Basic algebra: manipulating equations

This worksheet introduces the basic concepts of algebra: what variables are, rules and properties, and basic equation manipulation techniques.

The following equations are basic algebraic properties: rules that all real numbers adhere to.

Associative property:

$$a + (b + c) = (a + b) + c$$

$$a(bc) = (ab)c$$

Commutative property:

$$a + b = b + a$$

$$ab = ba$$

Distributive property:

$$a(b + c) = ab + bc$$

Properties of exponents:

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$(a^x)^y = a^{xy}$$

Properties of roots:

$$(\sqrt[x]{a})^x = a$$

$$\sqrt[x]{a^x} = a \text{ if } a \geq 0$$

$$\sqrt[x]{ab} = \sqrt[x]{a}\,\sqrt[x]{b}$$

$$\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$$

Questions

Question 1

A very important concept in algebra is the *variable*. What, exactly, is a variable, and why are they so useful to us?

file 00001

Question 2

What is the difference between these two variable expressions?

$$x^2$$

$$x_2$$

file 00015

Question 3

Suppose we begin with this mathematical statement:

$$3 = 3$$

Not very stunning, I know, but nevertheless absolutely true in a mathematical sense. If I were to add the quantity "5" to the left-hand side of the equation, though, the quantities on either side of the "equals" sign would no longer be equal to each other. To be proper, I would have to replace the "equals" symbol with a "not equal" symbol:

$$3 + 5 \neq 3$$

Assuming that we keep "$3 + 5$" on the left-hand side of the statement, what would have to be done to the right-hand side of the statement to turn it into an equality again?

file 00002

Question 4

Suppose we begin with this mathematical statement:

$$3 = 3$$

If I were to multiply the right-hand side of the equation by the number "7", the quantities on either side of the "equals" sign would no longer be equal to each other. To be proper, I would have to replace the "equals" symbol with a "not equal" symbol:

$$3 \neq 3 \times 7$$

Assuming that we keep "3×7" on the right-hand side of the statement, what would have to be done to the left-hand side of the statement to turn it into an equality again?

file 00003

Question 5

Suppose we begin with this mathematical statement:

$$3 \times 4 = 10 + 2$$

If I were to add the quantity "1" to the left-hand side of the equation, the quantities on either side of the "equals" sign would no longer be equal to each other. To be proper, I would have to replace the "equals" symbol with a "not equal" symbol:

$$(3 \times 4) + 1 \neq 10 + 2$$

What is the simplest and most direct change I can make to the right-hand side of this expression to turn it into an equality again?

file 00004

Question 6

Suppose we were given this mathematical equality:

$$x = y$$

If the variable x is truly the same value as the variable y, then this is a mathematically true statement. If, however, one were to take this equation and subtract the quantity "2" from the left-hand side, the quantities on either side of the "equals" sign would no longer be equal to each other. To be proper, I would have to replace the "equals" sign with a "not equal" symbol:

$$x - 2 \neq y$$

Without altering anything on the left-hand side of this new expression, what could be done to turn it back in to an equality again?

file 00005

Question 7

Suppose we were given this equation:

$$x + 10 = 33$$

If I were to subtract the quantity "10" from the left-hand side of the statement, it would no longer be an equality:

$$(x + 10) - 10 \neq 33$$

What would have to be done to the right-hand side of this statement to make both sides equal again?

file 00011

Question 8

Suppose we were told that this was a mathematically true statement:

$$x = y$$

In other words, variable x represents the exact same numerical value as variable y. Given this assumption, the following mathematical statements must also be true:

$$x + 8 = y + 8$$

$$-x = -y$$

$$9x = 9y$$

$$\frac{x}{25} = \frac{y}{25}$$

$$\frac{1}{x} = \frac{1}{y}$$

$$x^2 = y^2$$

$$\log x = \log y$$

Explain the general principle at work here. Why are we able to alter the basic equality of $x = y$ in so many different ways, and yet still have the resulting expressions be equalities?

file 00006

Question 9

Suppose we were given this equation and asked to solve for the value of x:

$$5 = x + 2$$

What could we do to this equation to isolate the variable x by itself, so that we could directly see the value of x revealed? Remember: it is "legal" to perform any mathematical operation we desire to an equation, so long as we apply the same operation to both sides of the equation in the exact same way!

file 00007

Question 10

Suppose we were given this equation and asked to solve for the value of x:

$$3x = 25$$

What could we do to this equation to isolate the variable x by itself, so that we could directly see the value of x revealed? Remember: it is "legal" to perform any mathematical operation we desire to an equation, so long as we apply the same operation to both sides of the equation in the exact same way!

file 00008

Question 11

Suppose we were given the following equation:

$$\frac{14}{y} = 2$$

What would the equation look like if both sides were multiplied by y?

file 00012

Question 12

Solve for the value of a in the following equations:

Equation 1: $a - 4 = 10$
Equation 2: $30 = a + 3$
Equation 3: $-2a = 9$
Equation 4: $\frac{a}{4} = 3.5$

file 00009

Question 13

Solve for n in the following equations:

Equation 1: $-56 = -14n$
Equation 2: $54 - n = 10$
Equation 3: $\frac{4}{n} = 12$
Equation 4: $28 = 2 - n$

file 00010

Question 14

What would this equation look like if both sides of it were *reciprocated* (inverted)?

$$\frac{10}{(x+2)} = -4$$

file 00013

Question 15

One of the most fundamental equations used in electrical circuit calculations is *Ohm's Law*, relating voltage, current, and resistance:

$$E = IR$$

Where,
$E =$ Voltage dropped across a resistance (volts)
$I =$ Current moving through the resistance (amperes)
$R =$ Resistance to the motion of electric current (ohms)

Manipulate this equation as many times as necessary to express it in terms of all its variables.
file 00019

Question 16

A common equation used in physics relates the kinetic energy, velocity, and mass of a moving object:

$$E_k = \frac{1}{2}mv^2$$

Where,
$E_k =$ Kinetic energy (Joules)
$m =$ Mass (kilograms)
$v =$ Velocity (meters per second)

Manipulate this equation as many times as necessary to express it in terms of all its variables.
file 00018

Question 17

The velocity necessary for a satellite to maintain a circular orbit around Earth is given by this equation:

$$v_s = \sqrt{g_c + h}$$

Where,
$v_s =$ Satellite velocity (feet per second)
$g_c =$ Acceleration of Earth gravity at sea level (32 feet per second squared)
$h =$ Orbit altitude, (feet)

Manipulate this equation as many times as necessary to express it in terms of all its variables.
file 00016

Question 18

The expected operating life of a rolling-contact bearing may be predicted by the following equation:

$$R_L = \left(\frac{C}{L}\right)^{\frac{10}{3}}$$

Where,
R_L = Operating life (millions of shaft revolutions)
C = Dynamic capacity of bearing (pounds)
L = Radial load applied to bearing (pounds)

Manipulate this equation as many times as necessary to express it in terms of all its variables.
file 00014

Question 19

Mechanical, chemical, and civil engineers must often calculate the size of piping necessary to transport fluids. One equation used to relate the water-carrying capacity of multiple, small pipes to the carrying capacity of one large pipe is as follows:

$$N = \left(\frac{d_2}{d_1}\right)^{2.5}$$

Where,
N = Number of small pipes
d_1 = Diameter of each small pipe
d_2 = Diameter of large pipe

Manipulate this equation as many times as necessary to express it in terms of all its variables.
file 00021

Question 20

The ratios of electrical transformer winding turns and impedances are related to each other as such:

$$\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}}$$

Where,
N_p = Number of primary winding turns
N_s = Number of secondary winding turns
Z_p = Impedance of primary circuit
Z_s = Impedance of secondary circuit

Manipulate this equation as many times as necessary to express it in terms of all its variables.
file 00113

Question 21

The amount of power required to propel a ship is given by this equation:

$$P = \frac{D^{\frac{2}{3}} V^3}{K}$$

Where,
P = Power required to turn propeller(s) (horsepower)

D = Vessel displacement (long tons)
V = Velocity (nautical miles per hour)
K = Admiralty coefficient (approximately 70 for a 30 foot long ship, load waterline)
Manipulate this equation as many times as necessary to express it in terms of all its variables.

file 00017

Question 22

Manipulate this equation to solve for a and for b:

$$\frac{1}{\sqrt[3]{b^2}} = ab^2$$

file 00028

Question 23

How do you add these two fractions (without using a calculator?)

$$\frac{1}{5} + \frac{2}{3}$$

file 00022

Question 24

Manipulate this equation as many times as necessary to solve for each of its variables:

$$\frac{x}{5} + \frac{y}{3} = z$$

file 00026

Question 25

In algebra, any equation may be manipulated in any way desired, so long as the same manipulation is applied to *both* sides of the equation equally. In this example, though, only one term on one side of the equation ($\frac{2}{x}$) is manipulated: we multiply it by the fraction $\frac{3x}{3x}$. Is this a "legal" thing to do? Why or why not?

$$y = \frac{2}{x} + \frac{5}{3x^2}$$

$$y = \frac{3x}{3x}\frac{2}{x} + \frac{5}{3x^2}$$

$$y = \frac{6x}{3x^2} + \frac{5}{3x^2}$$

$$y = \frac{6x+5}{3x^2}$$

file 00023

Question 26

Manipulate this equation so that it is expressed in terms of x (with all other variables and constants on the other side of the equals sign):

$$y = \frac{3}{2x} - \frac{7}{3y}$$

file 00114

Question 27

Equations with identical variables represented on *both* sides are often tricky to manipulate. Take this one, for example:

$$\frac{a}{x} = xy + xz$$

Here, the variable x is found on both sides of the equation. How can we manipulate this equation so as to "consolidate" these x variables together so that x is by itself on one side of the equation and everything else is on the other side?

file 00025

Question 28

In machining production technology, the time required to "broach" a hole in a plate may be estimated by the following equation:

$$T = \frac{L}{C} + \frac{L}{R_c}$$

Where,
T = Time to broach hole (minutes)
L = Length of stroke (feet)
C = Cutting speed (feet per minute)
R_c = Return speed (feet per minute)

Manipulate this equation as many times as necessary to express it in terms of all its variables.

file 00024

Question 29

The *Brinell hardness* of a metal specimen is measured by pressing a ball into the specimen and measuring the size of the resulting indentation. To calculate Brinell hardness, this equation is used:

$$H = \frac{F}{(\frac{\pi d_1}{2})(d_1 - \sqrt{d_1^2 - d_s^2})}$$

Where,
H = Hardness of specimen (Brinell units)
F = Force on ball (kg)
d_1 = Diameter of ball (mm)
d_s = Diameter of indentation (mm)

Manipulate this equation to solve for F and for d_s.

file 00027

Question 30

In digital electronic systems based on binary numeration, the number of possible *states* representable by the system is given by the following equation:

$$n_s = 2^{n_b}$$

Where,
n_s = Number of possible states
n_b = Number of binary "bits"

How could you manipulate this equation to solve for the number of binary bits necessary to provide a given number of states?

file 00020

Question 31

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$-5^2 = 25$$

file 00125

Question 32

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$-9^{-\frac{1}{2}} = 3$$

file 00131

Question 33

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$5\frac{a}{b} = \frac{5a}{5b}$$

file 00124

Question 34

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\frac{4y+8}{4y+3} = \frac{8}{3}$$

file 00116

Question 35

In this series of algebraic steps there is a **mistake**. Find the mistake, and correct it so that series leads to the correct answer:

$$5a - 6ac = 1 - 3a^2b$$

$$\frac{5a - 6ac}{a} = \frac{1 - 3a^2b}{a}$$

$$5 - 6c = 1 - 3ab$$

$$3ab + 5 - 6c = 1$$

file 00122

Question 36

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$(a+4)^2 = a^2 + 16$$

file 00117

Question 37

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$(4x+2)(2x+3) = 8x^2 + 6$$

file 00119

Question 38

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\frac{1}{c} + \frac{1}{d} = \frac{1}{c+d}$$

file 00118

Question 39

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\frac{6}{x+2} - \frac{x-4}{x+2} = \frac{6-x-4}{x+2}$$

file 00127

Question 40

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$-5y - 15 = -5(y-3)$$

file 00115

Question 41

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\sqrt[3]{\sqrt{x}} = \sqrt[5]{x}$$

file 00129

Question 42

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$(a^2 + b^{-1})^2 = a^4 + b^{-2}$$

file 00126

Question 43

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\sqrt{x^2 + x^4y^6} = \sqrt{x + x^2y^3}$$

file 00130

Question 44

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\log 3x^2 = 3x \log 2$$

file 00120

Question 45

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\log \frac{2x}{y} = \frac{\log 2x}{\log y}$$

file 00123

Question 46

In this series of algebraic steps there is a **mistake**. Find the mistake, and correct it so that series leads to the correct answer:

$$[(xy + x)y + x]y = z$$

$$[x^2y + xy + x]y = z$$

$$x^2y^2 + xy^2 + xy = z$$

file 00121

Question 47

In this series of algebraic steps there is a **mistake**. Find the mistake, and correct it so that series leads to the correct answer:

$$a^2 - 3a = 0$$

$$a^2 = 3a$$

$$a = 3$$

Note: this is a difficult problem to see. Although the answer, 3, does satisfy the original equation, there is still a mistake in the solution!

file 00128

Answers

Answer 1

Variables are alphabetical letters used to represent numerical quantities. Instead of writing numerical symbols (0, 1, -5, 2400, etc.), we may write letters (a, b, c, x, y, z), each of which representing a range of possible values.

Answer 2

Superscript numbers represent exponents, so that x^2 means "*x squared*", or x multiplied by itself. Subscript numbers are used to denote separate variables, so that the same alphabetical letter may be used more than once in an equation. Thus, x_2 is a distinct variable, different from x_0, x_1, or x_3.

Warning: sometimes subscripts are used to denote specific numerical values of a variable. For instance, x_2 could mean "the variable x when its value is equal to 2". This is almost always the meaning of subscripts when they are 0 (x_0 is the variable x, set equal to a value of 0). Confusing? Yes!

Answer 3

Without altering the left-hand side of this mathematical expression, the only way to bring both sides into equality again is to add the quantity "5" to the right-hand side as well:

$$3 + 5 = 3 + 5$$

Answer 4

Without altering the right-hand side of this mathematical expression, the only way to bring both sides into equality again is to multiply the left-hand side of the expression by "7" as well:

$$3 \times 7 = 3 \times 7$$

Answer 5

The simplest thing I can do to the right-hand side of the equation to make it equal once again to the left-hand side of the equation is to manipulate it in the same way that I just manipulated the left-hand side (by adding the quantity "1"):

$$(3 \times 4) + 1 = (10 + 2) + 1$$

Answer 6

To turn this statement into an equality again, all I would need to do is alter the right-hand side of the equation in that same way that the left-hand side had been altered. In this case, that means subtracting the quantity "2" from the right-hand side:

$$x - 2 = y - 2$$

Answer 7

Subtract "10" from the right-hand side as well:

$$(x + 10) - 10 = (33) - 10$$

This simplifies to the following equation:

$$x = 23$$

Answer 8

The principle at work is this: you may perform any mathematical operation you wish to an equation, provided you apply the *same* operation to both sides of the equation in the exact same way.

Answer 9

Subtract the quantity "2" from both sides:

$$5 = x + 2$$

$$(5) - 2 = (x + 2) - 2$$

$$3 = x$$

So, x is equal to 3.

Answer 10

Divide both sides of the equation by "3":

$$3x = 25$$

$$\frac{3x}{3} = \frac{25}{3}$$

$$x = 8.\overline{33}$$

Answer 11

$$14 = 2y$$

Answer 12

Equation 1: $a = 14$
Equation 2: $a = 27$
Equation 3: $a = -4.5$
Equation 4: $a = 14$

Answer 13

Equation 1: $n = 4$
Equation 2: $n = 44$
Equation 3: $n = 0.\overline{333}$
Equation 4: $n = -26$

Answer 14

$$\frac{(x + 2)}{10} = -\frac{1}{4}$$

Answer 15

$$I = \frac{E}{R}$$

$$R = \frac{E}{I}$$

Answer 16

$$m = 2\frac{E_k}{v^2}$$

$$v = \sqrt{2\frac{E_k}{m}}$$

Answer 17

$$g_c = v_s^2 - h$$

$$h = v_s^2 - g_c$$

Answer 18

$$C = L(R_L)^{\frac{3}{10}}$$

$$L = \frac{C}{(R_L)^{\frac{3}{10}}}$$

Answer 19

$$d_2 = d_1 N^{0.4}$$

$$d_1 = \frac{d_2}{N^{0.4}}$$

Answer 20

$$N_p = N_s\sqrt{\frac{Z_p}{Z_s}}$$

$$N_s = \frac{N_p}{\sqrt{\frac{Z_p}{Z_s}}}$$

$$Z_p = Z_s\left(\frac{N_p}{N_s}\right)^2$$

$$Z_s = \frac{Z_p}{\left(\frac{N_p}{N_s}\right)^2}$$

Answer 21

$$D = \left(\frac{PK}{V^3}\right)^{\frac{3}{2}}$$

$$V = \left(\frac{PK}{D^{\frac{2}{3}}}\right)^{\frac{1}{3}}$$

$$K = \frac{D^{\frac{2}{3}}V^3}{P}$$

Answer 22

$$a = \frac{1}{b^{\frac{8}{3}}} = b^{-\frac{8}{3}}$$

$$b = \frac{1}{a^{\frac{3}{8}}} = a^{-\frac{3}{8}}$$

Answer 23

In order to add fractions with unlike denominators, it is necessary to make the denominators common by multiplying both numerator and denominator by the same amount. In this case, the sum works out to be:

$$\frac{13}{15}$$

Answer 24

$$x = \frac{15z - 5y}{3}$$

$$y = \frac{15z - 3x}{5}$$

Answer 25

This type of manipulation is perfectly ”legal” to do, following the algebraic identity:

$$1a = a$$

Answer 26

$$x = \frac{9y}{6y^2 + 14}$$

Answer 27

$$x = \sqrt{\frac{a}{y + z}}$$

Answer 28

$$L = \frac{TCR_c}{R_c + C}$$

$$C = \frac{LR_c}{TR_c - L}$$

$$R_c = \frac{LC}{TC - L}$$

Answer 29

$$F = H\left(\frac{\pi d_1}{2}\right)\left(d_1 - \sqrt{d_1^2 - d_s^2}\right)$$

$$d_s = \sqrt{d_1^2 - \left(d_1 - \frac{2F}{H\pi d_1}\right)^2}$$

Answer 30

$$n_b = \frac{\log n_s}{\log 2}$$

Answer 31

Two alternative corrections:

$$-5^2 = -25$$

$$(-5)^2 = 25$$

Answer 32

Correction:

$$-9^{-\frac{1}{2}} = -\frac{1}{3}$$

Answer 33

Correction:

$$5\frac{a}{b} = \frac{5a}{b}$$

Answer 34

Correction:

$$\frac{(4y)(8)}{(4y)(3)} = \frac{8}{3}$$

Answer 35

A mistake was made between the second and third equations. Here is the correction:

$$5a - 6ac = 1 - 3a^2b$$

$$\frac{5a - 6ac}{a} = \frac{1 - 3a^2b}{a}$$

$$5 - 6c = \frac{1}{a} - 3ab$$

$$3ab + 5 - 6c = \frac{1}{a}$$

Answer 36

Correction:

$$(a + 4)^2 = a^2 + 8a + 16$$

Answer 37

Correction:

$$(4x + 2)(2x + 3) = 8x^2 + 16x + 6$$

Answer 38

Correction:

$$\frac{1}{c} + \frac{1}{d} = \frac{d}{cd} + \frac{c}{cd} = \frac{d + c}{cd}$$

Answer 39

Correction:

$$\frac{6}{x + 2} - \frac{x - 4}{x + 2} = \frac{6 - x + 4}{x + 2}$$

Answer 40

Two alternative corrections:

$$-5y + 15 = -5(y - 3)$$

. . . or . . .

$$-5y - 15 = -5(y + 3)$$

Answer 41

Correction:

$$\sqrt[3]{\sqrt{x}} = \sqrt[6]{x}$$

Answer 42

Correction:

$$(a^2 + b^{-1})^2 = a^4 + 2a^2b^{-1} + b^{-2}$$

Answer 43

Correction:

$$\sqrt{x^2 + x^4y^6} = x\sqrt{1 + x^2y^6}$$

Answer 44

Correction:

$$\log 3x^2 = 2\log 3x$$

Answer 45

Correction:

$$\log \frac{2x}{y} = \log 2x - \log y$$

Answer 46

There was a mistake between the first and second equations. Here is the correction:

$$[(xy + x)y + x]y = z$$

$$[xy^2 + xy + x]y = z$$

$$xy^3 + xy^2 + xy = z$$

Answer 47

Second-degree equations have *two* solutions, not just one!

$$a^2 - 3a = 0$$

$$a(a - 3) = 0$$

$$a = 0 \text{ or } a = 3$$

Notes

Notes 1

While this concept may be confusing to some, especially at first, it is an extremely powerful mathematical technique. It allows us to write *general* mathematical statements not restricted to one particular combination of numbers.

Notes 2

In the various sciences, certain letters have become standardized to represent specific quantities. In physics, for instance, *mass* is always represented by the variable m, *velocity* by the variable v, etc. What, then, do we do when we have to represent more than one mass or more than one velocity in a single equation? Using different letters (v = velocity 1, x = velocity 2, z = velocity 3 . . .) is not practical, since so many of the other letters are already reserved for different quantities in physics.

A practical solution to this dilemma is to use subscripts to denote different variables of the same quantity *type*. Subscript characters may be numbers, single letters, or even short words: v_1, m_Z, $T_{original}$.

Notes 3

One of the foundational principles of algebra is that any manipulation is allowed in an equation, so long as the same manipulation is applied to both sides. This question introduces students to this principle in a very gentle way, using numerical values rather than variables.

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Notes 5

One of the foundational principles of algebra is that any manipulation is allowed in an equation, so long as the same manipulation is applied to both sides. This question shows how this principle is valid even if the quantities on both sides of the equation are not identical in appearance or form (just so long as they are *equal* in value to each other.

Notes 6

One of the foundational principles of algebra is that any manipulation is allowed in an equation, so long as the same manipulation is applied to both sides. This question shows how this principle is valid even if the quantities on both sides of the equation are not identical in appearance or form (which is certainly the case if the quantities are represented by different letter variables.

Notes 7

The purpose of this question is to stealthily introduce the concept of applying identical mathematical operations to both sides of an equation as a method for isolating variables. Nowhere is this intent stated, but the student should realize the result at the end.

Notes 8

This principle is not only true, but proves to be extremely useful in simplifying and re-arranging algebraic expressions.

Notes 9

Here, the student may see how the principle of "any operation performed on both sides" is not just an interesting mathematical possibility, but rather a technique for simplifying equations.

Notes 10

Here, the student may see how the principle of "any operation performed on both sides" is not just an interesting mathematical possibility, but rather a technique for simplifying equations.

Notes 11

The purpose of this question is to show how a single operation, applied to both sides of the equation, will simplify its appearance, but fall short of isolating the variable y. Encourage students to manipulate the resulting equation one more time to solve for y.

Notes 12

For these equations, no hints are given for isolating the variables. Students will have to apply the principles gleaned from previous problems to solve these.

Notes 13

Equations 2 through 4 require two steps to solve for n. Equation 1 only requires a single step, but the two negative numbers may be a bit confusing to some.

Notes 14

Emphasize that reciprocation is also a valid mathematical operation to be performed on both sides of an equation, and may be used for isolating variables.

Notes 15

Nothing more complex than division is necessary to isolate I or R in this equation. Students need to realize the importance of being able to take an equation given to them in any arbitrary form, and isolate any one of the variables in that equation.

Notes 16

The most challenging portion of this problem is solving for v, which involves ”undoing” the exponent.

Notes 17

This question challenges students to ”undo” the square root before any isolation of g_c or h is done.

Taken from page 8-13 of the Standard Handbook of Engineering Calculations, Editor: Tyler G. Hicks, P.E., ISBN 0-07-028734-1.

Notes 18

This question challenges students to ”move” the $\frac{10}{3}$ exponent from one side of the equation to the other before any isolation of C or L may be done.

Taken from page 3-66 of the Standard Handbook of Engineering Calculations, Editor: Tyler G. Hicks, P.E., ISBN 0-07-028734-1.

Notes 19

The most challenging portion of this problem is ”undoing” the non-integer exponent.

Taken from page 3-378 of the Standard Handbook of Engineering Calculations, Editor: Tyler G. Hicks, P.E., ISBN 0-07-028734-1.

Notes 20

This problem is a good exercise in powers and roots of fractions.

Notes 21

This question provides good practice for students to cancel fractional exponents.

Taken from page 9-5 of the Standard Handbook of Engineering Calculations, Editor: Tyler G. Hicks, P.E., ISBN 0-07-028734-1.

Notes 22

It is important for students to be able to relate radicand (root) symbols with fractional exponents, and visa-versa.

Notes 23

Although this skill is taught at the primary school level, it is often forgotten by many students when they enter college. It can be a valuable tool for manipulating equations.

Notes 24

The ability to find a common denominator between the two fractions in the original equation is key to manipulating it.

Notes 25

Multiplying any quantity by 1 does not change the value of that quantity, and so performing this manipulation only on one side of an equation does not "imbalance" the equation, even if done to just a single term in the equation.

Notes 26

Several steps are required to solve for x in this equation, beginning with manipulation of the fractional terms to arrive at common denominators, and factoring x out of a polynomial expression in a later step.

Notes 27

The key here is to use *factoring* to consolidate the two x variables on the right-hand side of the equation, then it becomes easy to consolidate the single x variable on the left side with the single x variable on the right.

Notes 28

To solve this problem, you must not only make common the unlike denominators in the original equation, but you must also use factoring to consolidate variables.

Taken from page 3-139 of the Standard Handbook of Engineering Calculations, Editor: Tyler G. Hicks, P.E., ISBN 0-07-028734-1.

Notes 29

Taken from page 3-151 of the Standard Handbook of Engineering Calculations, Editor: Tyler G. Hicks, P.E., ISBN 0-07-028734-1.

Notes 30

Logarithms provide us a way to easily isolate variable exponents in an equation.

Notes 31

Proper order of operations holds that powers must be resolved prior to multiplications. In this case, the negative sign may be taken as a negative 1 (-1) multiplied by 5^2, in which case the 5^2 must be calculated before the -1 is multiplied.

Notes 32

Proper order of operations holds that powers must be resolved prior to multiplications. In this case, the negative sign may be taken as a negative 1 (-1) multiplied by $9^{-\frac{1}{2}}$, in which case the $9^{-\frac{1}{2}}$ must be calculated before the -1 is multiplied.

Notes 33

Remember when multiplying fractions by non-fractions, the non-fraction is actually a fraction with a denominator of 1 ($5 = \frac{5}{1}$).

Notes 34

Improper cancellations in complex fractions are another common mistake that algebra students make.

Notes 35

Students often make the mistake of incomplete division of terms: when additive or subtractive terms are divided by a single term in the denominator, the division extends to each term in the numerator, not just one!

Notes 36

When an additive term is squared, the result is a polynomial! Students must think of $(a+4)^2$ in this sense: $(a+4)(a+4)$.

Notes 37

A good rule to remember when multiplying additive terms is "**FOIL**": **F**irst, **O**utside, **I**nside, **L**ast.

Notes 38

Remember that fractions added together must have common denominators before they may be combined into a single fraction!

Notes 39

Remember to distribute the "-" sign! This is a *very* common mistake of algebra students.

Notes 40

It is very important to ensure that signs are properly distributed along with variables and constants!

Notes 41

An easier way to approach this problem is to represent the roots as fractional exponents instead. Then, the proper combination of powers becomes apparent.

Notes 42

When an additive term is squared, the result is a polynomial!

Notes 43

You cannot distribute a root to added terms, any more than you can distribute a power to added terms.

Notes 44

Logarithms are powerful in their ability to "undo" exponentiation problems by transforming them into multiplication problems, but you have to remember which portions of the original exponentiation problems go where!

Notes 45

Remember that logarithms act as *transform functions*. That is, they transform complex arithmetic operations into simpler operations. In this case, the transformation is from division ($\frac{2x}{y}$) to subtraction ($2x - y$).

Notes 46

Proper distribution of terms is paramount to success when "undoing" layers of parentheses.

Notes 47

Although it may not be apparent at first, the reason the original solution sequence is incorrect is because a division by zero is taking place.