

fft Fast Fourier Transform

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Data Sheet

Features

- fft MegaCore function implementing a fast Fourier transform (FFT)
- Optimized for the Altera[®] FLEX[®] 10K architecture
- Block-floating-point to provide the best accuracy
- Parameterized data width, twiddle width, and number of points
- Dual memory architecture
- Complex data in and data out, decimation in frequency (DIF) FFT
- Flexible memory interface with data and twiddle memories that can be implemented in internal and/or external memory

General Description

The fft MegaCore function implements a fast Fourier transform (FFT), which is used to separate a signal into its constituent frequencies. This function is useful for a variety of digital signal processing (DSP) applications, including wireless communications, voice recognition, spectrum analysis, and noise analysis. Figure 1 shows the symbol for the fft MegaCore function.



To optimize throughput, the function uses a dual memory architecture in which data is read from one memory and written to the other memory. This dual-memory architecture consists of a right and left memory. The function also uses a third memory, known as the twiddle memory, that must be kept separate from the left and right memories to maximize throughput. All three memories can be implemented in internal memory, using FLEX 10K embedded array blocks (EABs), and/or external RAM.

Altera Corporation

AHDL Function Prototype

The Altera Hardware Description Language (AHDL) Function Prototype for the fft is shown below:

```
FUNCTION fft (clock, start_fft,
    data_left_in_re[WIDTH_DATA-1..0],
    data left in im[WIDTH DATA-1..0],
    data_right_in_re[WIDTH_DATA-1..0],
    data_right_in_im[WIDTH_DATA-1..0],
    twiddle_re[WIDTH_TWIDDLE-1..0],
    twiddle_im[WIDTH_TWIDDLE-1..0])
  WITH (WIDTH_DATA, WIDTH_TWIDDLE, PIPE_DATA,
    PIPE_TWIDDLE, WIDTH_EXPONENT, FFT_DIRECTION,
    WIDTH_ADD, EXPONENT_INITIAL_VALUE)
  RETURNS (done, data_direction, we_left,
    add left[WIDTH ADD-1..0], we right,
    add_right[WIDTH_ADD-1..0],
    add_twiddle[WIDTH_ADD-2..0],
    data_out_re[WIDTH_DATA-1..0],
    data_out_im[WIDTH_DATA-1..0],
    exponent[WIDTH_EXPONENT-1..0]);
```

VHDL Component Declaration

The VHDL Component Declaration for the fft is shown below:

```
COMPONENT fft
  GENERIC(
    WIDTH_DATA : POSITIVE;
    WIDTH_TWIDDLE : POSITIVE;
    PIPE DATA : INTEGER;
    PIPE_TWIDDLE : INTEGER;
    WIDTH_EXPONENT : POSITIVE;
    WIDTH ADD : POSITIVE;
    EXPONENT_INITIAL_VALUE : INTEGER);
  PORT(
    clock : IN STD_LOGIC := '0';
    start_fft : IN STD_LOGIC;
    data_left_in_re, data_left_in_im,
    data_right_in_re, data_right_in_im : IN
    STD_LOGIC_VECTOR(WIDTH_DATA-1 DOWNTO 0);
    twiddle_re, twiddle_im : IN
    STD LOGIC VECTOR(WIDTH TWIDDLE-1 DOWNTO 0);
    done, data_direction, we_left : OUT STD_LOGIC;
    add_left : OUT STD_LOGIC_VECTOR(WIDTH_ADD-1
```

```
DOWNTO 0);
we_right : OUT STD_LOGIC;
add_right : OUT STD_LOGIC_VECTOR(WIDTH_ADD-1
DOWNTO 0);
add_twiddle : OUT STD_LOGIC_VECTOR(WIDTH_ADD-2
DOWNTO 0);
data_out_re, data_out_im : OUT
STD_LOGIC_VECTOR(WIDTH_DATA-1 DOWNTO 0);
exponent : OUT_STD_LOGIC_VECTOR(WIDTH_EXPONENT-1
DOWNTO 0));
END COMPONENT;
```

Ports

Table 1 describes the ports for the fft.

Table 1. fft Ports										
Name	Туре	Required	Description							
clock	Input	Yes	Clock signal.							
start_fft	Input	Yes	Starts the fft after data is loaded.							
data_left_in_re[]	Input	Yes	Yes Real data input from the left memory.							
<pre>data_left_in_im[]</pre>	Input	Yes	Imaginary data input from the left memory.							
data_right_in_re[]	Input	Yes	Real data input from the right memory.							
data_right_in_im[]	Input	Yes	Imaginary data input from the right memory.							
<pre>twiddle_re[]</pre>	Input	Yes	Real twiddle input from the twiddle memory.							
<pre>twiddle_im[]</pre>	Input	Yes	Imaginary data input from the twiddle memory.							
done	Output	Yes	Goes high when the fft has completed the calculation.							
data_direction	Output	Yes	When high, the fft reads data from the left memory and							
			writes data to the right memory. When low, the inverse							
			operation is performed.							
we_left	Output	Yes	Write enable for the left memory.							
we_right	Output	Yes	Write enable for the right memory.							
add_left[]	Output	Yes	Address bus for the left memory.							
add_right[]	Output	Yes	Address bus for the right memory.							
add_twiddle[]	Output	Yes	Address bus for the twiddle memory.							
data_out_re[]	Output	Yes	Real data output to both left and right memories.							
data_out_im[]	Output	Yes	Imaginary data output to both left and right memories.							
exponent[]	Output	Yes	Exponent of the resultant data. Valid after done goes high.							
			Exponent of the block-floating-point notation. All data							
			should be scaled by 2 ^{exponent[]} .							

Parameters

Table 2. fft Parameters								
Name	Value	Description						
PIPE_DATA	Integer	Number of clock cycles after add_left[] or add_right[] becomes active that the data will be valid at data_left_in_re[], data_left_in_im[], data_right_in_re[], or data_right_in_im[].						
PIPE_TWIDDLE	Integer	The number of clock cycles after add_twiddle[] becomes active that twiddle_re[] or twiddle_im[] becomes active.						
WIDTH_ADD	Integer	Width of the data address buses. The number of points in the fft is $2^{\tt WIDTH_ADD}.$						
WIDTH_DATA	Integer	Data width.						
WIDTH_EXPONENT	Integer	Number of bits in the exponent.						
WIDTH_TWIDDLE	Integer	Twiddle width.						

Table 2 describes the parameters for the fft.

Functional Description

The fft function implements a decimation in frequency (DIF) algorithm and contains all the core logic functions necessary to compute an FFT. To maximize flexibility, the fft function does not include a memory or I/O interface. The memory and I/O interface is dependent upon the end application, and thus must be customized to fit each application.

Figure 2 shows the basic butterfly operation performed by the DIF algorithm.



Blue lines indicate negative numbers.



Figure 3 shows a sample 8-point DIF FFT algorithm as implemented by the fft function. The input data addresses are in normal order, and the output data addresses are in bit-reversed order. However, dedicated hardware can be easily added to address the memories so that data appears in normal order for both inputs and outputs.





The fft_on_chip reference design includes a memory and I/O interface as well as dedicated hardware for addressing data in normal order for both inputs and outputs. Example 1 on page 8 and Example 2 on page 10 describe the implementation of the fft_on_chip reference design.

When processing data, the fft alternates which memory it reads from and writes to on each pass. For example, on Pass 1 in Figure 3, the fft simultaneously reads data from the right memory, performs the butterfly operation, and writes the result to the left memory. On Pass 2, the fft simultaneously reads the result from the left memory, performs the next step of the DIF algorithm, and writes the result to the right memory. The fft continues this process until the calculation is complete. The number of points in the fft determines which memory the fft reads first. When the number of points is an odd power of 2 (e.g., $2^3 = 8$, $2^5 = 32$), data is always read from the right memory on the first pass through the fft. When the number of points is an even power of 2 (e.g., $2^4 = 16$, $2^6 = 64$), the fft switches which memory it reads first after each complete calculation. The data_direction output indicates which memory the fft is reading from and writing to.

Twiddle Generation

The data is multiplied by twiddles (i.e., coefficients) each time it passes through the fft. Each twiddle (W) is calculated by the equation:

$$W_N^k = e^{(-j2\pi k)/N} = \cos(2\pi k/N) - j\sin(2\pi k/N)$$

where: $N =$ Number of points in fft = $2^{\text{WIDTH}_A\text{ADD}}$
 $k = 0 \text{ to } (N/2 - 1)$
 $j = \sqrt{-1}$

The real part of the twiddle is $\cos(2\pi k/N)$ and the imaginary part is $-\sin(2\pi k/N)$. Both twiddle outputs must be stored in the twiddle memory.

The **twiddle** utility program distributed with the fft MegaCore function automatically generates an EAB Memory Initialization File (**.mif**) that contains all the twiddle data. This MIF can be easily translated to other formats for use in an external ROM.

IP I

For help on the syntax, type twiddle from the UNIX or DOS command prompt.

Number Representation

All buses in and out of the fft are in two's complement fractional notation, in which the numbers range from -1 to 1. The input data is always in two's complement fractional notation, with one bit to the left of the binary point and WIDTH_DATA – 1 bits to the right. For example, the decimal number 0.5 is represented as 0.1000000_B , and -0.5 is 1.1000000_B . The smallest 8-bit wide number that can be represented in two's complement fractional notation is -1.0 and the largest number is:

 $\frac{2^{\text{WIDTHTWIDDLE}-1}-1}{2^{\text{WIDTHTWIDDLE}-1}} = \frac{127}{128} = 0.9922$

Twiddle data is in inverted two's complement notation (i.e., the decimal number 0.5 is represented as 1.1000000_B , and -0.5 is 0.1000000_B). The twiddle data uses this notation because the most used twiddle—twiddle zero (W_N^0)—has the value of 1.0 + j0 and less error is accumulated when W_N^0 can be represented exactly. Although the accuracy of $W_{N/4}^0$ suffers slightly, the final result is more accurate because this twiddle is used less in the FFT calculation.

Memory Interface

The fft contains the core functions necessary to compute an FFT and does not include memory or an I/O interface. The function does not include a memory or I/O interface so the user has the most flexibility in every application. FLEX 10K devices contain a finite amount of memory, so not all memory configurations can be accommodated on-chip. Table 3 shows the available memory configurations for EPF10K100 and EPF10K50 devices.

Table 3. Available Configurations in EPF10K100 & EPF10K50 Devices Note (1)													
Device	EABs Availabl e	Data Width (Bits)	Twiddle Width (Bits)	Memory Implementation									
				Twiddles & Data in EABs		Twiddles in EABs & Data in External RAM		Twiddles in External RAM & Data in EABs					
				Points	EABs	Points	EABs	Points	EABs				
EPF10K100	12	≤ 8	≤8	512	10	2,048	8	512	8				
		9 to 16	≤8	256	10	2,048	8	256	8				
		9 to 16	9 to 16	256	12	1,024	8	256	8				
		> 16	> 16	Note (2)	Note (2)	Note (2)	Note (2)	Note (2)	Note (2)				
EPF10K50	10	≤ 8	≤8	512	10	2,048	8	512	8				
		9 to 16	≤8	256	10	2,048	8	256	8				
		9 to 16	9 to 16	Note (2)	Note (2)	1,024	8	256	8				
		> 16	> 16	Note (2)	Note (2)	Note (2)	Note (2)	Note (2)	Note (2)				

Notes:

(1) If both the data and twiddles are in external RAM, the design can use an unlimited number of points.

(2) The device cannot support this configuration because the RAM required exceeds the available memory in the device.

Pipelining

The designer can choose the number of pipeline stages in the memory path, allowing a trade-off between size and speed for a particular memory interface. For example, the fft_on_chip function described in "Example 1: On-Chip RAM, Odd Number of Passes, with No Data Buffering" uses a data pipeline delay of three, and a twiddle pipeline delay of two. Because the twiddle memory in the fft_on_chip function is a synchronous ROM with registered inputs and outputs, the PIPE_TWIDDLE parameter is set to two. The data memory is a synchronous RAM with registered inputs (address) and outputs (data), and with multiplexers on the data, address, and write enable inputs. Because these multiplexers are pipelined to maximize performance, the delay from the address/control valid to data valid on the data ports is three. Therefore, PIPE_DATA is set to three.

Example Implementations

When using the fft, the majority of the design effort will be spent on creating the memory and I/O interface. The user can choose from a wide variety of memory and I/O interface schemes; this section describes four of these schemes:

- Example 1: On-chip RAM, odd number of passes (even number of address bits), with no data buffering
- Example 2: On-chip RAM, even number of passes, with no data buffering
- Example 3: Off-chip RAM, odd number of passes, with no data buffering
- Example 4: Off-chip RAM, odd number of passes, with data buffering

Example 1: On-Chip RAM, Odd Number of Passes, with No Data Buffering

This example describes the same memory architecture provided by the fft_on_chip reference design. The right, left, and twiddle memories are all implemented in FLEX 10K EABs. Data is loaded into the right memory and processed by the fft. When the fft is not processing data, data can be loaded and unloaded from both memories simultaneously. Figure 4 shows a block diagram of this implementation.



Figure 4. Block Diagram of Example 1

Memory & I/O Interface

Because data passes through the fft an odd number of times, the fft always reads new data from the right memory and writes the result to the left memory. This scheme can cause a problem if both the fft and the I/O interface require access to the right and left memory control and data buses. However, this problem can be avoided by placing multiplexers on the data and address buses. Because these multiplexers can cause a performance bottleneck if they are not pipelined, a register is placed after the data and address multiplexers. Since the EAB is used with both registered inputs and outputs to maintain performance, the delay from address valid to data valid is 3. Therefore, PIPE_DATA is set to 3.

Load & Unload Addressing

The incoming data is in normal bit order, and the outgoing data (as placed in the left memory) is in bit-reversed order. Therefore, the designer can use either one or two counters to load and unload the memories. If one counter is used, the right memory receives the load counter output in normal bit order, and the left memory receives the load counter output in bit-reversed order. When the I/O interface has one address counter, loads and unloads must be completed simultaneously. If two counters are used (as shown in Figure 4), data can be individually loaded and unloaded once done is asserted.

Twiddle Memory

The twiddle memory interface is simple to design because the twiddle memory is always read-only. The twiddle memory does not require a write enable signal or an I/O interface, and thus the twiddle memory is connected directly to the fft. The twiddle memory has registered inputs and outputs, so the PIPE_TWIDDLE parameter is set to 2.

Usage

To load data into the right memory, assert load_nrun for N + 1 clock cycles, and assert load_bank_enable for N cycles while placing data on the data_in port. The load_nrun input is then deasserted to return control to the fft. On the next clock cycle, assert start_fft for one clock cycle to begin computation. After the fft has finished processing data and has written the result to the left memory, the fft asserts the done output.

Example 2: On-Chip RAM, Even Number of Passes, with No Data Buffering

Example 2 is similar to Example 1, except the data passes through the fft an even number of times. To allow maximum throughput in this scheme, the fft alternates the first memory it reads after each complete FFT calculation (i.e., for the first FFT calculation, fft reads new data from the right memory; for the second FFT calculation, the fft reads new data from the left memory.) When the data passes through the fft an even number of times, the fft reads new data from one memory, processes the data, and writes the result back to the same memory. Because each memory is storing either new data or the result of a calculation, the I/O interface should be able to load new data into one memory and unload the result from the other memory. Figure 5 shows a block diagram of Example 2. This example uses a multiplexing scheme to allow both load and unload addresses to access the right and left memories. The multiplexing scheme also allows data to be loaded and unloaded to and from both memories. To maintain performance, the multiplexers are pipelined, thus adding a stage of delay between the address counters and the left and right memories. An extra stage of delay is added in the data_in path so that load_bank_enable, data_in, left_address, and right_address are all active during the same clock cycles.



Figure 5. Block Diagram of Example 2

Example 3: Off-Chip RAM, Odd Number of Passes, without Data Buffering

Using off-chip RAM allows the fft to be longer than 256 points. An even longer fft can be created by using external synchronous static RAM (SSRAM), while still maintaining a high clock rate. The SSRAM used in this example is a synchronous cache RAM intended for use with high-speed microprocessors. This type of RAM is available from a variety of vendors.

This architecture is similar to the architectures used in Examples 1 and 2, except for the use of off-chip RAM. The address, data multiplexing, and pipelining schemes remain the same as Examples 1 and 2. The only difference is that the RAM does not have separate data_in and data_out ports and instead uses bidirectional ports. Therefore, this design must use bidirectional pins on the FLEX 10K device that are used as output pins when writing and input pins at all other times.

Example 4: Off-Chip RAM, Odd Number of Passes, with Data Buffering

In examples 1, 2, and 3, new data cannot be captured when the fft is processing data because all of the RAM is in use. Through the use of four memories (i.e., right 1, right 2, left 1, and left 2 memories), this example allows new data to be captured while the fft is processing the current data. While the fft is processing data from the right 1 and left 1 memories, new data is being loaded into the left 2 memory. When both the left 2 memory is full and the fft has finished processing data, the left 1 and left 2 memories are swapped and the right 1 and right 2 memories are swapped. Then, the start_fft signal is immediately asserted to begin processing data from the left 2 and right 2 memories. While the fft is processing data from the left 2 and right 2 memories are loaded and unloaded.



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