative, put a negative sign in front of the 72.