

# 4 : MOSFET



1. MOSFET I-V , ( , , )

2. 가 , 가

3. ( )

4. - , ,

5. - ,

## Introduction

History of FETs

the basic concept is known in the 1930s

the device became practical in the 1960s

MOSFET has been popular since the late 1970s

Types of FETs

**MOSFET**      **enhancement mode**

depletion mode

JFET

MESFET

Features of MOSFETs (compared to BJTs)

small area

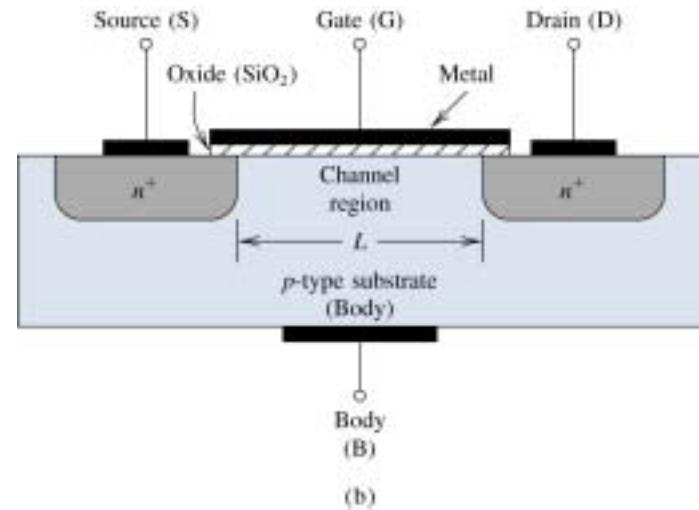
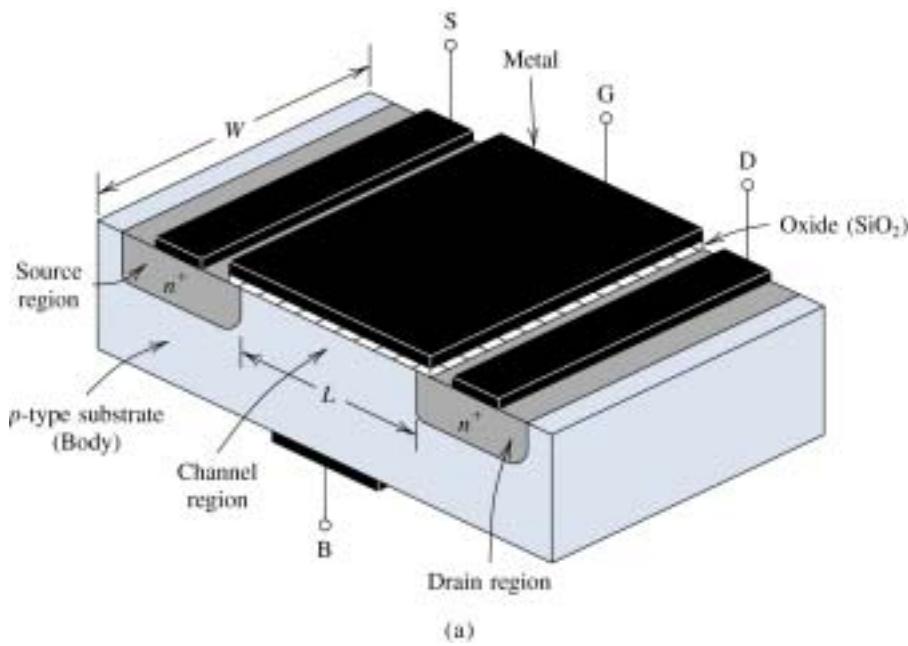
simple process

low power

digital logic and memory functions using MOSFETs only

most VLSI circuits are made using MOS technology

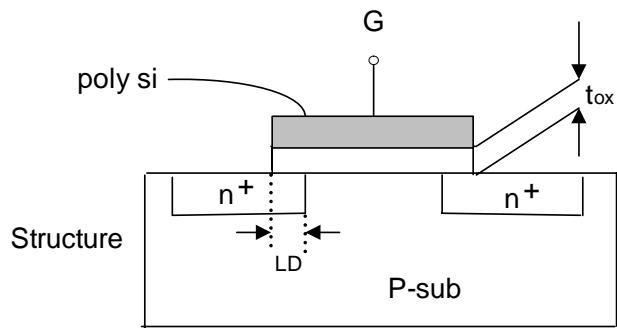
## Device Structure



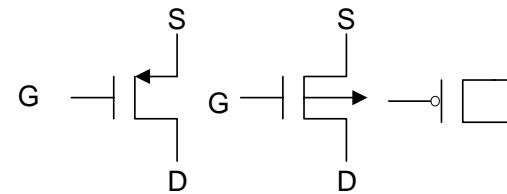
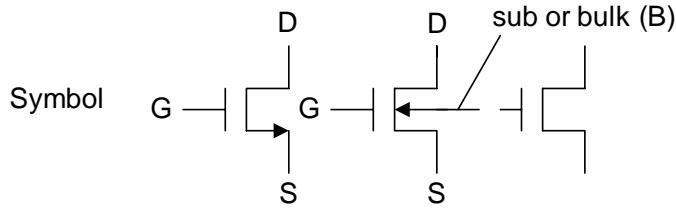
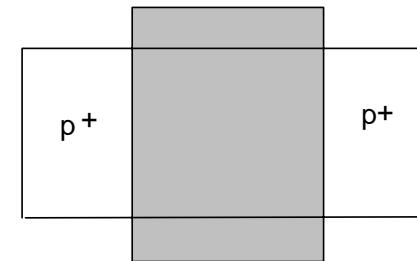
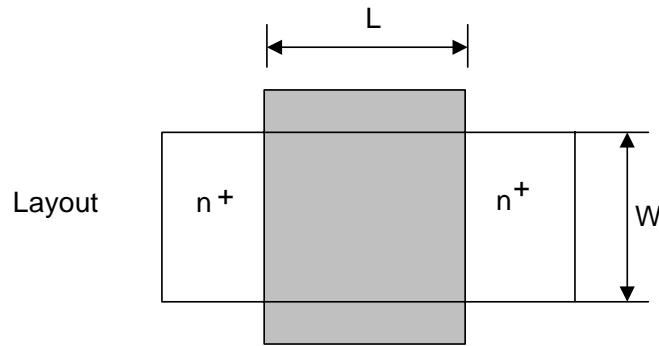
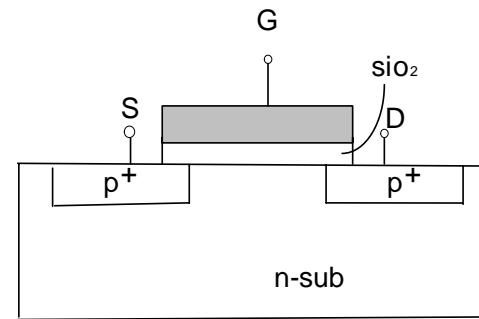
4 terminals : Drain(D), Gate(G), Source(S), Body(B)  
symmetrical (S & D can be interchanged)  
silicon gate (polysilicon)  
another name : IGFET(insulated-gate FET)

## Layout & Symbol

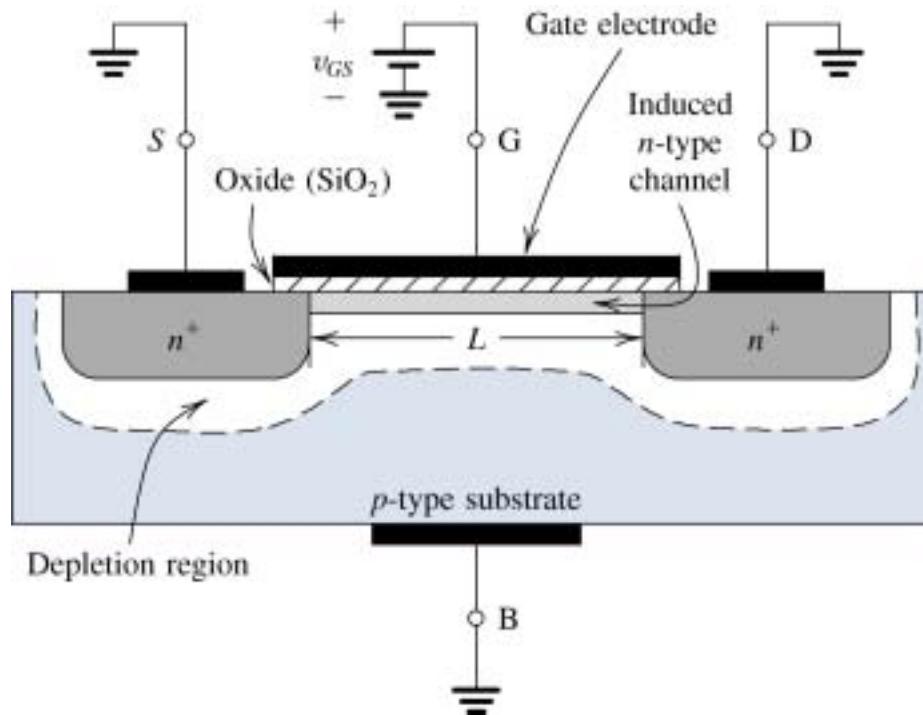
(NMOS)



( PMOS)



## Creating a channel



positive  $v_{GS}$

repel the free holes    a carrier depletion region

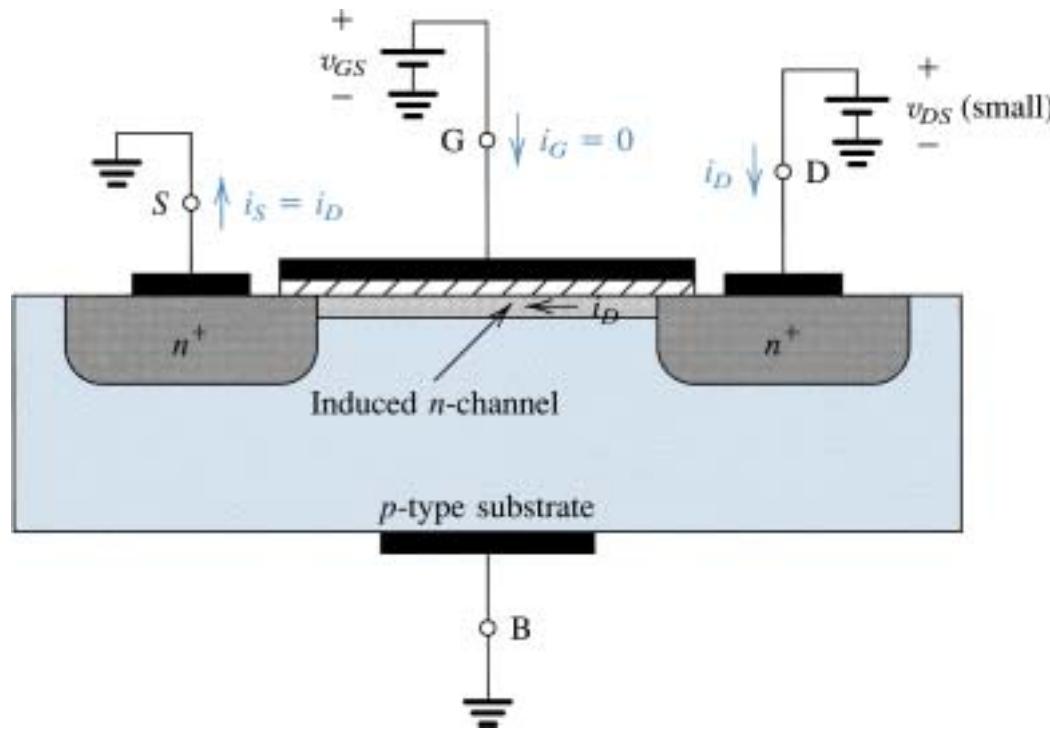
attract electrons from the S & D in the channel region

induced n region    n-channel    inversion layer

threshold voltage  $V_t$

$v_{GS}$  at which a sufficient electrons accumulate in the channel

## Applying a small $v_{DS}$



magnitude of  $i_D$

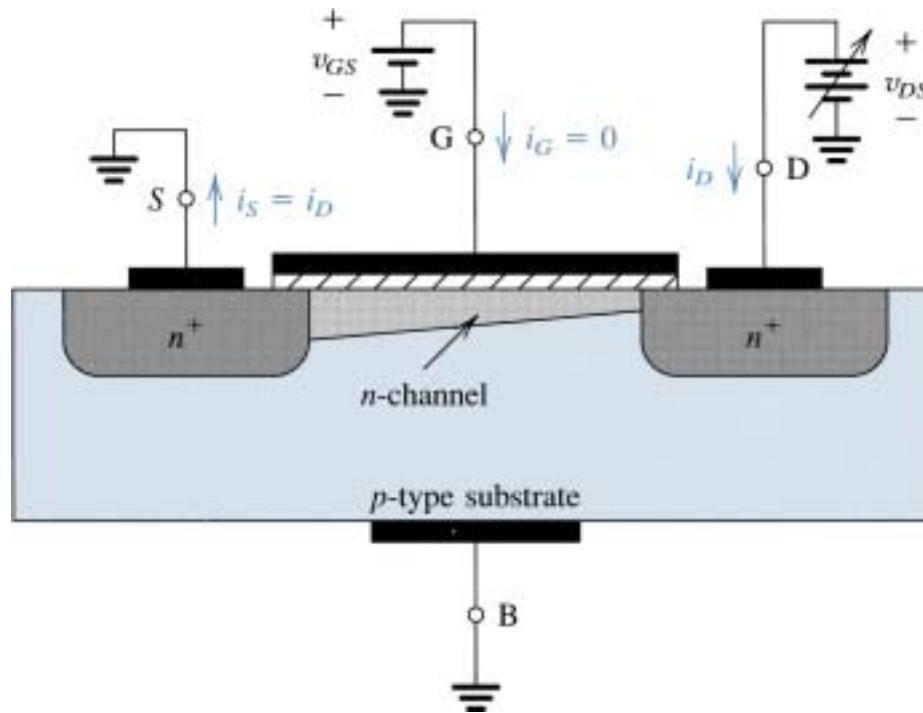
depends on the density of electrons in the channel &  $v_{GS}$

$v_{GS} = V_t$  the channel is just induced  $i_D$  is negligibly small

$v_{GS}$  channel electrons channel depth channel resistance

channel conductance &  $i_D \propto v_{GS} - V_t$  : excess gate voltage

## Operation as $v_{DS}$ is increased



Gate-to-channel voltage

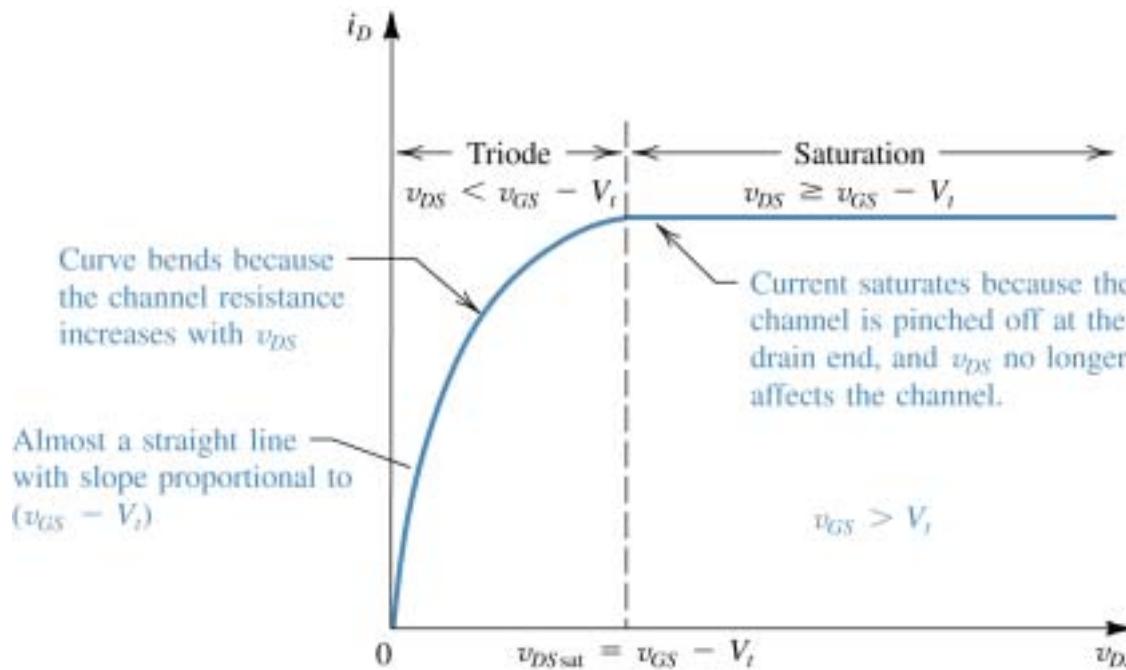
$v_{GS}$  at the source end      deep channel

$v_{GS} - v_{DS}$  at the drain end      shallow channel

$v_{DS}$       channel becomes more tapered      resistance

# 1. MOS FET I-V

( , , )



$v_{GD} = V_t$  or  $v_{DS} = v_{GS} - V_t$   
 the channel depth at the drain end becomes zero      pinched off

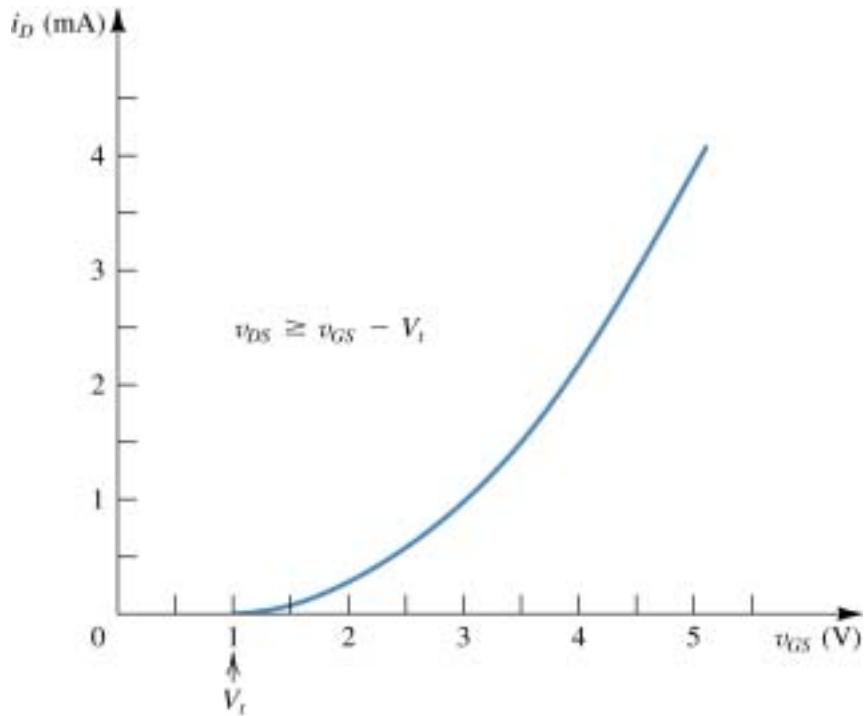
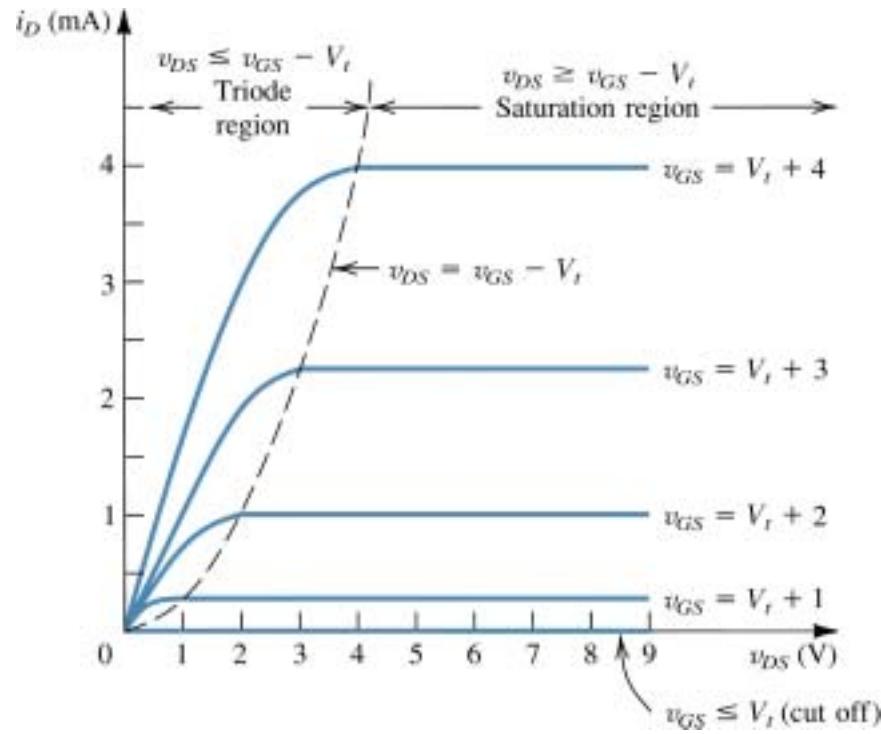
$$v_{DS} > v_{GS} - V_t$$

has little effect on the channel shape

$i_D$  remains constant at the value reached for  $v_{DS} = v_{GS} - V_t$

$i_D$  saturates at this value      saturation region

## I-V characteristics



## Operation mode (NMOS)

<cutoff region>

condition :  $v_{GS} < V_{tn}$

no current flows ideally

<triode region>

condition :  $v_{GS} > V_{tn}$  &  $v_{GD} > V_{tn}$        $v_{DS} < v_{GS} - V_{tn}$

$$i_{DS} = \beta_n \left[ (v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2 \right] \quad \beta_n = k_n' \frac{W}{L} \quad k_n' = \mu_n C_{ox}$$

act as a linear resistor :  $r_{DS} \approx 1/\beta_n(v_{GS} - V_{tn})$

<saturation region>

condition :  $v_{GS} > V_{tn}$  &  $v_{GD} < V_{tn}$        $v_{DS} > v_{GS} - V_{tn}$

$$i_{DS} = \frac{\beta_n}{2} (v_{GS} - V_{tn})^2$$

act as an ideal current source

its value is controlled by  $v_{GS}$

## Operation mode (PMOS)

<cutoff region>

condition :  $v_{SG} < |V_{tp}|$   
no current flows ideally

<triode region>

condition :  $v_{SG} > |V_{tp}| \text{ & } v_{DG} > |V_{tp}| \quad v_{SD} < v_{SG} - |V_{tp}|$   
 $i_{SD} = \beta_p \left[ (v_{SG} - |V_{tp}|)v_{SD} - \frac{1}{2}v_{SD}^2 \right] \quad \beta_p = k_p \frac{W}{L} \quad k_p = \mu_p C_{ox}$

act as a linear resistor :  $r_{SD} \approx 1/\beta_p(v_{SG} - |V_{tp}|)$

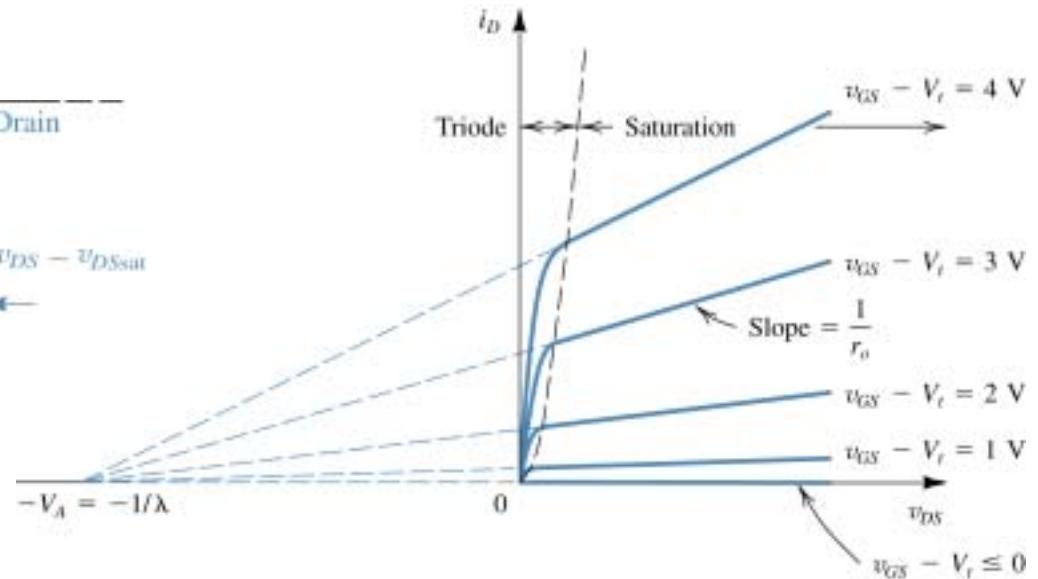
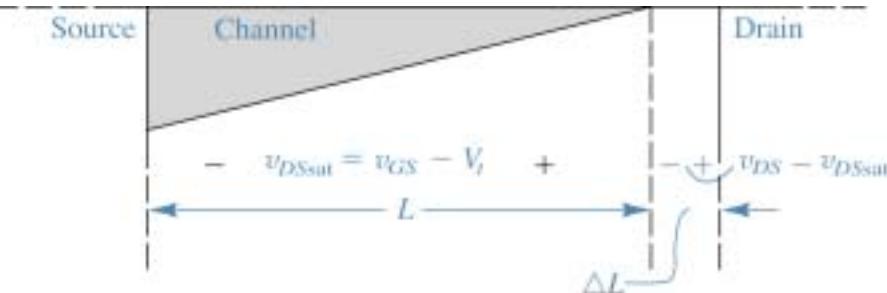
<saturation region>

condition :  $v_{SG} > |V_{tp}| \text{ & } v_{DG} < |V_{tp}| \quad v_{SD} > v_{SG} - |V_{tp}|$   
 $i_{SD} = \frac{\beta_p}{2} (v_{SG} - |V_{tp}|)^2$

act as a ideal current source

its value is controlled by  $v_{SG}$

## Channel-length modulation effect

 $v_{DS}$  $L$  $i_{DS}$ 

$$i_{DS} = \frac{\beta_n}{2} (v_{GS} - V_{tn})^2 (1 + \lambda v_{DS})$$

channel-length modulation factor:  $\lambda = \frac{1}{V_A} \propto \frac{1}{L}$   
 output resistance

$$r_o = \left[ \frac{\partial i_{DS}}{\partial v_{DS}} \right]_{v_{GS}}^{-1} \approx \frac{1}{\lambda I_{DS}} = \frac{V_A}{I_{DS}}$$

## Body effect

$V_{SB} \neq 0$       depletion region      channel depth  
 bulk charge                   $V_t$

$$V_t = V_{t0} + \gamma \left( \sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right)$$

$V_{t0}$  : threshold voltage for  $V_{SB} = 0$

$\phi_f$  : Fermi potential

$\gamma$  : body-effect parameter (process parameter)

$$\gamma = \frac{\sqrt{2qN_A\epsilon_s}}{C_{ox}}$$

for PMOS

$$|V_t| = |V_{t0}| + \gamma \left( \sqrt{2\phi_f + V_{BS}} - \sqrt{2\phi_f} \right)$$

## Temperature effects

$$V_t : -2 \text{mV}/^\circ\text{C} \quad \& \quad \mu \propto T^{-1.5}$$

$$T \quad V_t \quad \& \quad k' \quad i_{DS}$$

## Breakdown

junction breakdown

50 – 100V

occurs in the pn-junction bet. D & B

punch-through : 20V

occurs in short-channel devices

the depletion region(D) extends to the S

does not result in permanent damage

gate-oxide breakdown : 50V

occurs in gate oxide

results in permanent damage

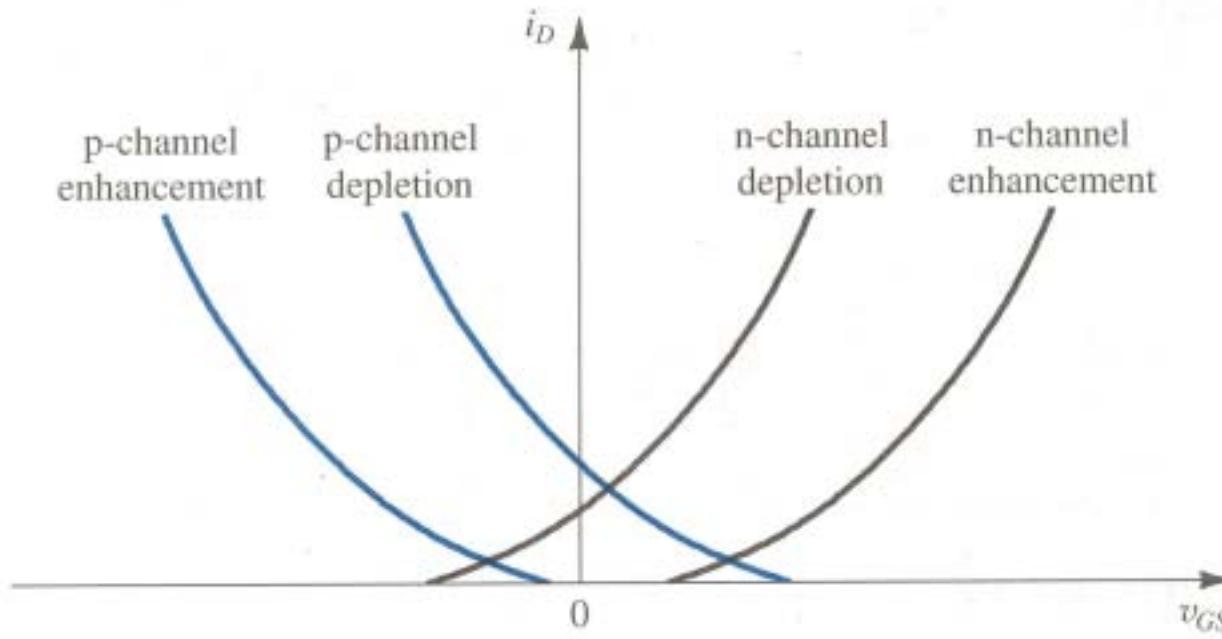
small static charge can cause very high input impedance

gate protection devices are required

## Depletion-type MOSFET

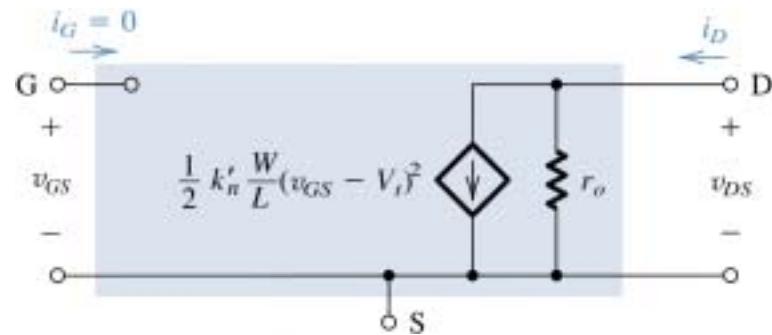
has a physically implanted channel

$$V_{tn} < 0 \quad i_{DS} > 0 \quad \text{for } v_{GS} = 0$$

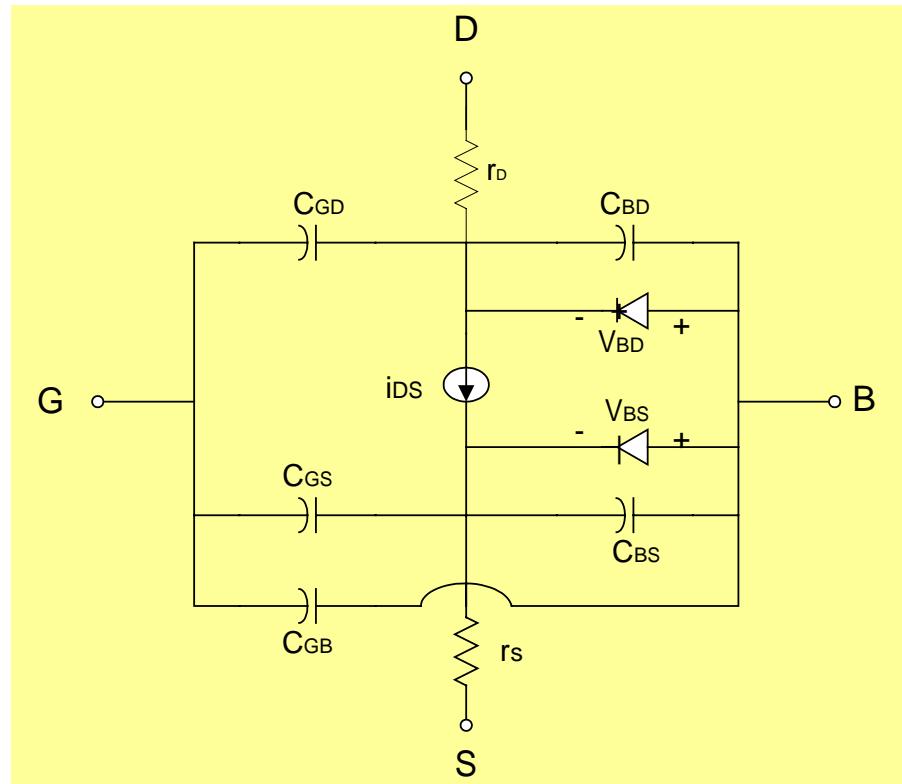


## Large-signal equivalent circuit

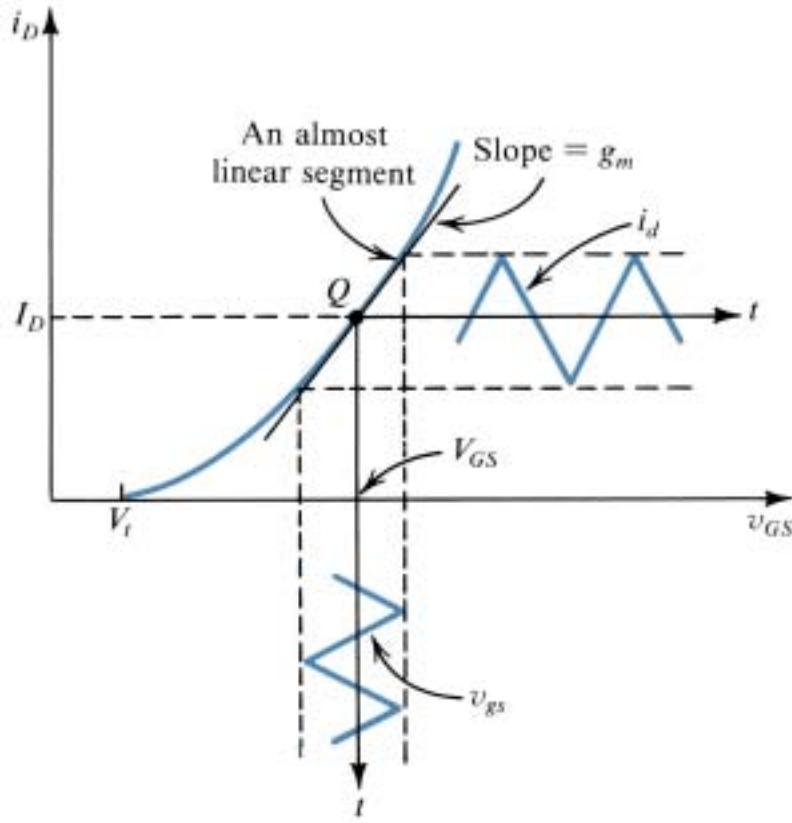
simple model in saturation



complete model



## Small-signal approximation



total current

$$i_D \approx I_D + i_d$$

dc component

$$I_D = \frac{\beta_n}{2} (V_{GS} - V_t)^2$$

ac signal component

$$i_d = \left[ \frac{\partial i_D}{\partial v_{GS}} \right]_{v_{GS}=V_{GS}} \cdot v_{gs} = \beta_n (V_{GS} - V_t) \cdot v_{gs}$$

small-signal condition

$$v_{GS} = V_{GS} + v_{gs}$$

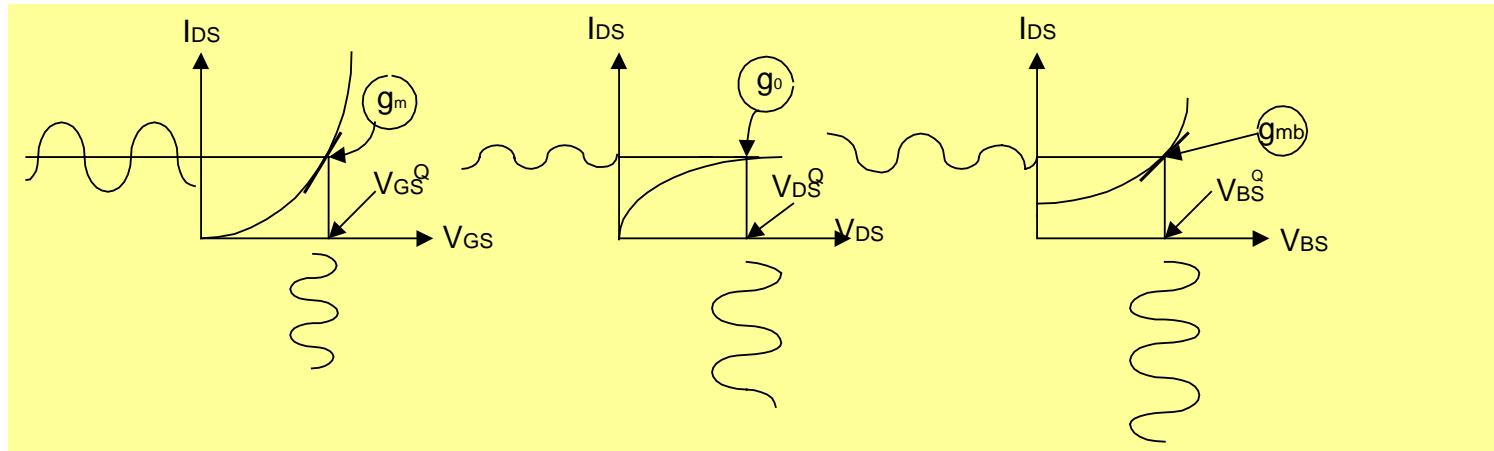
$$\begin{aligned} i_D &= \frac{\beta_n}{2} (V_{GS} + v_{gs} - V_t)^2 \\ &= \frac{\beta_n}{2} (V_{GS} - V_t)^2 + \beta_n (V_{GS} - V_t) v_{gs} + \frac{\beta_n}{2} v_{gs}^2 \\ &= I_D + i_d + i_{d(nl)} \end{aligned}$$

$$\beta_n (V_{GS} - V_t) v_{gs} \gg \frac{\beta_n}{2} v_{gs}^2$$

$$v_{gs} \ll 2(V_{GS} - V_t) = 2V_{OV} \quad (V_{OV} = V_{GS} - V_t)$$

## MOSFET small-signal model

$$\begin{aligned}
 i_{DS} &= \frac{\beta_n}{2} (v_{GS} - V_t)^2 (1 + \lambda v_{DS}) \\
 &= \frac{\beta_n}{2} [v_{GS} - V_{t0} - \gamma (\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})]^2 (1 + \lambda v_{DS}) \\
 &= f(v_{GS}, v_{DS}, v_{BS})
 \end{aligned}$$

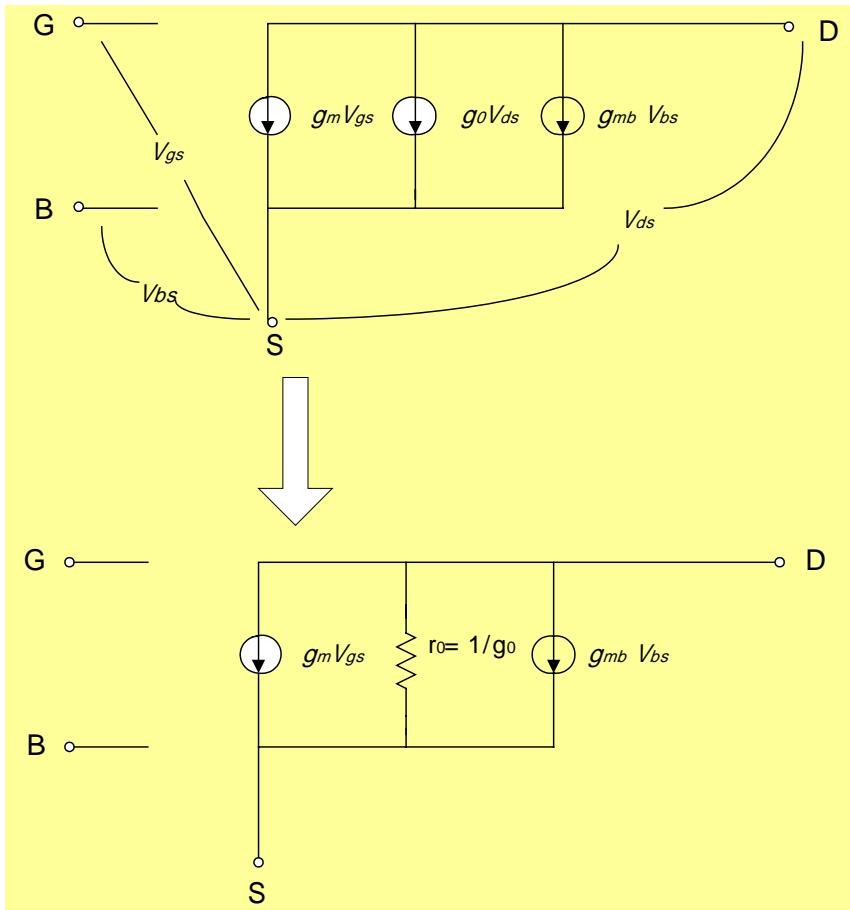


$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$\begin{aligned}
 i_{ds} &\approx \left[ \frac{\partial i_{DS}}{\partial v_{GS}} \Big|_{v_{GS}=V_{GS}} \right] \cdot v_{gs} + \left[ \frac{\partial i_{DS}}{\partial v_{DS}} \Big|_{v_{DS}=V_{DS}} \right] \cdot v_{ds} + \left[ \frac{\partial i_{DS}}{\partial v_{BS}} \Big|_{v_{BS}=V_{BS}} \right] \cdot v_{bs} \\
 &= g_m \cdot v_{gs} + g_o \cdot v_{ds} + g_{mb} \cdot v_{bs}
 \end{aligned}$$

## MOSFET small-signal equivalent circuit

$$i_{ds} = g_m \cdot v_{gs} + g_o \cdot v_{ds} + g_{mb} \cdot v_{bs}$$



transconductance

$$g_m = \beta_n (V_{GS} - V_{tn}) = \sqrt{2\beta_n I_{DS}} = \frac{2I_{DS}}{V_{GS} - V_{tn}}$$

output conductance

$$g_o = \frac{\lambda I_{DS}}{1 + \lambda I_{DS}} \approx \lambda I_{DS}$$

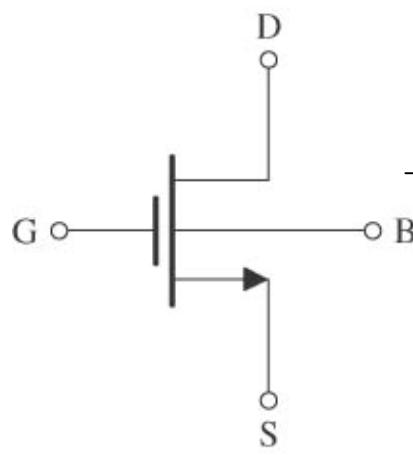
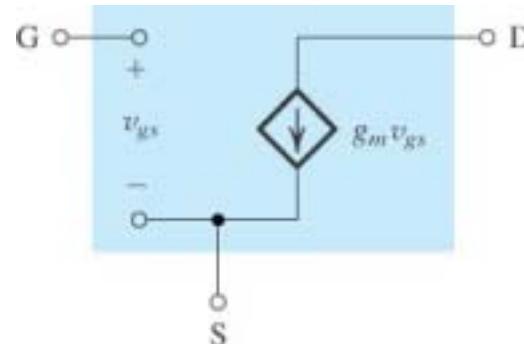
body transconductance

$$g_{mb} = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} = \chi \cdot g_m \quad (\chi < 1)$$

$$g_m > g_{mb} > g_o$$

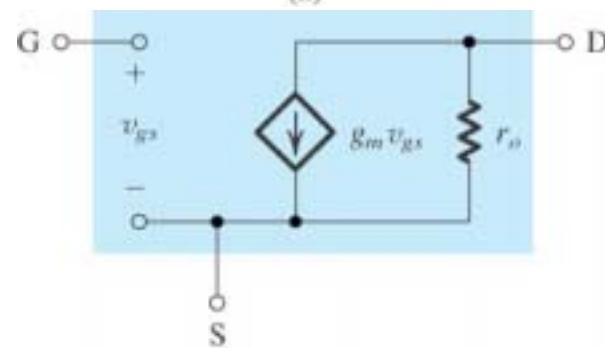


$$\lambda = 0 \quad \& \quad v_{SB} = 0$$

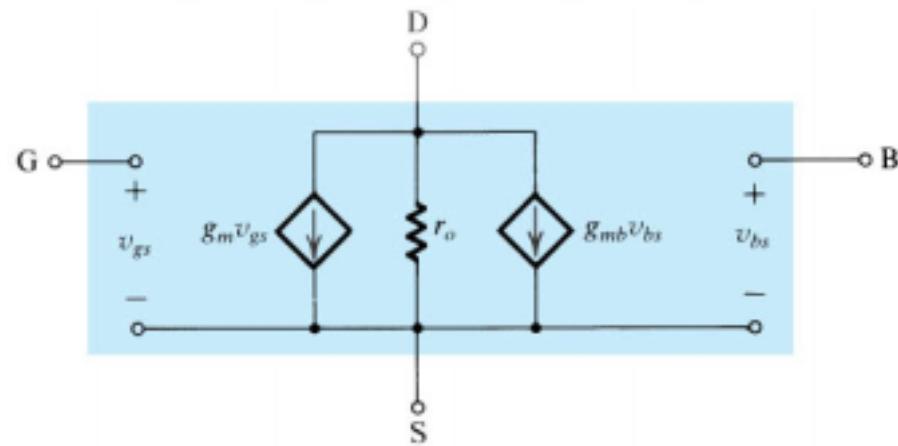


(a)

$$\lambda \neq 0 \quad \& \quad v_{SB} = 0$$

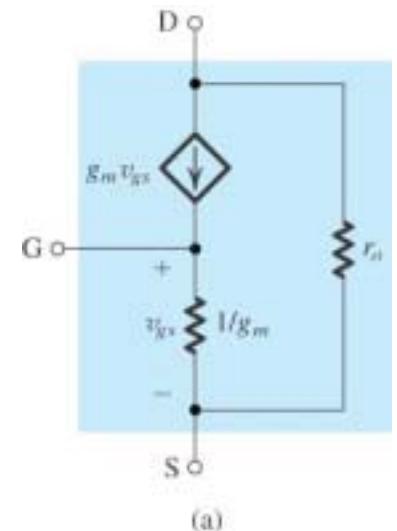
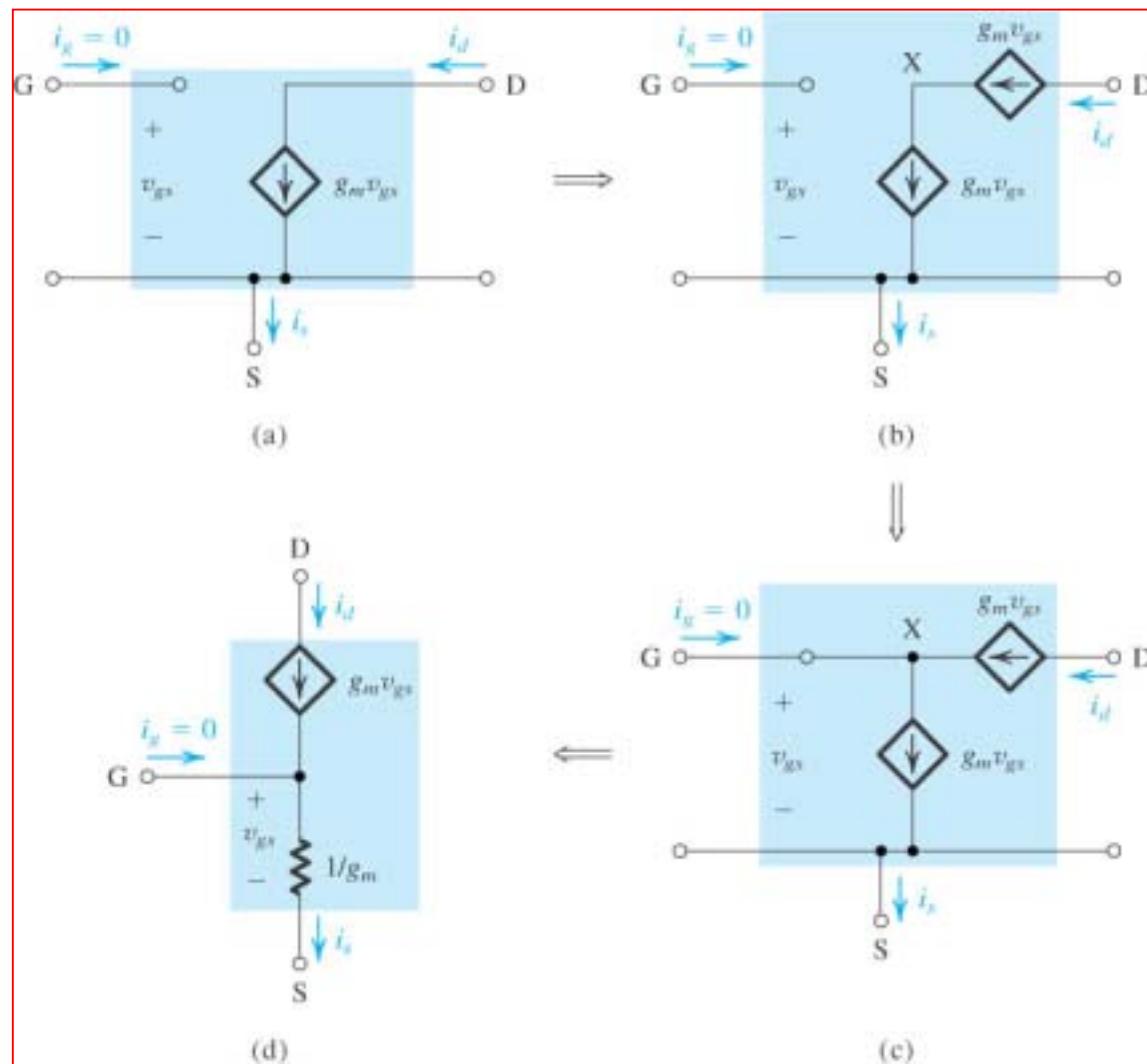


$$\lambda \neq 0 \quad \& \quad v_{SB} \neq 0$$



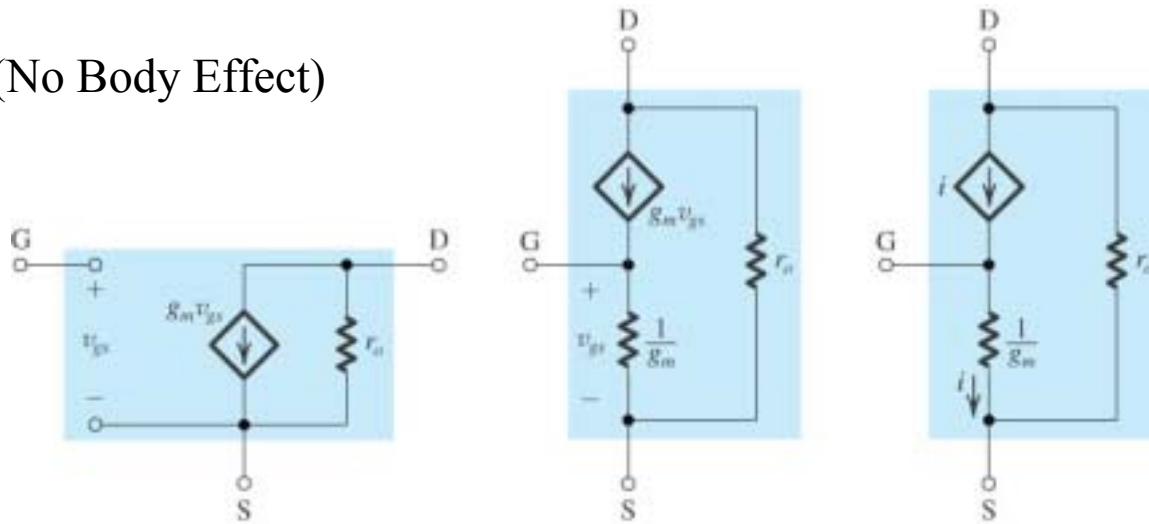
(b)

## T-equivalent circuit

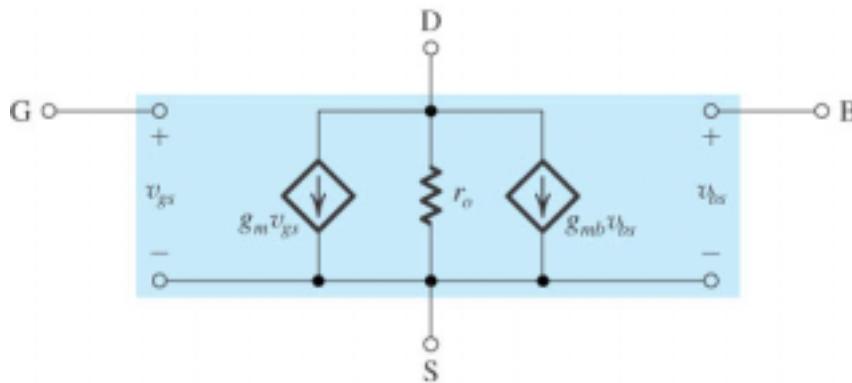


# Summary (small-signal low-freq. equivalent circuit)

$|V_{SB}|=0$  (No Body Effect)



$|V_{SB}| \neq 0$  (including the Body Effect)



$$g_m = \beta_n V_{OV} = \sqrt{2\beta_n I_D} = \frac{2I_D}{V_{OV}}$$

$$r_o = \frac{1}{M_D} = \frac{V_A}{I_D}$$

$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} g_m$$

## Transconductance

$$g_m = \beta_n (V_{GS} - V_{tn}) = k'_n (W/L) (V_{GS} - V_{tn})$$

$$\propto (V_{GS} - V_{tn})$$

$$V_{GS} \qquad \qquad g_m$$

signal swing

$$g_m = \sqrt{2\beta_n I_{DS}} = \sqrt{2k'_n (W/L) I_{DS}}$$

$$\propto \sqrt{I_{DS}} \text{ for a given MOSFET } (\text{BJT: } \propto I_{DS})$$

$$\propto \sqrt{W/L} \text{ for a given bias current}$$

$$I_{DS} = 1\text{mA}, k'_n = 20\mu\text{A/V}^2 \rightarrow g_m = 0.2\text{mA/V} \text{ for } W/L = 1$$

$$\rightarrow g_m = 2\text{mA/V} \text{ for } W/L = 100$$

$$(\text{BJT: } g_m = 40\text{mA/V} \text{ for } I_C = 1\text{mA})$$

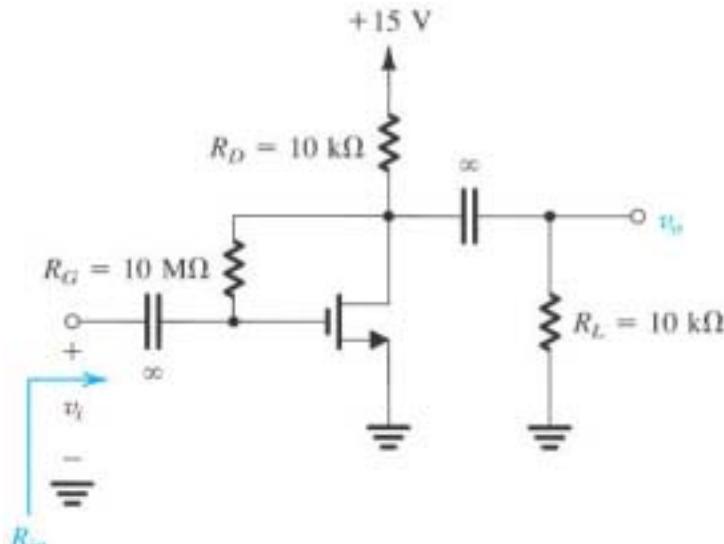
$$g_m = \frac{2I_{DS}}{V_{GS} - V_{tn}} = \frac{I_{DS}}{(V_{GS} - V_{tn})/2} \quad (\text{BJT: } g_m = \frac{I_C}{V_T})$$

$$g_m = \beta \cdot V_{OV} = \sqrt{2\beta I_{DS}} = \frac{2I_{DS}}{V_{OV}} \quad (V_{OV} = V_{GS} - V_{tn})$$

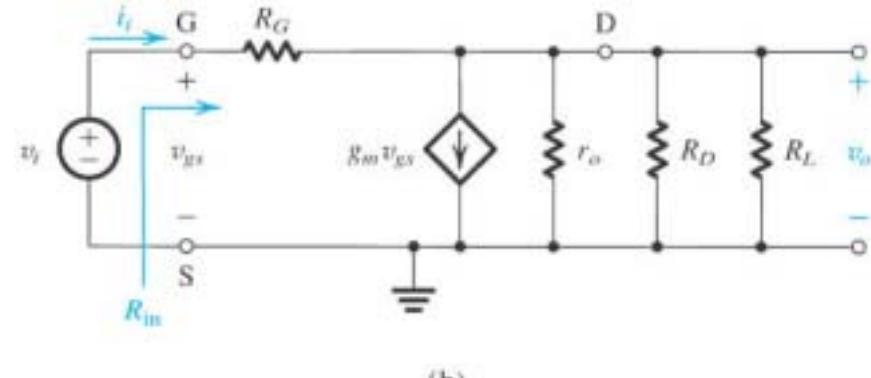
# Example 2.1

Q) small-signal voltage gain & input resistance & the largest input signal?

$$V_{tn} = 1.5\text{V}, \quad \beta_n = k_n(W/L) = 0.25\text{mA/V}^2, \quad V_A = 1/\lambda = 50\text{V}$$



(a)



(b)

<dc analysis>

$$I_D = \frac{\beta_n}{2} (V_{GS} - V_{tn})^2 = \frac{0.25}{2} (V_D - 1.5)^2$$

$$V_D = 15 - R_D I_D = 15 - 10 I_D$$



$$\begin{aligned} I_D &= 1.06\text{mA} \\ V_D &= 4.4\text{V} \end{aligned}$$

<ac analysis>

$$g_m = \beta_n V_{OV} = 0.25(4.4 - 1.5) = 0.725 \text{mA/V}$$

$$r_o = \frac{1}{M_D} = \frac{V_A}{I_D} = \frac{50}{1.06} = 47 \text{k}\Omega$$

voltage gain

$$A_v = \frac{v_o}{v_i} = -g_m (R_D // R_L // r_o) = -3.3 \text{V/V}$$

input resistance

$$i_i = \frac{v_i - v_o}{R_G} = \frac{v_i}{R_G} \left( 1 - \frac{v_o}{v_i} \right) = \frac{v_i}{R_G} [1 - (-3.3)] = \frac{4.3v_i}{R_G} \quad R_{in} = \frac{v_i}{i_i} = \frac{R_G}{4.3} = 2.33 \text{M}\Omega$$

the largest input signal

$$v_{DS} \geq v_{GS} - V_{tn}$$

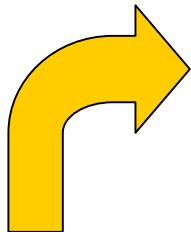
$$\rightarrow v_{DS\min} = v_{GS\max} - V_{tn}$$

$$V_{DS} - |A_v| v_{i\max} = V_{GS} + v_{i\max} - V_{tn}$$

$$4.4 - 3.3v_{i\max} = 4.4 + v_{i\max} - 1.5$$

$$v_{i\max} = 0.34 \text{V}$$

# PSPICE Example



$$V_t = 1.5V \rightarrow VTO = 1.5, \text{ GAMMA} = 0$$

$$\beta_n = 0.25\text{mA/V}^2 \rightarrow KP = 250\text{U}$$

$$L = 10\text{U}, W = 10\text{U}$$

$$V_A = 50V \rightarrow \lambda = 0.02$$

$$\rightarrow \text{LAMBDA} = 0.02$$

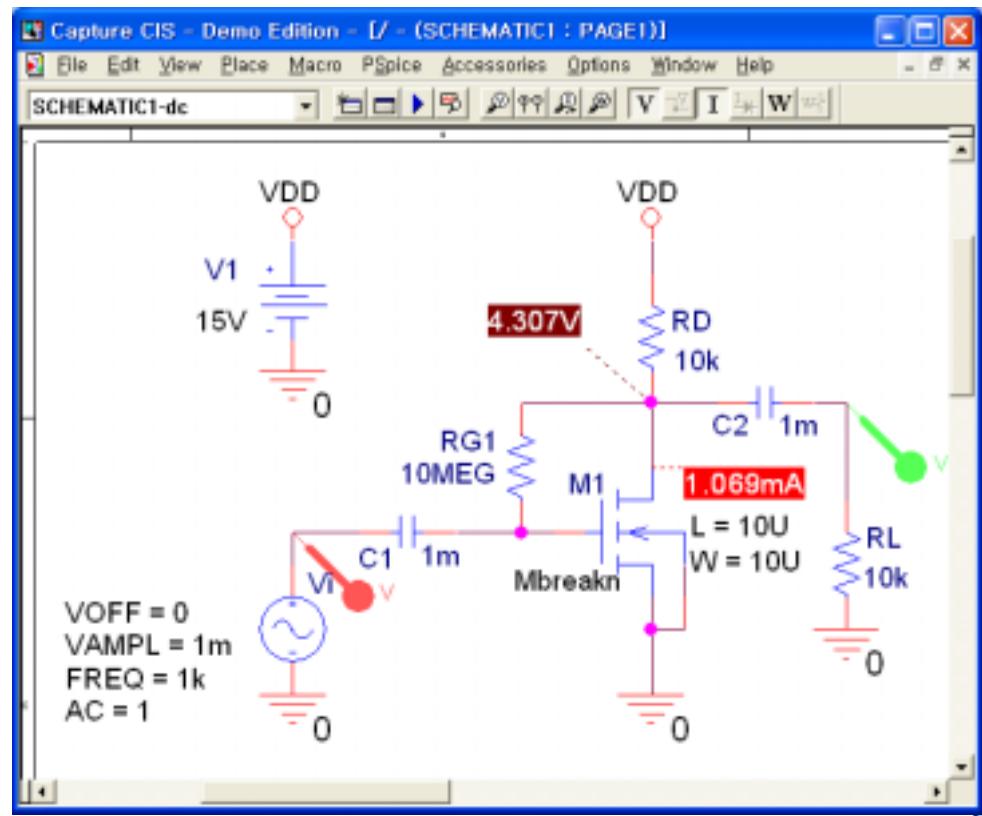
ch4\_ex4\_10:Mbreakn - PSpice Model Editor Demo - [...]

File Edit View Model Plot Tools Window Help

Models List

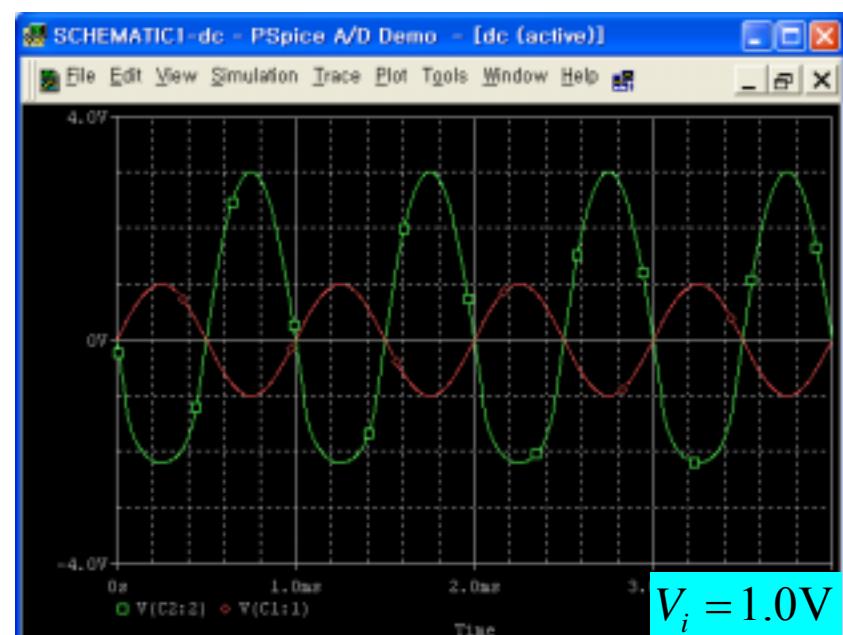
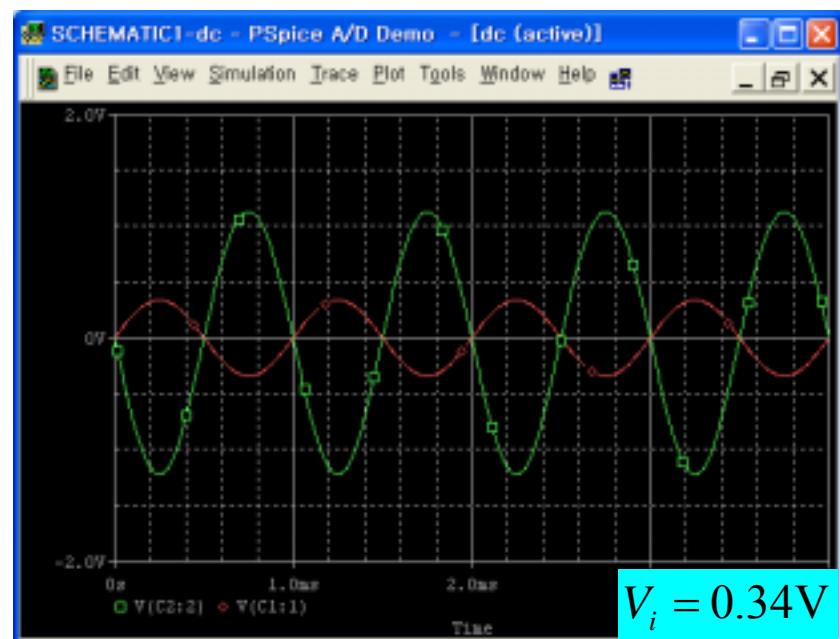
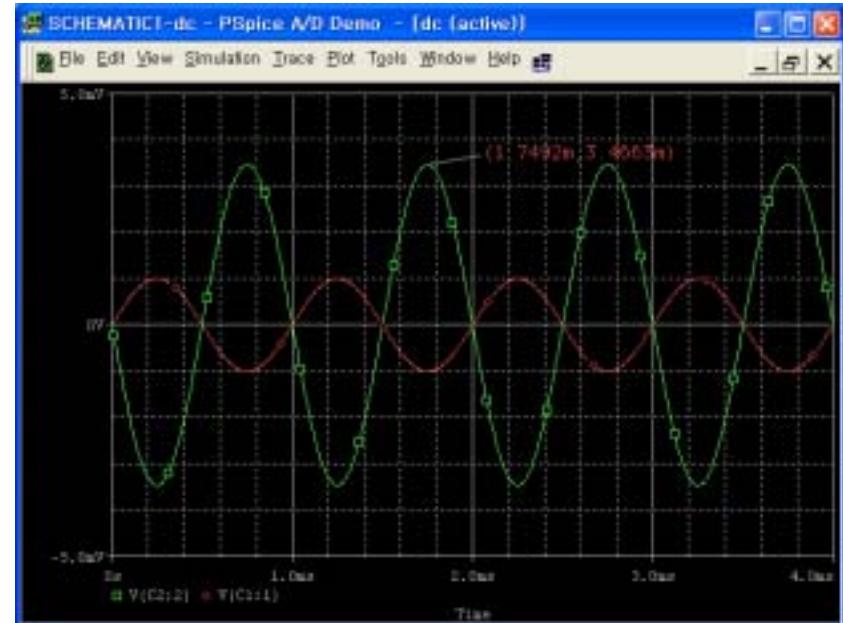
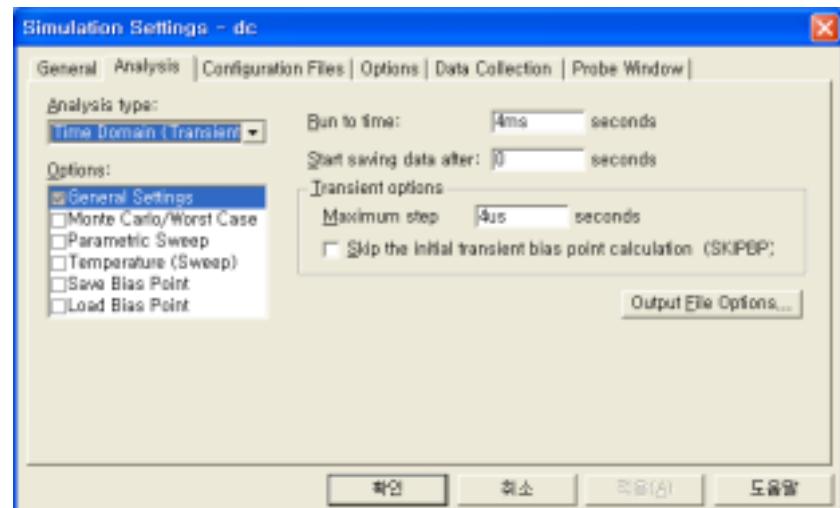
```
.model Mbreakn NMOS LEVEL=1 VTO=1.5 KP=250U
+ LAMBDA=0.02 GAMMA=0
```

Ready

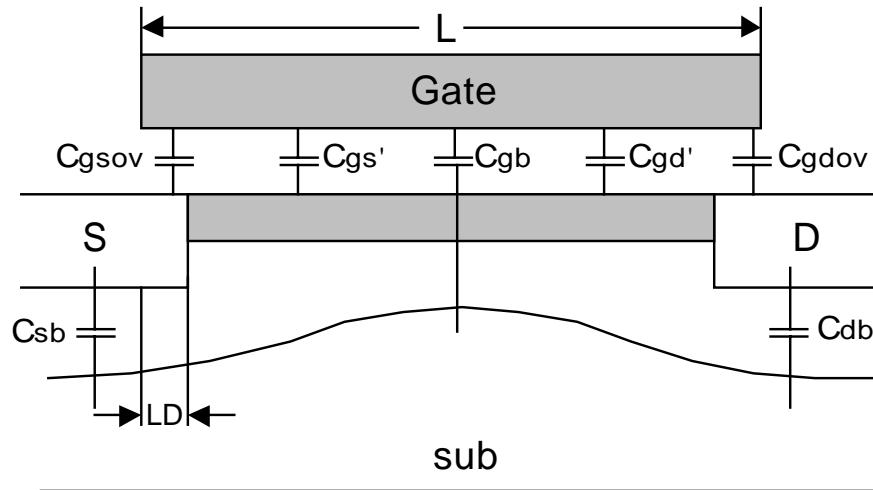


2.

가 , 가



## Capacitors in MOSFET



$$L_{\text{eff}} = L' = L - 2LD, \quad LD = L_{\text{ov}}$$

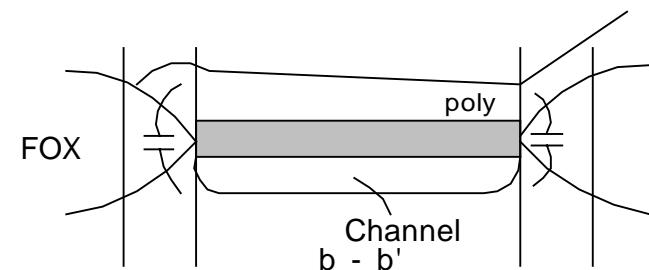
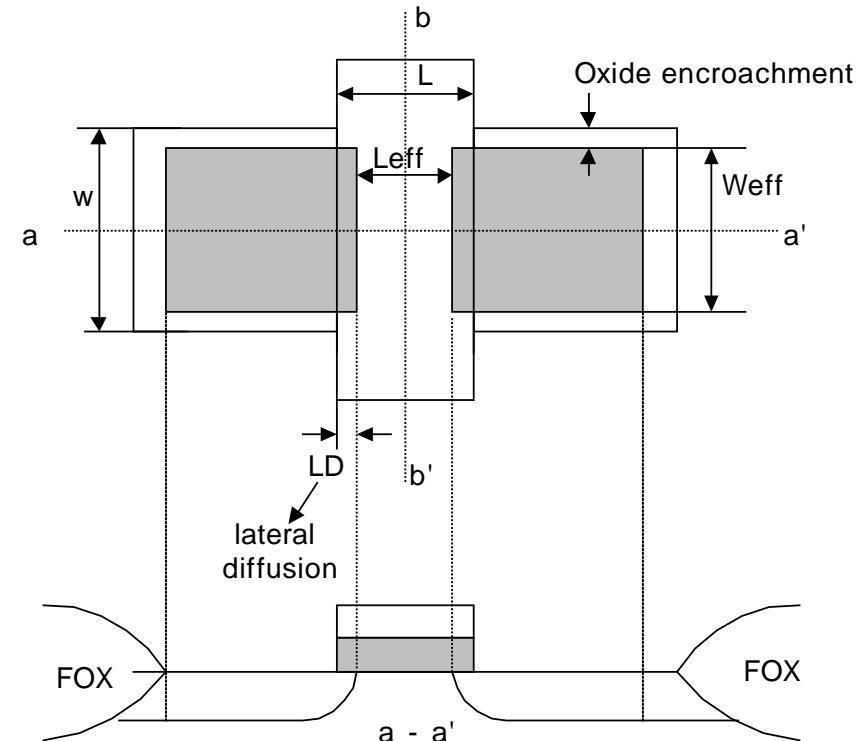
$C_{gs} = C_{gs'} + C_{gsov}$  : gate to source capacitance

$C_{gd} = C_{gd'} + C_{gdov}$  : gate to drain capacitance

$C_{gb}$  : gate to bulk capacitance ( $C_{gbov}$  is negligible)

$C_{sb}$  : source to substrate capacitance

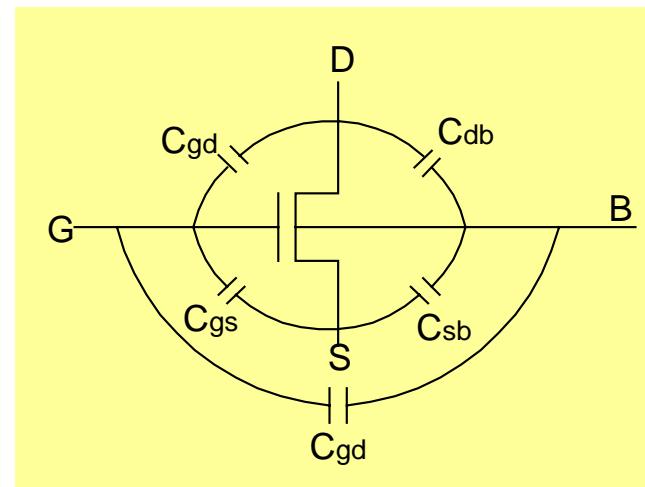
$C_{db}$  : drain to substrate capacitance



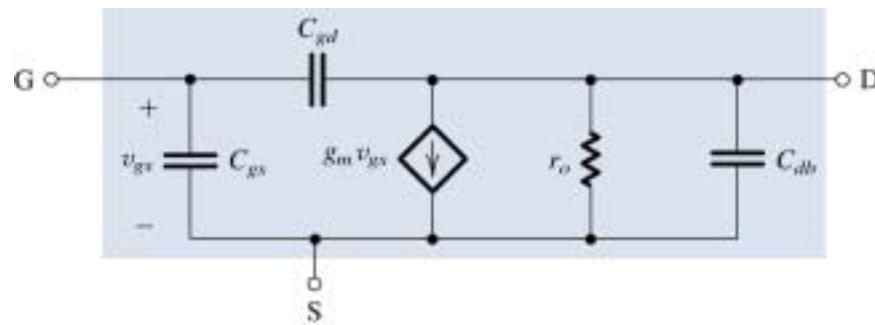
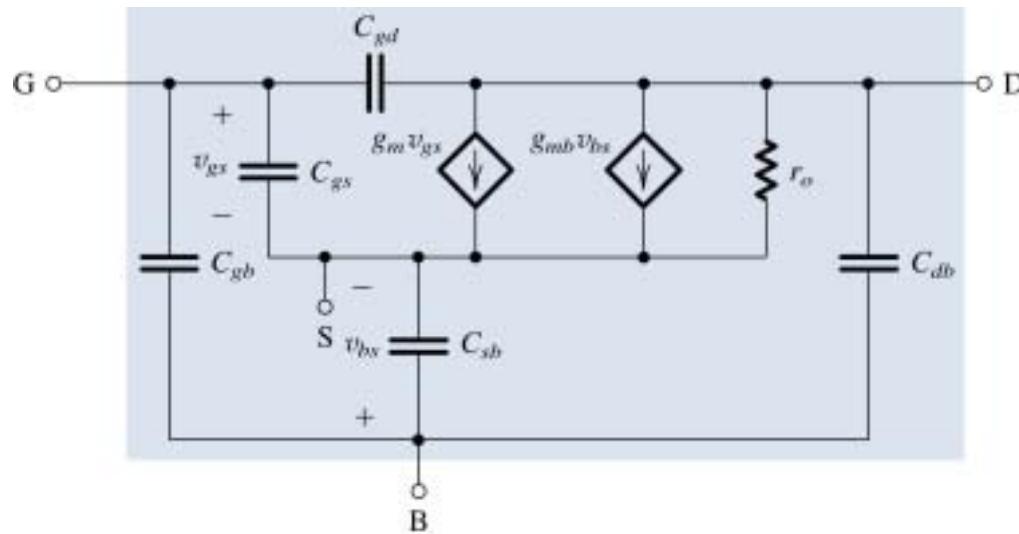
Region	Cutoff	Saturation	Linear
$C_{gs}$	$WL_{ov}C_{ox}$	$WC_{ox}[L_{ov} + (2/3)L']$	$WC_{ox}[L_{ov} + (1/2)L']$
$C_{gd}$	$WL_{ov}C_{ox}$	$WL_{ov}C_{ox}$	$WC_{ox}[L_{ov} + (1/2)L']$
$C_{gb}$	$WL'C_{ox}$	$(1/3)WL'(C_{ox}/C_{dep})$	0
$C_{sb}$	$A_e C_{ja}(V_{sb}) + P_e C_{jp}(V_{sb})$	$A_e C_{ja}(V_{sb}) + P_e C_{jp}(V_{sb}) + (2/3)WL'C_{ja}(V_{sb})$	$A_e C_{ja}(V_{sb}) + P_e C_{jp}(V_{sb}) + (1/2)WL'C_{ja}(V_{sb})$
$C_{db}$	$A_d C_{ja}(V_{db}) + P_d C_{jp}(V_{db})$	$A_d C_{ja}(V_{db}) + P_d C_{jp}(V_{db}) + (1/2)WL'C_{ja}(V_{db})$	$A_d C_{ja}(V_{db}) + P_d C_{jp}(V_{db}) + (1/2)WL'C_{ja}(V_{db})$
$C_g$	$WLC_{ox}$		$WLC_{ox}$

Total gate capacitance

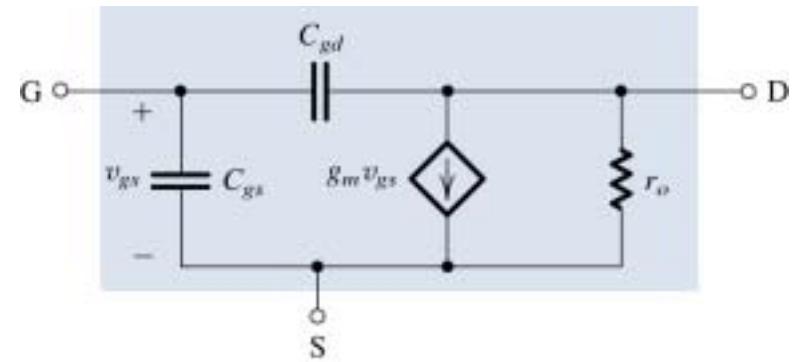
$$C_g = C_{gs} + C_{gd} + C_{gb}$$



## MOSFET high-frequency equivalent circuit

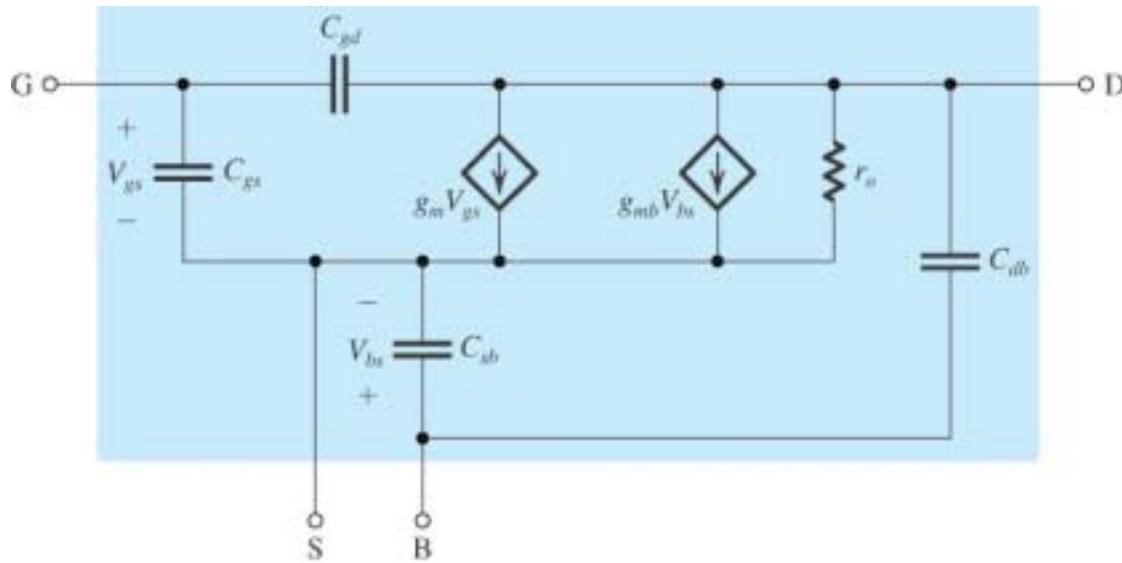


(b)



(c)

# Summary (small-signal high-freq. equivalent circuit)



$$g_m = \beta_n V_{OV} = \sqrt{2\beta_n I_D} = \frac{2I_D}{V_{OV}}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{V_A}{I_D}$$

$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} g_m$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox}$$

$$C_{gd} = WL_{ov} C_{ox}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}} \quad C_{db} = -\frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}}$$

### Example 3.1

Design the circuit  $R_D$ ,  $R_S$ ?

$$I_D = 0.4\text{mA}, V_D = 1\text{V}$$

$$V_t = 2\text{V}, \mu_n C_{ox} = 20\mu\text{A/V}^2$$

$$L = 10\mu\text{m}, W = 400\mu\text{m}, \lambda = 0$$

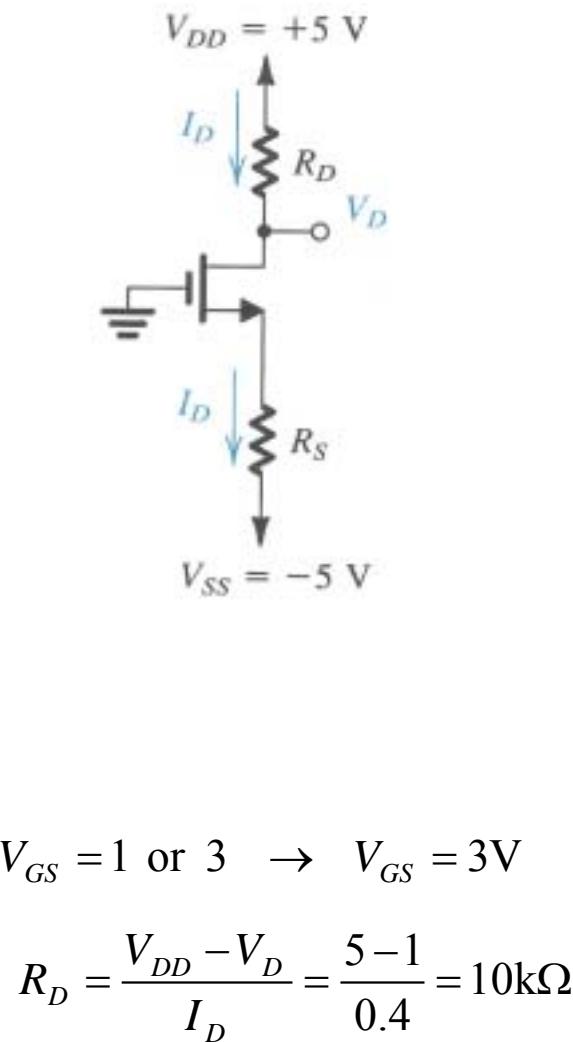
### Solution

$$V_{GD} = -1\text{V} < V_t \quad \therefore \text{saturation}$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2$$

$$0.4 = \frac{20 \times 10^{-3}}{2} \frac{400}{10} (V_{GS} - 2)^2 \quad \rightarrow \quad V_{GS} = 1 \text{ or } 3 \quad \rightarrow \quad V_{GS} = 3\text{V}$$

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-3 - (-5)}{0.4} = 5\text{k}\Omega$$



$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 1}{0.4} = 10\text{k}\Omega$$

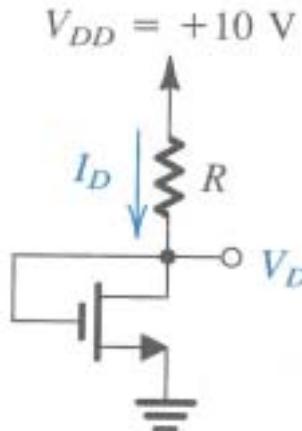
**Example 3.2**

Design the circuit  $R$ ,  $V_D$  ?

$$I_D = 0.4\text{mA}$$

$$V_t = 2\text{V}, \mu_n C_{ox} = 20\mu\text{A/V}^2$$

$$L = 10\mu\text{m}, W = 100\mu\text{m}, \lambda = 0$$

**Solution**

$$V_{GD} = 0\text{V} < V_t \therefore \text{saturation}$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2$$

$$0.4 = \frac{20 \times 10^{-3}}{2} \frac{100}{10} (V_{GS} - 2)^2 \rightarrow V_{GS} = 4 \text{ or } 0 \rightarrow V_{GS} = V_D = 4\text{V}$$

$$R = \frac{V_{DD} - V_D}{I_D} = \frac{10 - 4}{0.4} = 15\text{k}\Omega$$

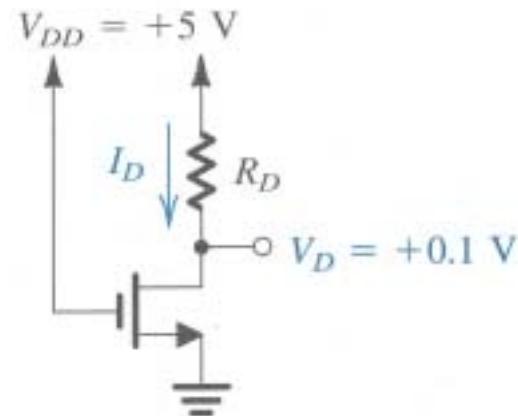
**Example 3.3**

Design the circuit  $R_D$  ?

$$V_D = 0.1\text{V}$$

$$V_t = 1\text{V}, \beta_n = \mu_n C_{ox} (W/L) = 1\text{mA/V}^2$$

Effective resistance between D & S,  $r_D$  ?

**Solution**

$$V_{GD} = 4.9\text{V} > V_t \quad \therefore \text{triode}$$

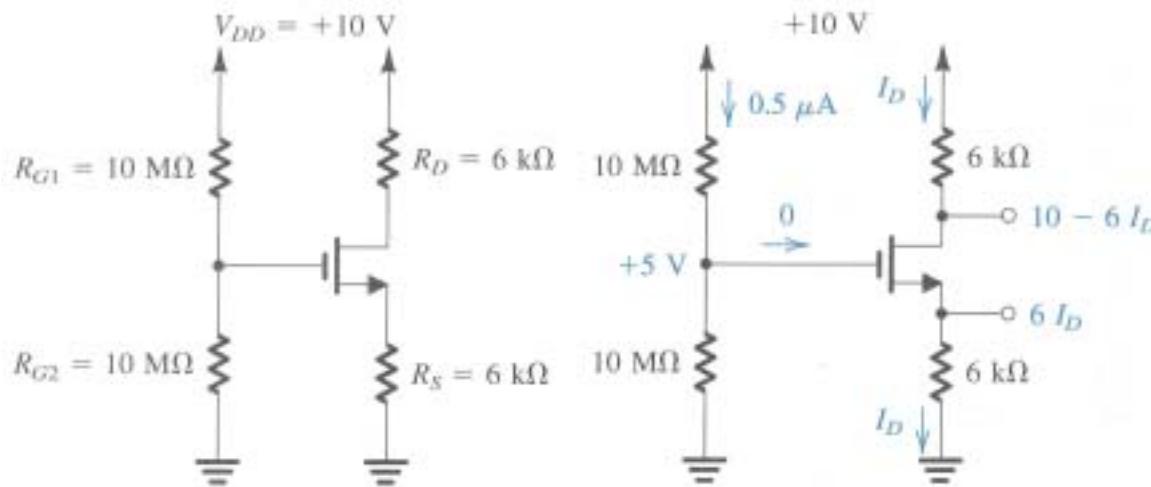
$$I_D = \beta_n \left[ (V_{GS} - V_t)V_{DS} - \frac{V_{DS}^2}{2} \right] = 1 \times \left[ (5 - 1) \times 0.1 - \frac{0.01}{2} \right] = 0.359\text{mA}$$

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 0.1}{0.395} = 12.4\text{k}\Omega$$

$$r_{DS} = \frac{V_{DS}}{I_D} = \frac{0.1}{0.395} = 253\Omega$$

### Example 3.4

Determine the node voltages and the branch currents ( $V_t = 1\text{V}$ ,  $\beta_n = 1\text{mA/V}^2$ ,  $\lambda = 0$ )



### Solution

Assume saturation:

$$V_{GS} = V_G - 6I_D = 5 - 6I_D \quad I_D = \frac{\beta_n}{2}(V_{GS} - V_t)^2 = \frac{1}{2}(5 - 6I_D - 1)^2$$

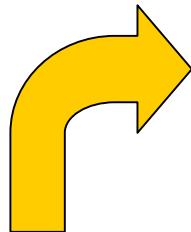
$$18I_D^2 - 25I_D + 8 = 0 \rightarrow I_D = 0.89\text{mA} \text{ or } 0.5\text{mA}$$

$$I_D = 0.89\text{mA} \rightarrow V_S = 0.89 \times 6 = 5.34\text{V} \rightarrow \text{cutoff} \quad \therefore I_D = 0.5\text{mA}$$

$$V_S = 3\text{V}, V_{GS} = 2\text{V}, V_D = 10 - 6 \times 0.5 = 7\text{V}$$

$$V_{GD} = -2\text{V} < V_t \quad \therefore \text{saturation}$$

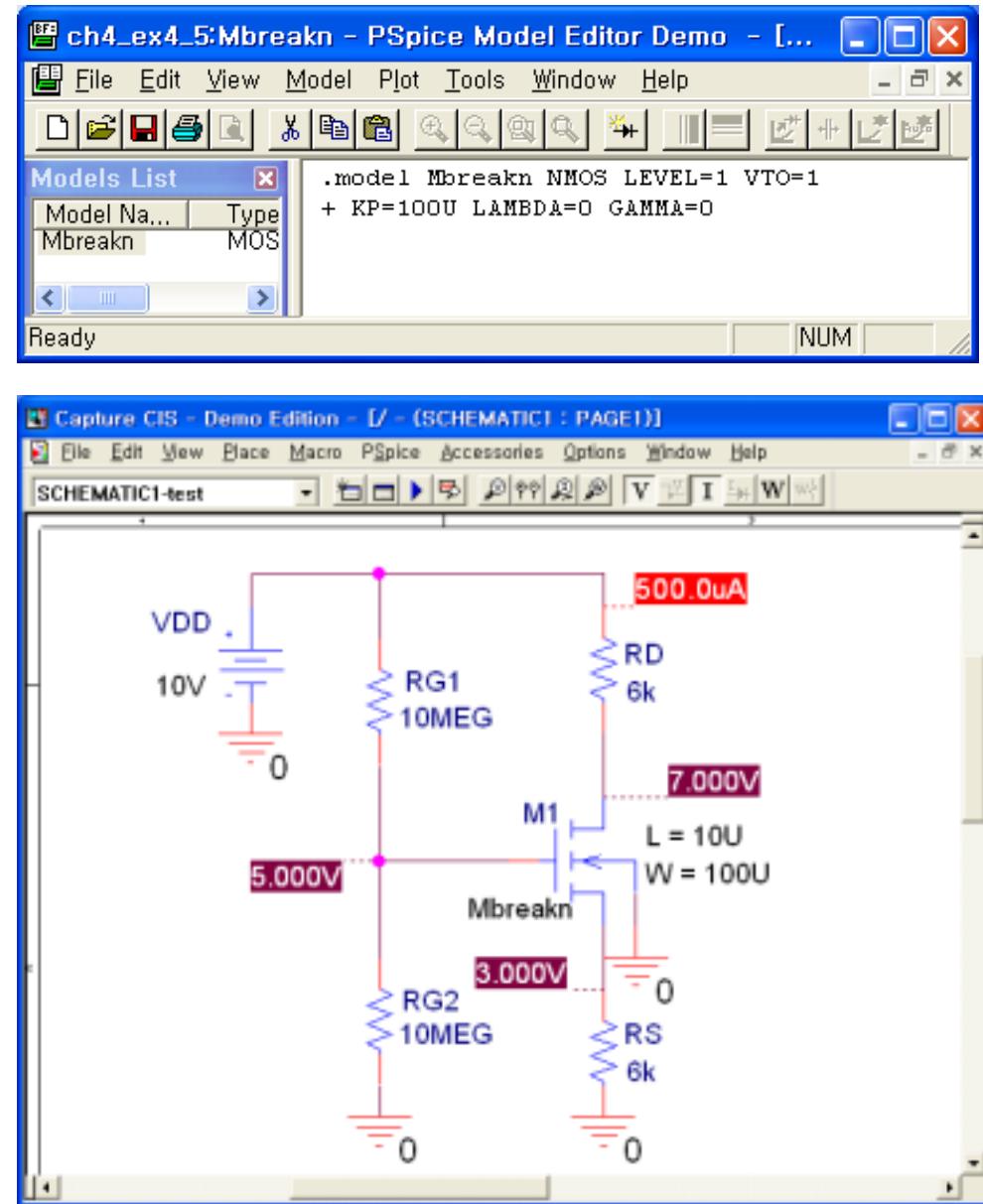
# PSPICE Example



$$V_t = 1V \rightarrow VTO = 1, \text{ GAMMA} = 0$$

$$\beta_n = 1\text{mA/V}^2 \rightarrow KP = 100E - 6 \\ L = 10\text{U}, W = 100\text{U}$$

$$\lambda = 0 \rightarrow \text{LAMBDA} = 0$$



**Example 3.5**

Design the circuit (saturation)

$$I_D = 0.5 \text{ mA}, V_D = 3 \text{ V}$$

$$V_t = -1 \text{ V}, \beta_p = 1 \text{ mA/V}^2, \lambda = 0$$

Largest  $R_D$  for saturation ?

Solution

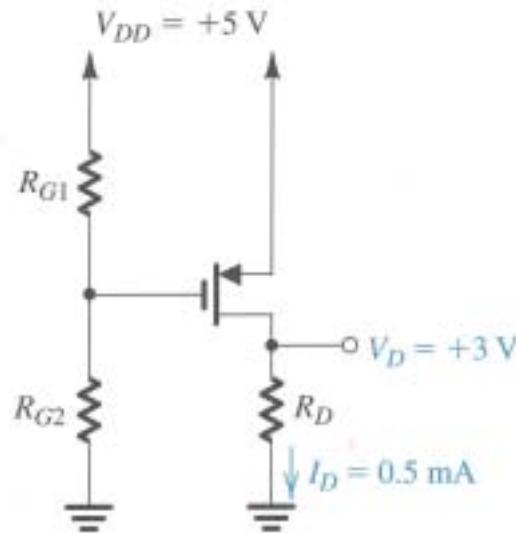
$$I_{SD} = \frac{\beta_p}{2} (V_{SG} - |V_t|)^2$$

$$0.5 = \frac{1}{2} (V_{SG} - 1)^2 \rightarrow V_{SG} = 2 \text{ or } 0 \rightarrow V_{SG} = 2 \text{ V}$$

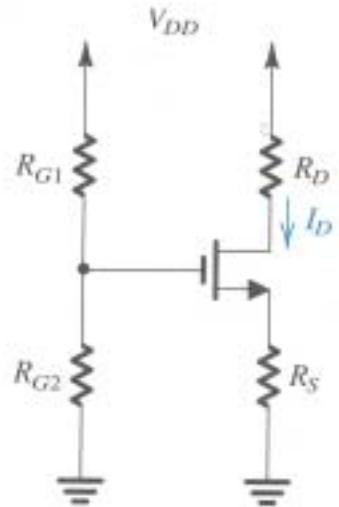
$$\therefore V_G = 3 \text{ V} \rightarrow R_{G1} = 2 \text{ M}\Omega \text{ & } R_{G2} = 3 \text{ M}\Omega, \quad R_D = V_D / I_D = 6 \text{ k}\Omega$$

For saturation :

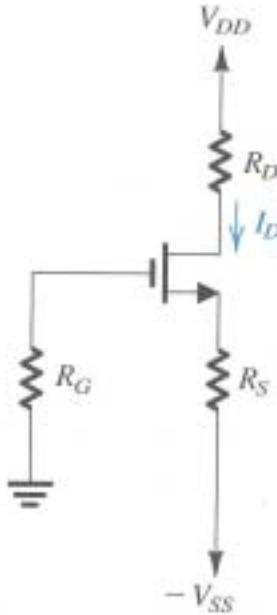
$$V_{DG} \leq |V_t| \rightarrow V_D \leq V_G + |V_t| = 4 \text{ V} \quad \therefore V_{D,\max} = 4 \text{ V}, \quad R_{D,\max} = 4 / 0.5 = 8 \text{ k}\Omega$$



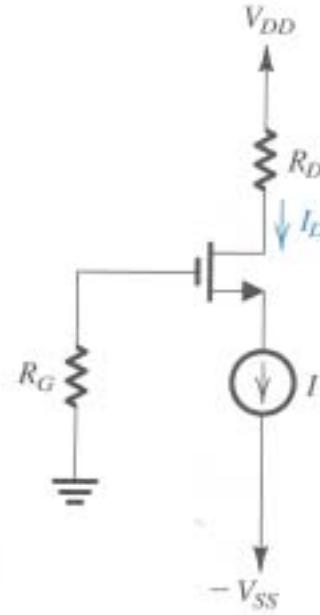
## Biasing in discrete circuits



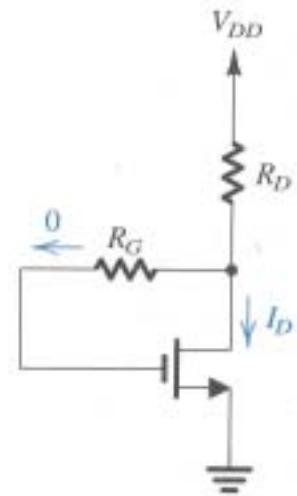
(a)



(b)



(c)



(d)

### Biassing or Bias design

establishment of a dc operating point for the transistors  
stable and predictable  $I_D$  and  $V_{DS}$

somewhat easier     $I_G = 0$      $R_{G1} \& R_{G2}$  can be very high    high  $R_{in}$   
self-bias resistor  $R_S$     provides negative feedback    stabilize  $I_D$   
 $R_D$  should be large for high gain but small enough for desired signal swing

## Biassing in integrated circuits

the use of resistors are discouraged in MOS IC design

- requires relatively large area

- exhibits large tolerances

the use of large coupling and bypass capacitors is impossible

MOSFETs are utilized

- can be fabricated in a small area

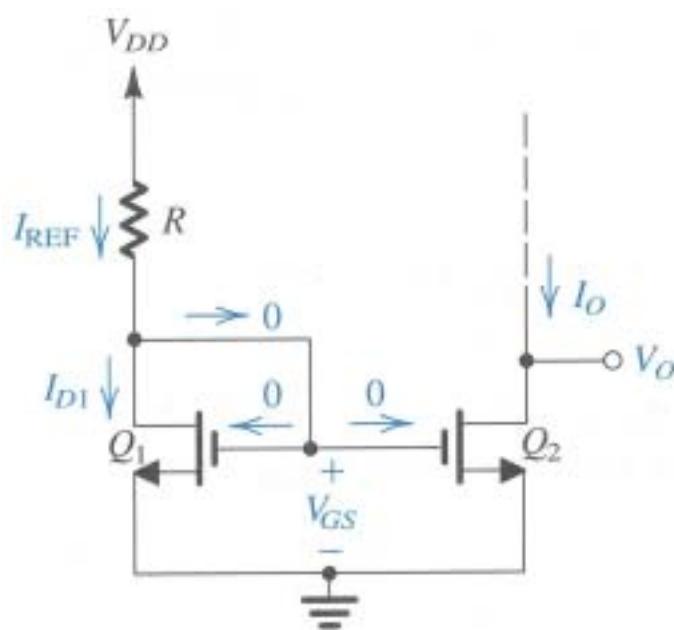
- its parameters are relatively well controlled

biassing of IC MOS amplifiers utilizes constant current sources

## Basic MOSFET current source

a simple MOS current source or current mirror  
resistor R would be outside the IC chip

assume  $\lambda = 0$



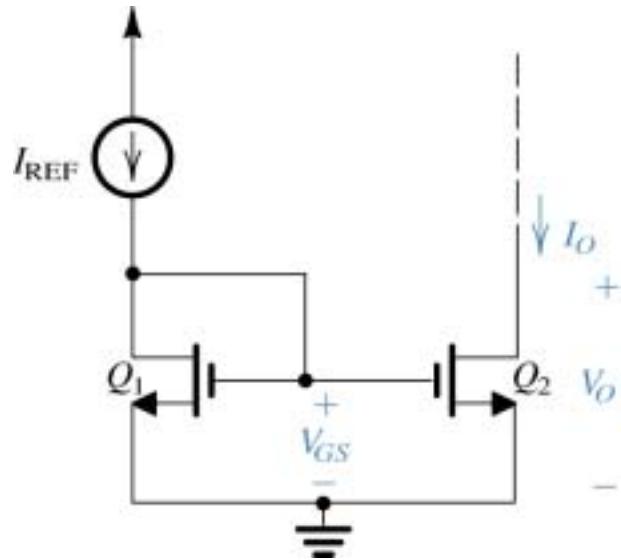
$$I_{D1} = \frac{k_n'}{2} \left( \frac{W}{L} \right)_1 (V_{GS} - V_t)^2$$

$$I_{D1} = I_{REF} = \frac{V_{DD} - V_{GS}}{R}$$

$$I_{D2} = I_O = \frac{k_n'}{2} \left( \frac{W}{L} \right)_2 (V_{GS} - V_t)^2$$

$$\frac{I_O}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1}$$

## Basic MOSFET current mirror



Q2 should operate in saturation

$$V_O \geq V_{GS} - V_t$$

channel-length modulation effect

$$\frac{I_O}{I_{REF}} = \frac{(W/L)_2(1 + \lambda V_{DS2})}{(W/L)_1(1 + \lambda V_{DS1})}$$

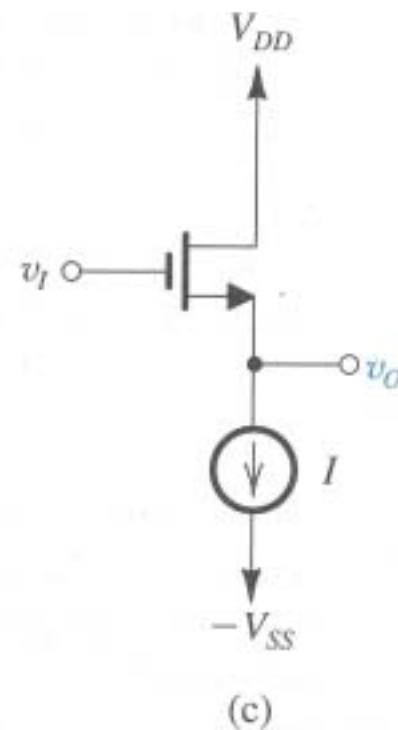
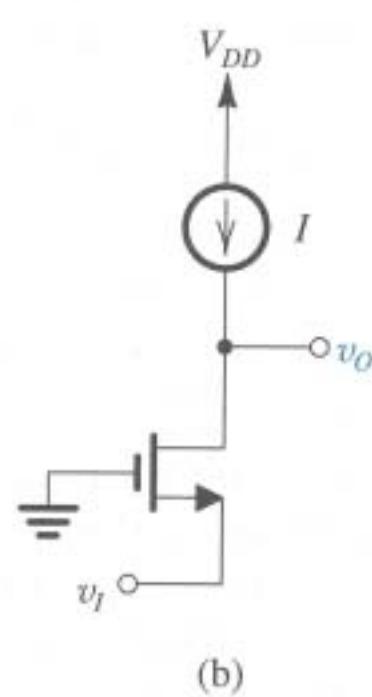
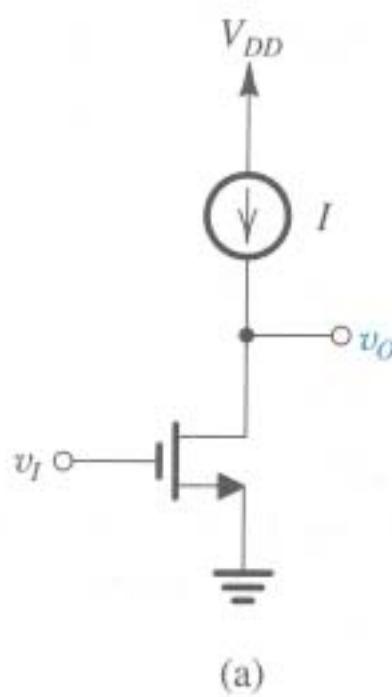
finite output resistance

$$R_o = \frac{\Delta V_O}{\Delta I_O} = r_{o2} = \frac{V_{A2}}{I_O}$$

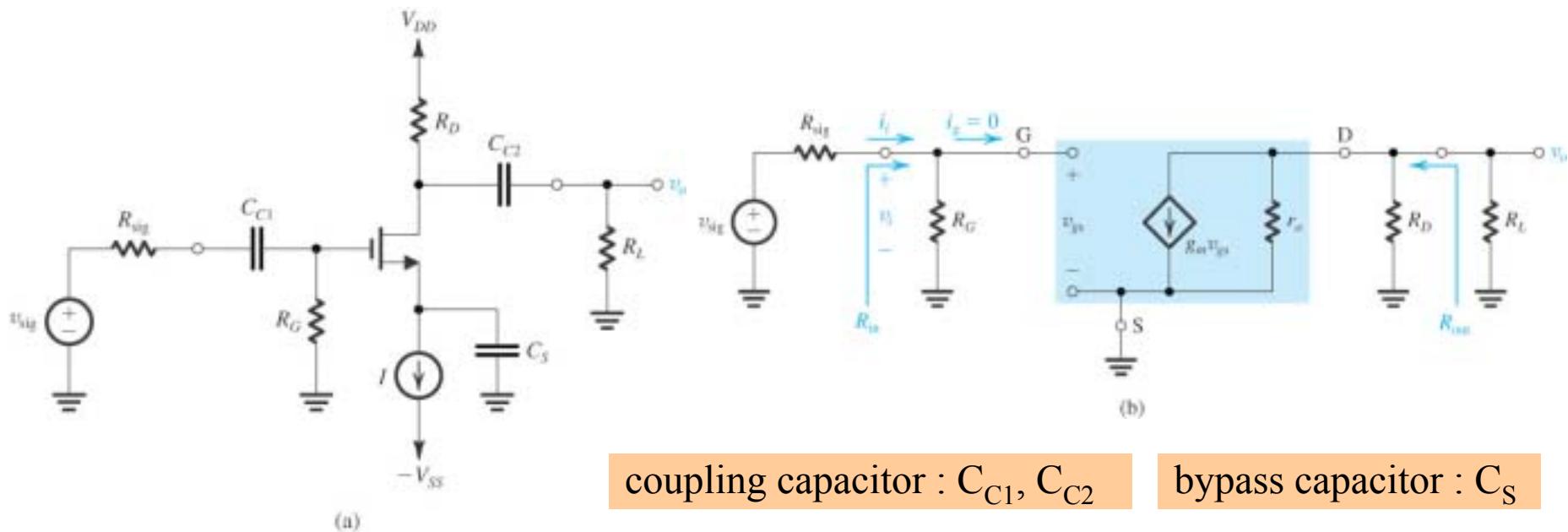
$$V_A \propto L \quad \therefore L \uparrow \rightarrow R_o \uparrow$$

## Basic configurations of single-stage IC MOS amplifiers

- (a) Common-source (CS)
- (b) Common-gate (CG)
- (c) Common-drain (CD)



# CMOS common-source amplifier



input resistance :  $R_{in} = R_G$

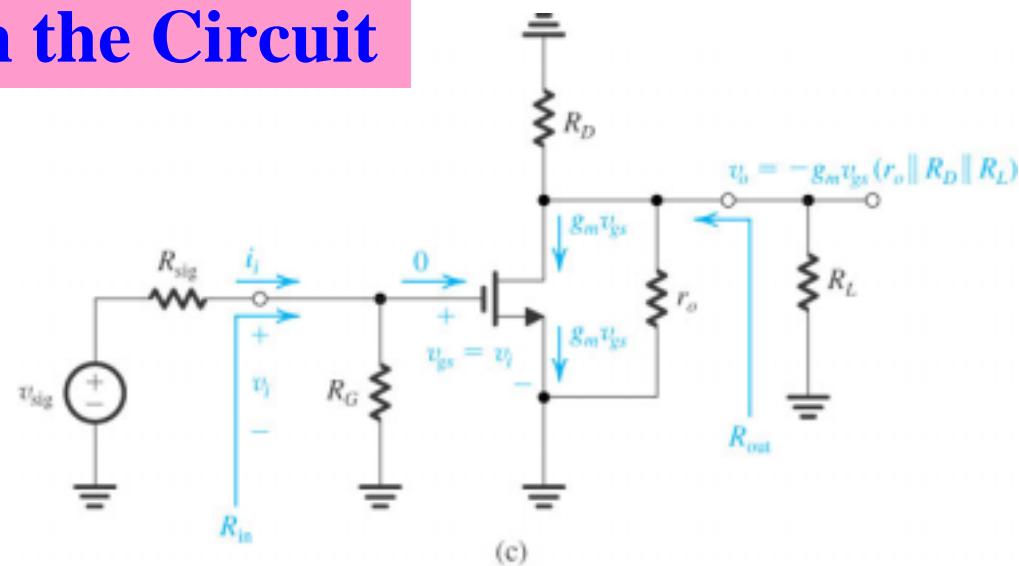
**output resistance** :  $R_{out} = r_o // R_D$

**voltage gain :**  $A_v = -g_m(r_o \parallel R_D \parallel R_L)$

**overall voltage gain :**  $G_v = -\frac{R_G}{R_G + R_{sig}} g_m (r_o // R_D // R_L)$

**applications** : part of a larger amplifier circuit

## Directly on the Circuit



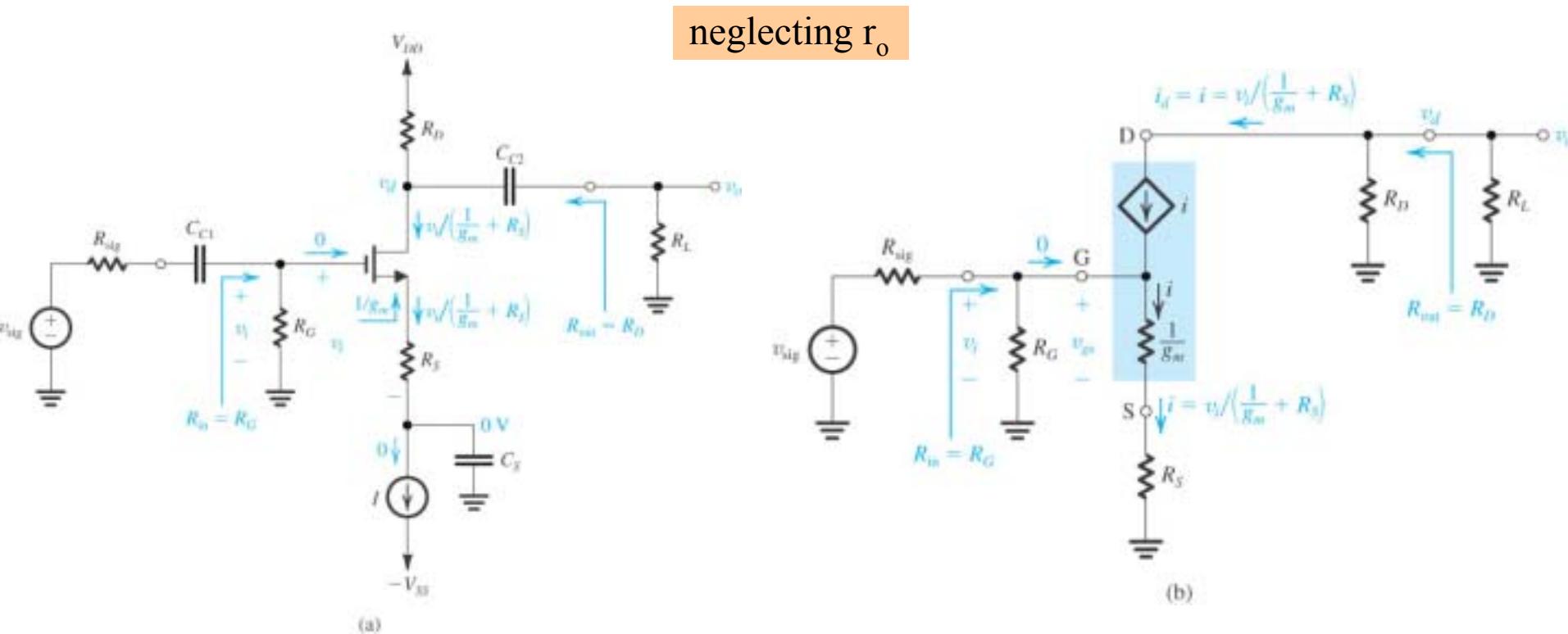
$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} = \frac{R_G}{R_{sig} + R_G}$$

$$G_m \equiv \left. \frac{i_o}{v_i} \right|_{R_L=0} = -g_m \quad R_{out,total} = r_o // R_D // R_L$$

$$A_v = \frac{v_o}{v_i} = G_m R_{out,total} = -g_m (r_o // R_D // R_L)$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \frac{v_o}{v_i} = -\frac{R_G}{R_G + R_{sig}} g_m (r_o // R_D // R_L)$$

## CMOS common-source amplifier with a source resistance



input resistance :  $R_{in} = R_G$

output resistance :  $R_{out} = R_D$

voltage gain :

$$v_{gs} = \frac{1/g_m}{(1/g_m) + R_S} v_i = \frac{v_i}{1 + g_m R_S}$$

$$i_d = i = \frac{v_i}{(1/g_m) + R_S} = \frac{g_m v_i}{1 + g_m R_S}$$

$$v_o = -i_d (R_D // R_L) = -\frac{g_m (R_D // R_L)}{1 + g_m R_S} v_i$$

$$A_v = -\frac{g_m (R_D // R_L)}{1 + g_m R_S}$$

overall voltage gain :  $G_v = -\frac{R_G}{R_G + R_{sig}} \frac{g_m (R_D // R_L)}{1 + g_m R_S}$

source degeneration resistor  $R_S$  : reduction of the gain by  $1 + g_m R_S$ )

## Directly on the Circuit

$$v_i = v_{gs} + R_S i_d = i_d / g_m + R_S i_d = (1/g_m + R_S) i_d$$

$$G_m \equiv \left. \frac{i_o}{v_i} \right|_{R_L=0} = -\frac{i_d}{v_i} = -\frac{1}{1/g_m + R_S}$$

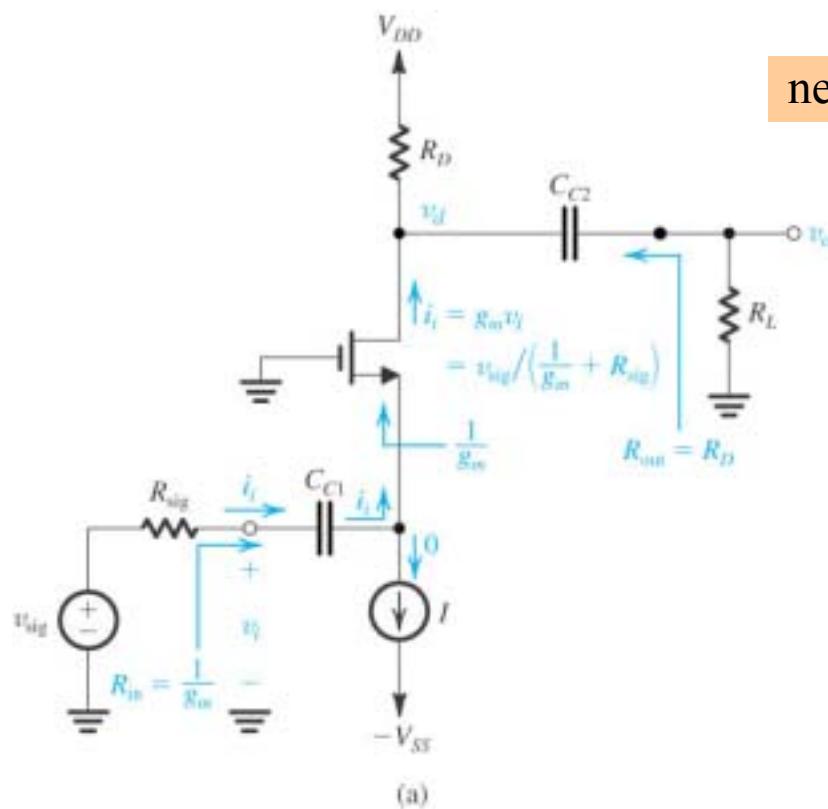
$$R_{out,total} = R_D // R_L$$

$$A_v = G_m R_{out,total} = -\frac{R_D // R_L}{1/g_m + R_S} = -\frac{g_m (R_D // R_L)}{1 + g_m R_S}$$

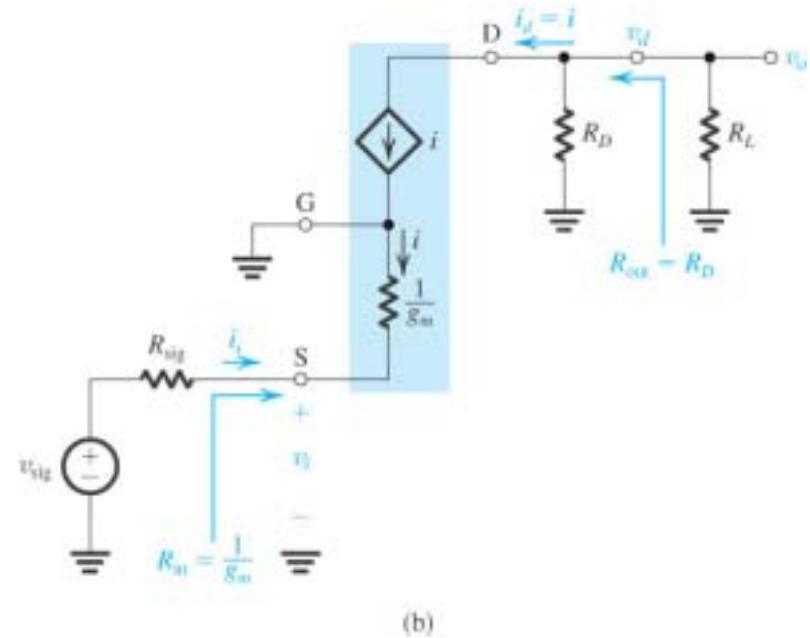
useful interpretation :

$$A_v = -\frac{\text{total resistance in the drain}}{\text{total resistance in the source}} = -\frac{R_D // R_L}{1/g_m + R_S}$$

## CMOS common-gate amplifier



neglecting  $r_o$



input resistance :  $R_{in} = 1/g_m$

output resistance :  $R_{out} = R_D$

voltage gain :  $i_i = -i = -i_d = \frac{v_i}{R_{in}} = \frac{v_i}{1/g_m} = g_m v_i$

$$v_o = -i_d (R_D // R_L) = g_m (R_D // R_L) v_i$$

$$A_v = g_m (R_D // R_L)$$

overall voltage gain :  $G_v = \frac{1/g_m}{1/g_m + R_{sig}} A_v = \frac{g_m (R_D // R_L)}{1 + g_m R_{sig}}$

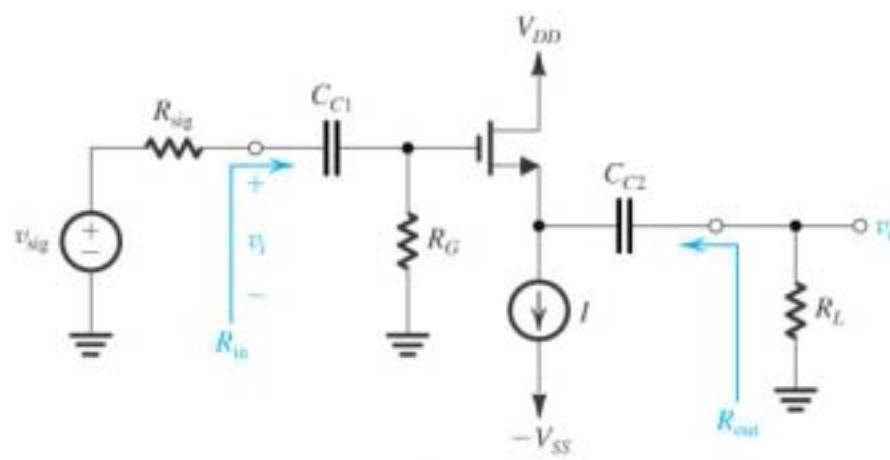
applications : unity-gain current amplifier, cascode circuit

## Directly on the Circuit

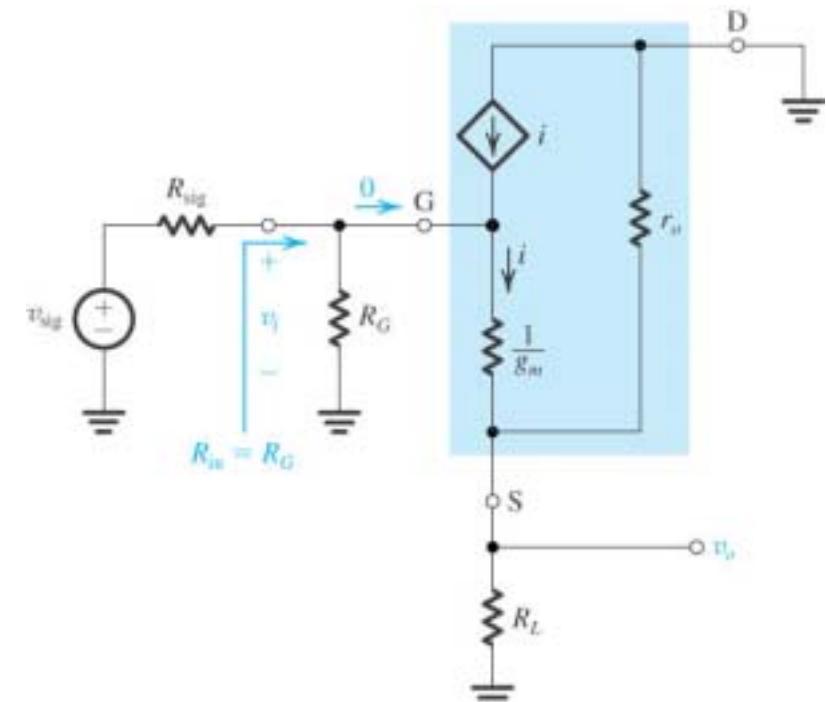
$$v_i = -v_{gs} = -i_d / g_m \quad G_m \equiv \left. \frac{i_o}{v_i} \right|_{R_L=0} = -\frac{i_d}{v_i} = g_m \quad R_{out,total} = R_D // R_L$$

$$A_v = \frac{v_o}{v_i} = G_m R_{out,total} = g_m (R_D // R_L)$$

## CMOS common-drain amplifier (Source follower)



(a)



(b)

input resistance :  $R_{in} = R_G$

voltage gain :  $v_o = \frac{R_L // r_o}{(R_L // r_o) + 1/g_m} v_i$

$$A_v = \frac{R_L // r_o}{(R_L // r_o) + 1/g_m} \cong \frac{R_L}{R_L + 1/g_m}, \quad \text{for } r_o \gg R_L$$

overall voltage gain :

$$G_v = \frac{R_G}{R_G + R_{sig}} \frac{R_L // r_o}{(R_L // r_o) + 1/g_m} \cong 1, \quad \text{for } R_G \gg R_{sig}, r_o \gg 1/g_m, r_o \gg R_L$$

output resistance :

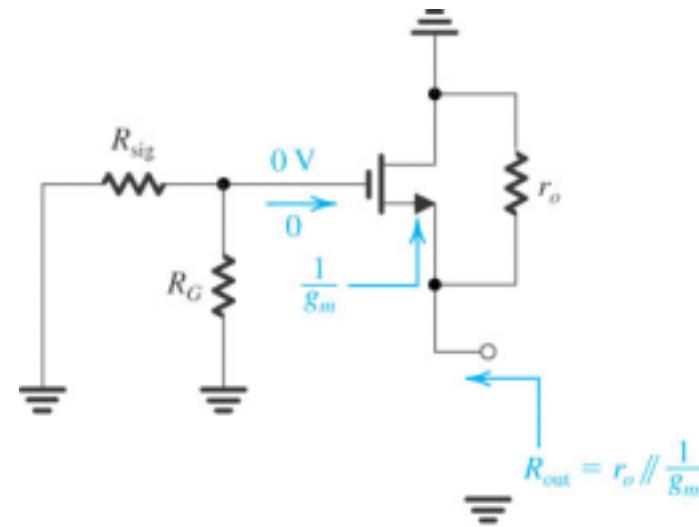
$$R_{out} = r_o // \frac{1}{g_m} \cong \frac{1}{g_m}$$

applications :

unity-gain voltage amplifier

or voltage buffer

output stage in a multistage amplifier



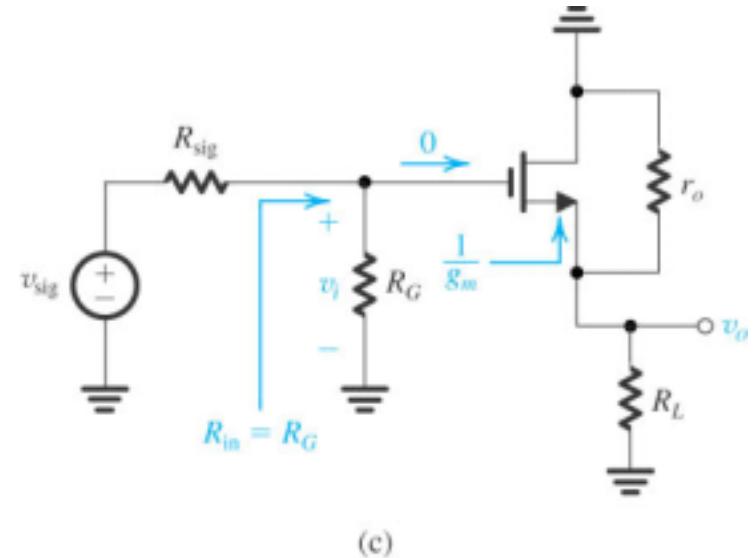
(d)

## Directly on the Circuit

$$G_m \equiv \left. \frac{i_o}{v_i} \right|_{R_L=0} = \frac{i_d}{v_{gs}} = g_m$$

$$R_{out,total} = R_L // r_o // \frac{1}{g_m} = \frac{(R_L // r_o) \frac{1}{g_m}}{(R_L // r_o) + \frac{1}{g_m}}$$

$$A_v = \frac{v_o}{v_i} = G_m R_{out,total} = \frac{R_L // r_o}{(R_L // r_o) + 1/g_m}$$



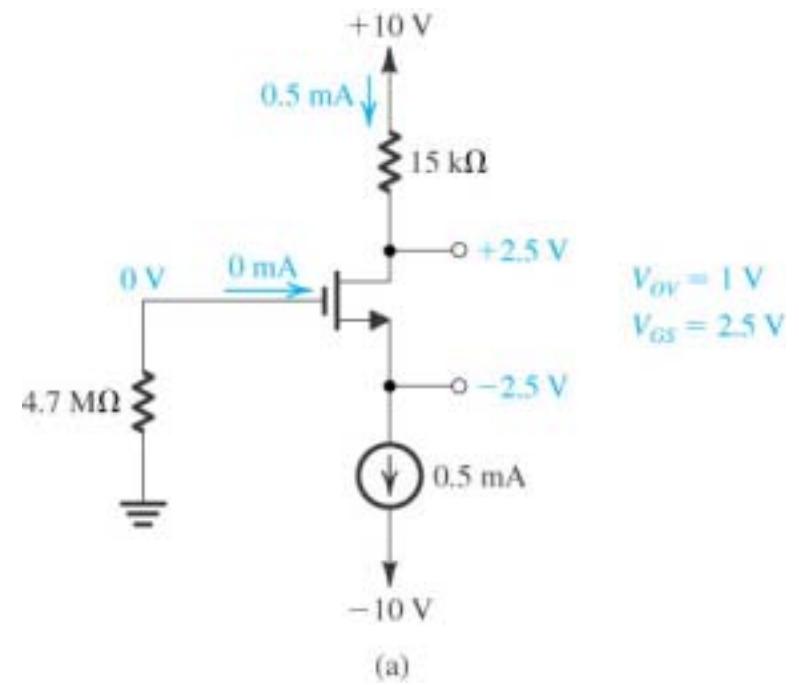
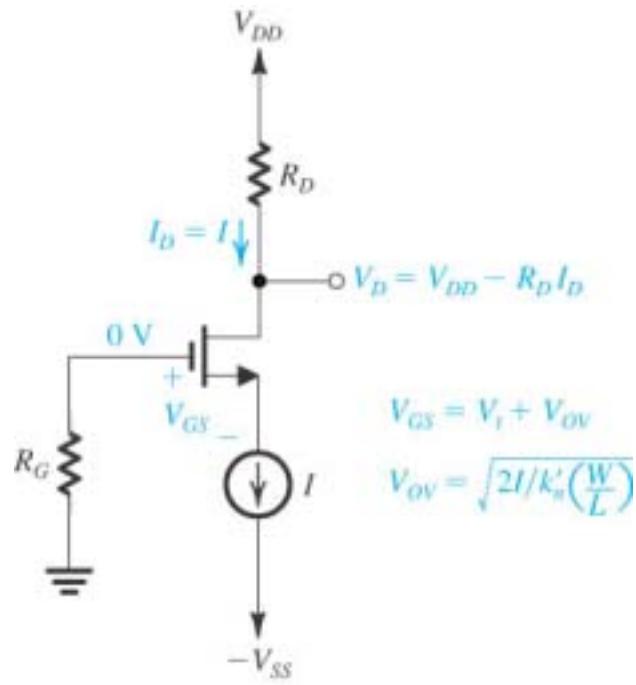
## Comparison

Neglecting Body Effect &  $r_o$

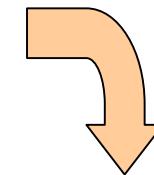
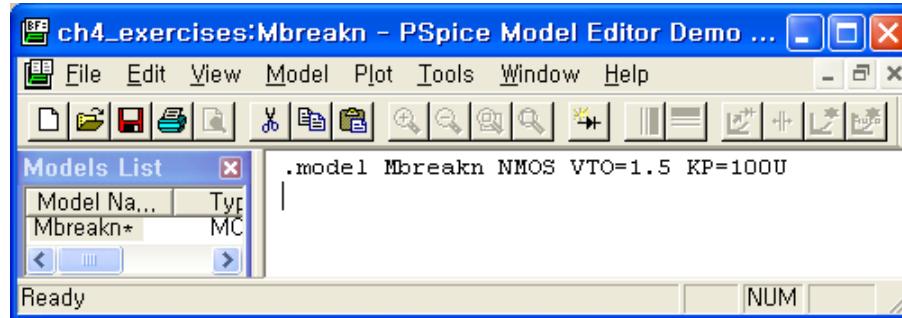
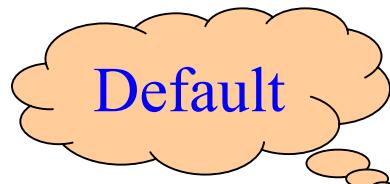
	CS	CS_ $R_S$	CG	CD
$R_{in}$	$R_G$	$R_G$	$\frac{1}{g_m}$	$R_G$
$R_{out}$	$R_D$	$R_D$	$R_D$	$\frac{1}{g_m}$
$A_{vo}$	$-g_m R_D$	$-\frac{g_m R_D}{1 + g_m R_S}$	$g_m R_D$	1

# Example 4.1

$$V_{tn} = 1.5 \text{ V}, \quad \beta_n = k'_n (W/L) = 1 \text{ mA/V}^2, \quad \lambda = 0, \quad \gamma = 0$$



$$g_m = \beta_n V_{OV} = 1 \text{ mA/V}^2 \times 1 \text{ V} = 1 \text{ mA/V}$$



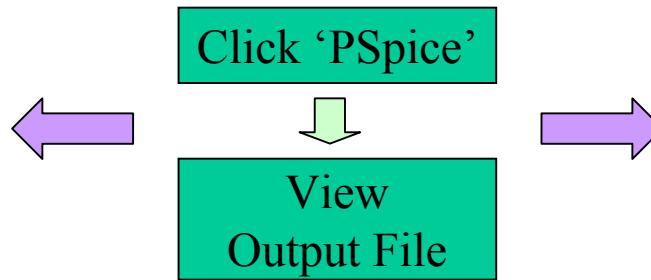
Capture CIS - Demo Edition... File Edit Options Window Help

```
exercise4_30-dc
Mbreakn
NMOS
LEVEL 1
L 100.000000E-06
W 100.000000E-06
VTO 0
KP 20.000000E-06
GAMMA 0
PHI .6
LAMBDA 0
IS 10.000000E-15
JS 0
PB .8
PBSW .8
CJ 0
CJSW 0
CGSO 0
CGDO 0
CGBO 0
TOX 0
XJ 0
UCRIT 10.000000E+03
DIOMOD 1
VFB 0
LETA 0
WEWA 0
UD 0
TEMP 0
VDD 0
XPART 0
```

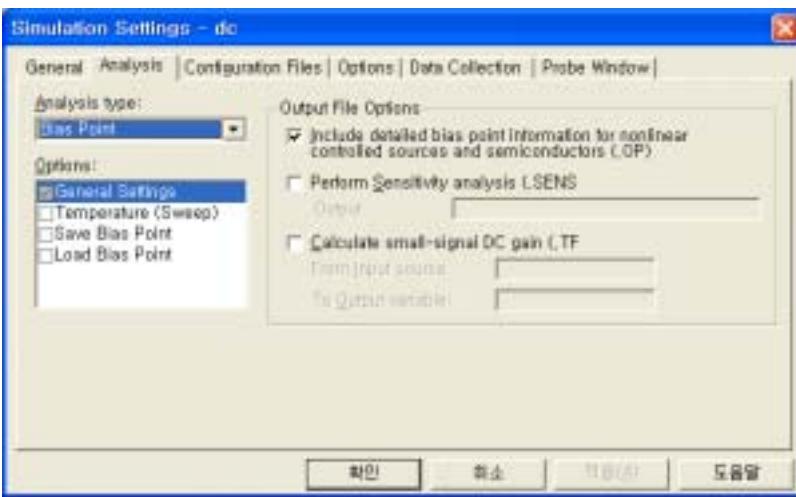
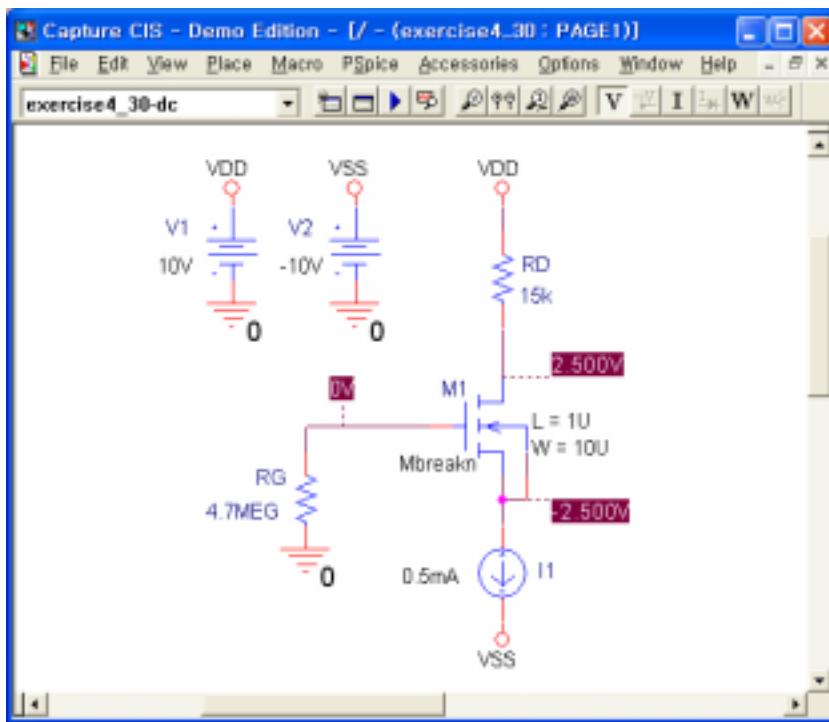
$V_t = 1.5V \rightarrow VTO = 1.5$   
 $\beta_n = \text{ImA/V}^2 \rightarrow KP = 100E - 6$   
 $L = 1U, W = 10U$

Capture CIS - Demo Edition... File Edit Options Window Help

```
SCHEMATIC1-dc
Mbreakn
NMOS
LEVEL 1
L 100.000000E-06
W 100.000000E-06
VTO 1.5
KP 100.000000E-06
GAMMA 0
PHI .6
LAMBDA 0
IS 10.000000E-15
JS 0
PB .8
PBSW .8
CJ 0
CJSW 0
CGSO 0
CGDO 0
CGBO 0
TOX 0
XJ 0
UCRIT 10.000000E+03
DIOMOD 1
VFB 0
LETA 0
WEWA 0
UD 0
TEMP 0
VDD 0
XPART 0
```



4.



Capture CIS - Demo Editio... File Edit Options Window Help

SCHEMATIC1-dc

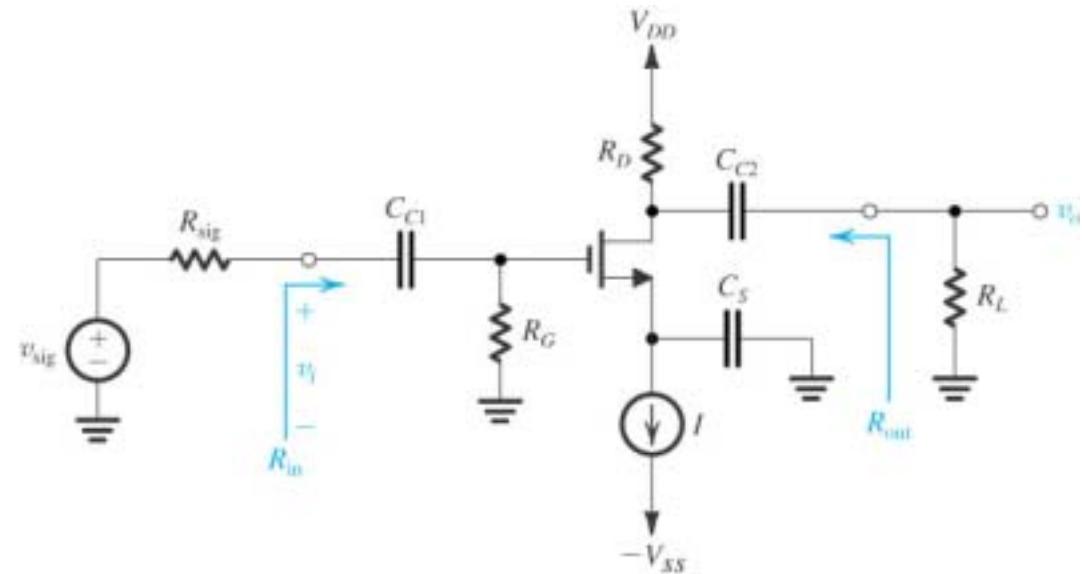
139:	
140: NAME	M_M1
141: MODEL	Mbreakn
142: ID	5.00E-04
143: VGS	2.50E+00
144: VDS	5.00E+00
145: VBS	0.00E+00
146: VTH	1.50E+00
147: VDSAT	1.00E+00
148: Lin0/Sat1	-1.00E+00
149: if	-1.00E+00
150: ir	-1.00E+00
151: TAU	-1.00E+00
152: GM	1.00E-03
153: GDS	0.00E+00
154: GMB	0.00E+00
155: CBD	0.00E+00
156: CBS	0.00E+00
157: CGSOV	0.00E+00
158: CGDOV	0.00E+00
159: CGBOV	0.00E+00
160: CGS	0.00E+00
161: CGD	0.00E+00
162: CGB	0.00E+00
163:	

## Example 4.2 <CS>

$$V_{tn} = 1.5V, \lambda = 0, \gamma = 0$$

$$\beta_n = k_n(W/L) = 1mA/V^2$$

$$R_{sig} = 100k\Omega, R_L = 15k\Omega$$



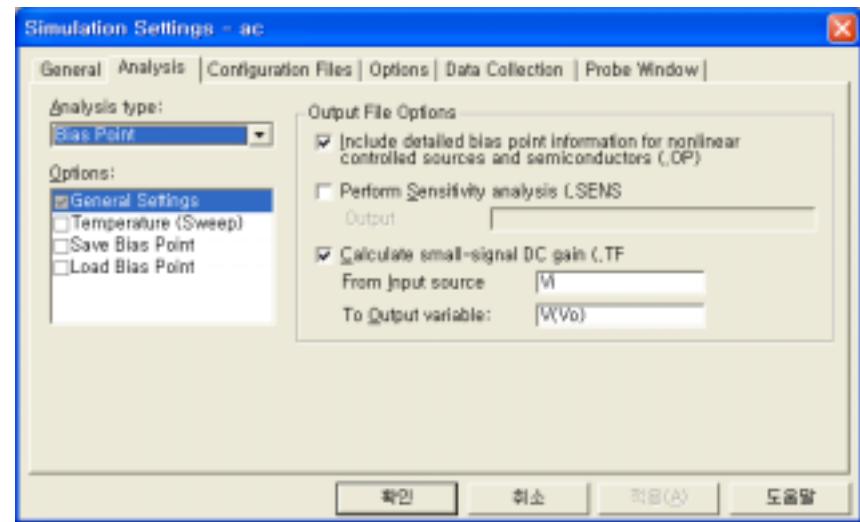
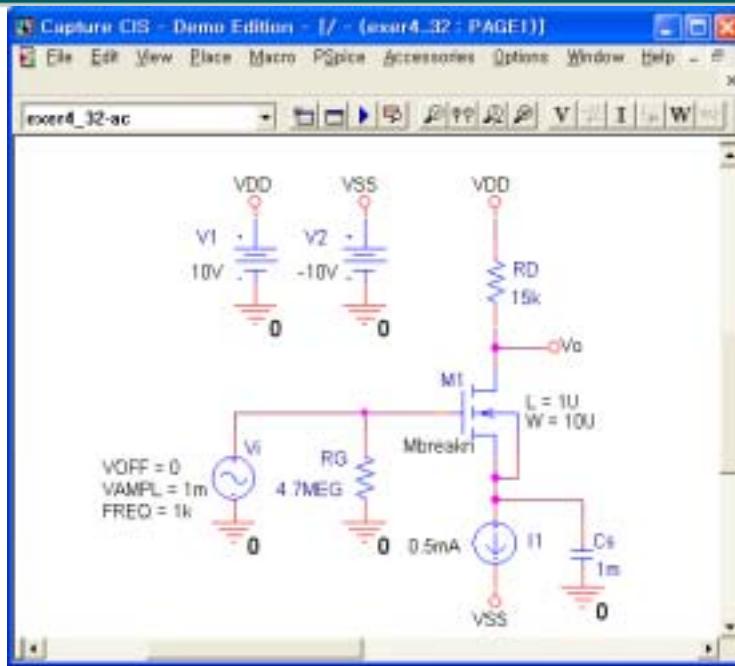
$$R_{in} = R_G = 4.7M\Omega$$

$$R_{out} = R_D = 15k\Omega$$

$$A_{vo} = -g_m R_D = -1m \times 15k = -15 V/V$$

$$G_v = -\frac{R_G}{R_G + R_{sig}} g_m (R_D // R_L) = -7.34 V/V$$

4.

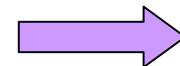


```

Capture CIS - Demo Edition - [I:\WUSER\WCOUR...
File Edit Options Window Help
exer4_32-ac
170:
171: ***** SMALL-SIGNAL CHARACTERISTICS
172:
173:
174: V(VO)/V_Vi = -1.500E-08
175:
176: INPUT RESISTANCE AT V_Vi = 4.700E+06
177:
178: OUTPUT RESISTANCE AT V(VO) = 1.500E+04
179:

```

$A_{vo} = -1.5E-08$ : incorrect



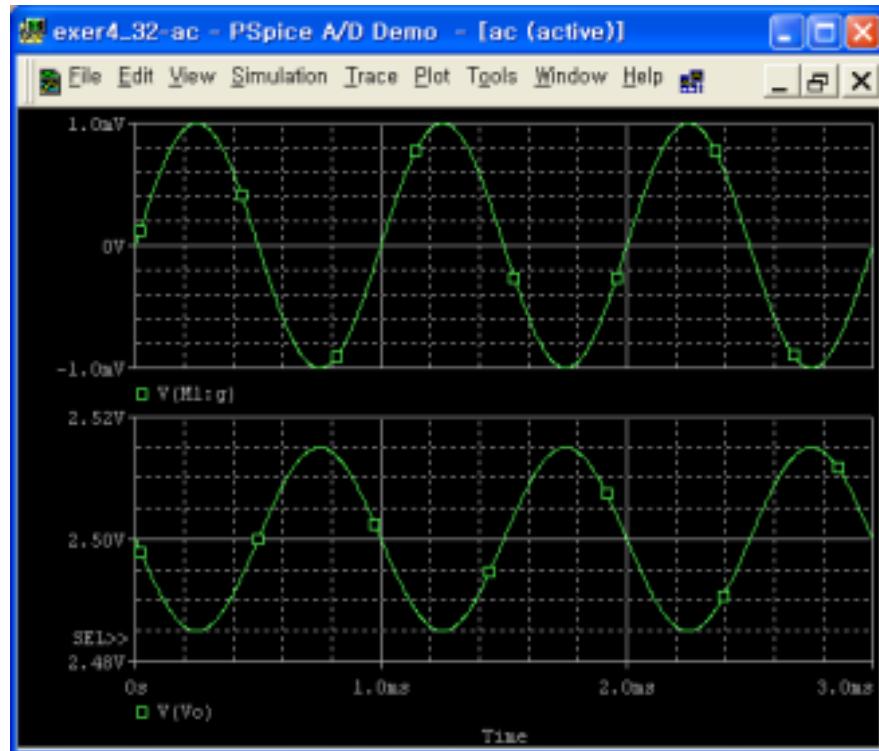
Due to dc gain

Why?

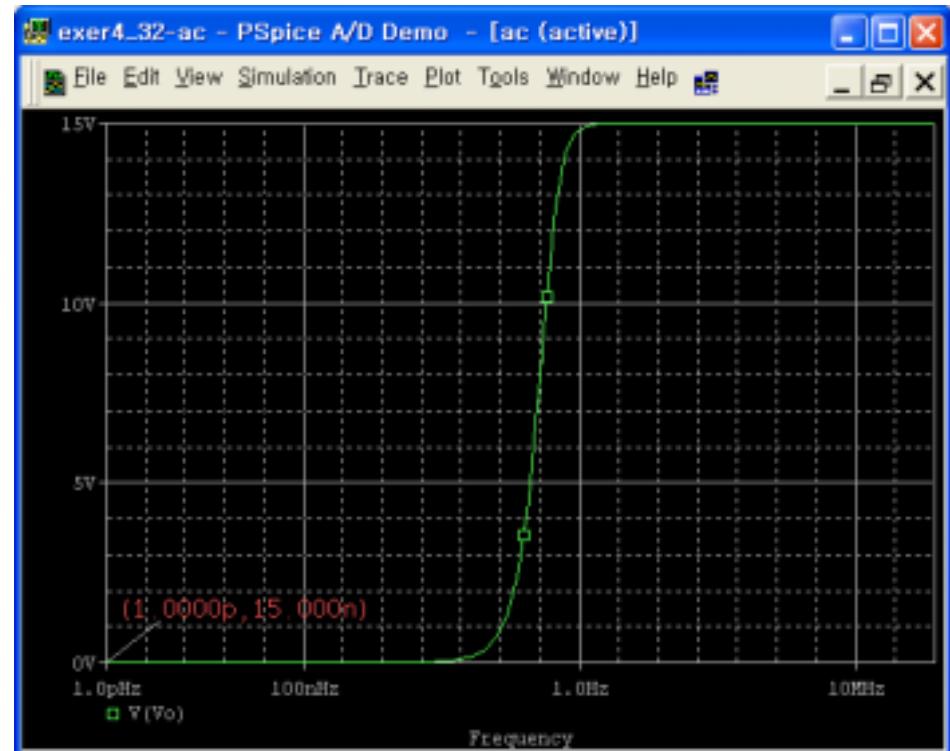
$$R_{in} = 4.7M\Omega$$

$$R_{out} = 15k\Omega$$

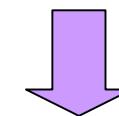
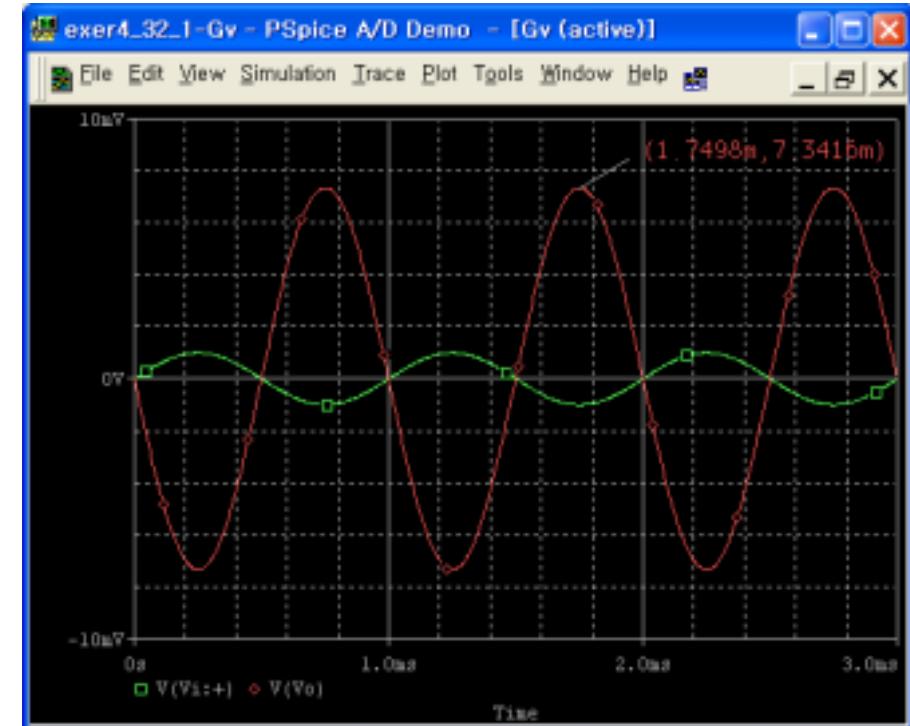
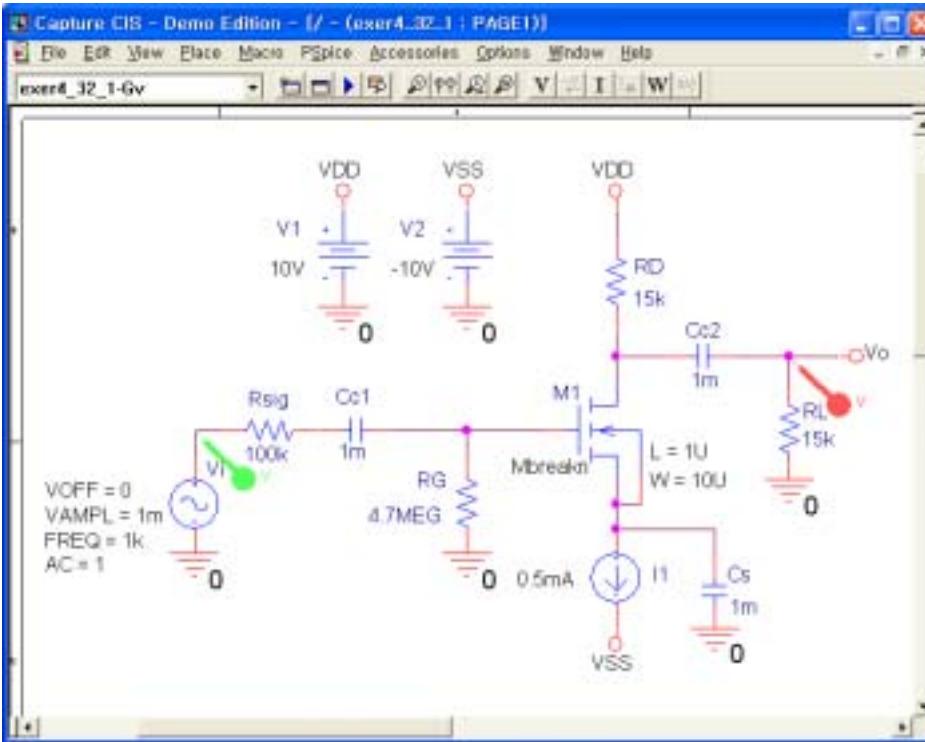
## Transient analysis



## AC analysis



$$A_{vo} = -15 \text{ V/V}$$



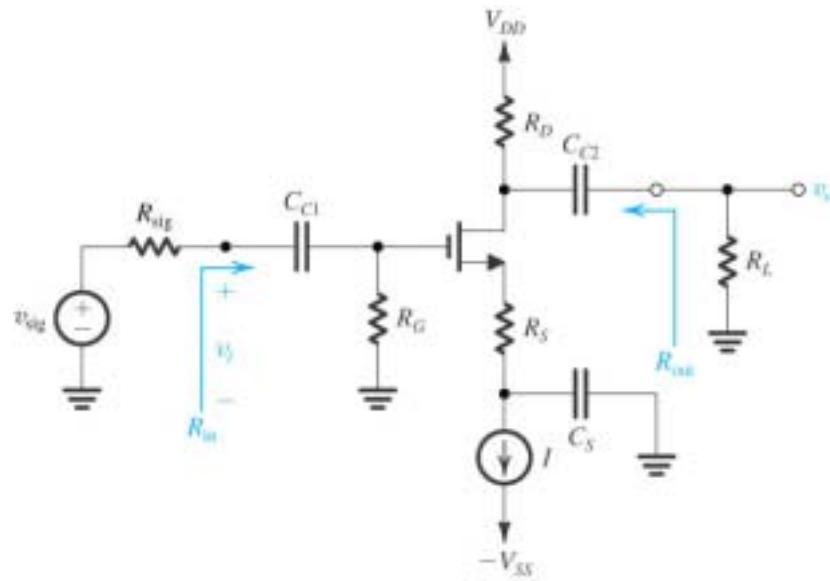
$$G_v = -7.34 \text{ V/V}$$

## 1

1. degeneration  
PSPICE

Common-Source

$$R_{in}, \quad R_{out}, \quad A_{vo} = \left. \frac{v_o}{v_i} \right|_{R_L=\infty}, \quad G_v = \frac{v_o}{v_{sig}}$$



$$V_{tn} = 1.5V, \quad \lambda = 0, \quad \gamma = 0$$

$$\beta_n = k_n (W/L) = 1\text{mA/V}^2, \quad I = 0.5\text{mA}$$

$$R_G = 4.7\text{M}\Omega, \quad R_D = 15\text{k}\Omega, \quad R_S = 2\text{k}\Omega$$

$$R_{sig} = 100\text{k}\Omega, \quad R_L = 15\text{k}\Omega$$

$$C_{C1}, \quad C_{C2}, \quad C_S \rightarrow \infty$$

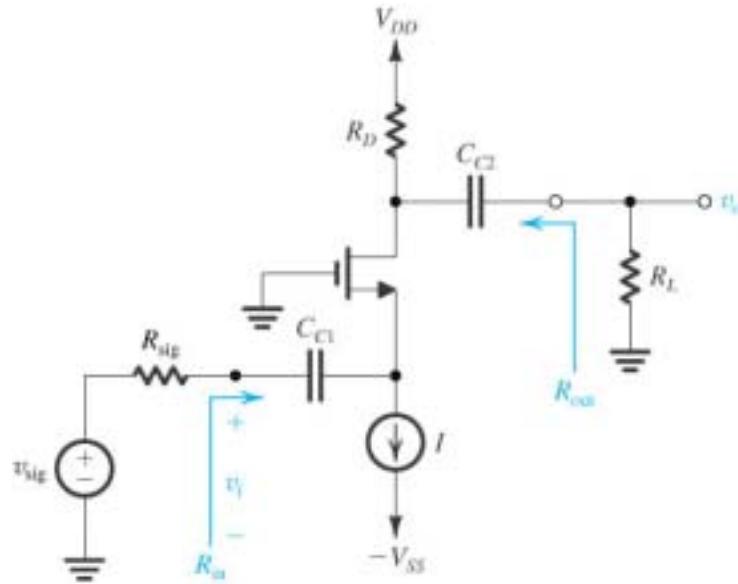
2.  $\lambda = 0.02, \quad R_S = 0\Omega$

## 2

## 1. Common-Gate

## PSPICE

$$R_{in}, \quad R_{out}, \quad A_{vo} = \left. \frac{v_o}{v_i} \right|_{R_L=\infty}, \quad G_v = \frac{v_o}{v_{sig}}$$



$$V_{tn} = 1.5V, \quad \lambda = 0, \quad \gamma = 0$$

$$\beta_n = k_n (W/L) = 1mA/V^2$$

$$I = 0.5mA, \quad R_D = 15k\Omega$$

$$R_{sig} = 1k\Omega, \quad R_L = 15k\Omega$$

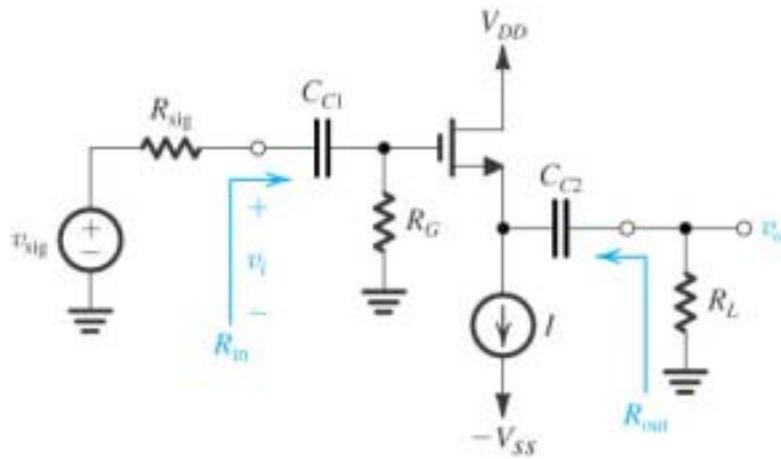
$$C_{C1}, \quad C_{C2} \rightarrow \infty$$

$$2. \quad R_{sig} = 10k\Omega, \quad 100k\Omega$$

## 1. Common-Drain

PSPICE

$$R_{in}, \quad R_{out}, \quad A_{vo} = \left. \frac{v_o}{v_i} \right|_{R_L=\infty}, \quad G_v = \frac{v_o}{v_{sig}}$$



$$V_m = 1.5V, \quad \lambda = 0, \quad \gamma = 0$$

$$\beta_n = k_n (W/L) = 1\text{mA/V}^2$$

$$I = 0.5\text{mA}, \quad R_G = 4.7\text{M}\Omega$$

$$R_{sig} = 100\text{k}\Omega, \quad R_L = 15\text{k}\Omega$$

$$C_{C1}, \quad C_{C2} \rightarrow \infty$$

$$2. \quad \lambda = 0.02, \quad R_{sig} = 1\text{M}\Omega$$

## Open-circuit time constants

for determining the upper 3-dB frequency

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \dots}{1 + b_1 s + b_2 s^2 + \dots}$$

$$b_1 = \frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}} + \dots + \frac{1}{\omega_{Pn_H}}$$

$$b_1 = \sum_{i=1}^{n_H} C_i R_{io}$$

If a dominant pole exists (say,  $P_1$ ), then

$$b_1 \cong \frac{1}{\omega_{P1}} \quad \text{and} \quad \omega_H \cong \omega_{P1}$$

Thus,

$$\omega_H \cong 1 \sqrt{\sum_{i=1}^{n_H} C_i R_{io}}$$

the method for determining  $b_1$   
is exact

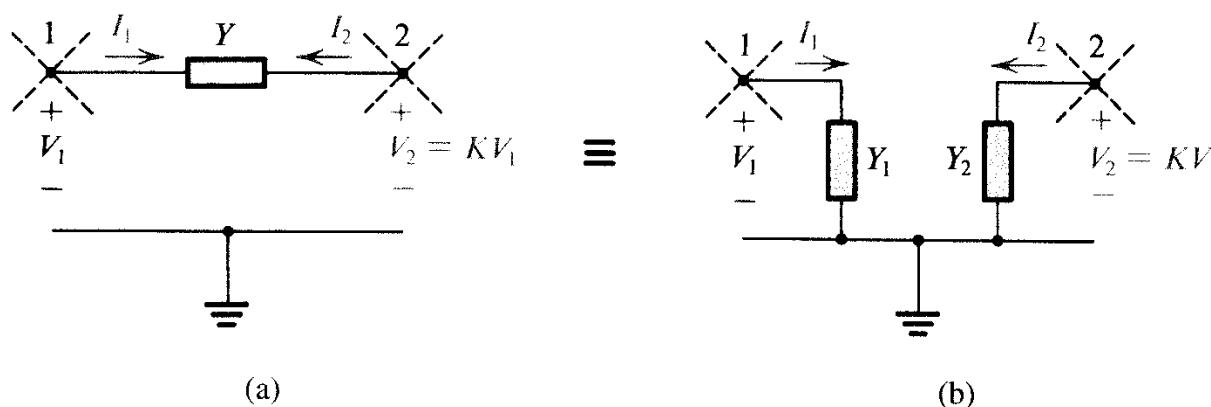
$R_{i0}$  can be determined by

$$C_k = 0 \text{ (open) for } k \neq i$$

$$V_s = 0 \text{ & } I_s = 0$$

determine the resistance  
seen by  $C_i$

## Miller's theorem



$$\begin{aligned} Y_1 &= Y(1-K) \\ Y_2 &= Y\left(1 - \frac{1}{K}\right) \\ K &= \frac{V_2}{V_1} \end{aligned}$$

**Fig. 7.18** Miller's theorem.

## An important caution

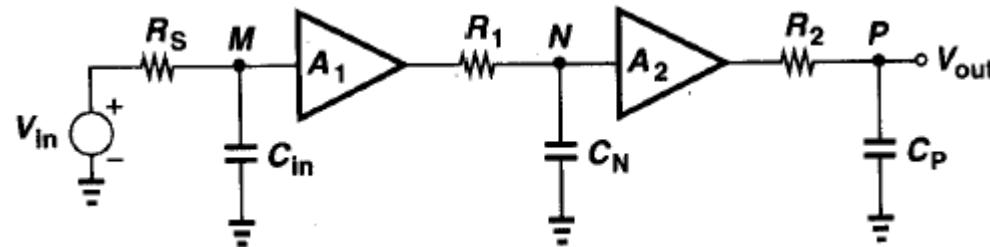
the Miller equivalent circuit is valid only as long as the conditions that existed in the network when  $K$  was determined are not changed

Miller's theorem is very useful in determining the input impedance and the gain of an amplifier

it cannot be used to determine its output resistance  
the change of  $K$

## Association of poles with nodes

each node contributes one pole

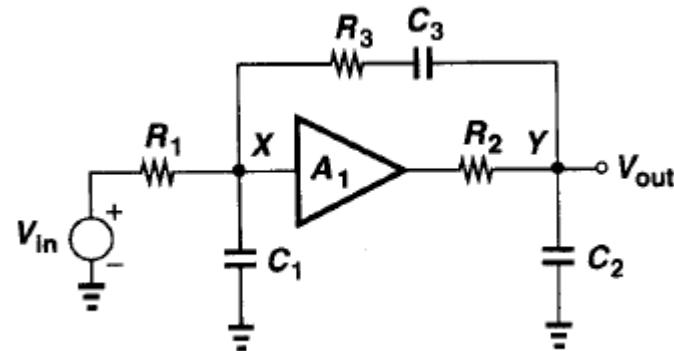


$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + \frac{s}{\omega_M}} \cdot \frac{A_2}{1 + \frac{s}{\omega_N}} \cdot \frac{1}{1 + \frac{s}{\omega_P}}$$

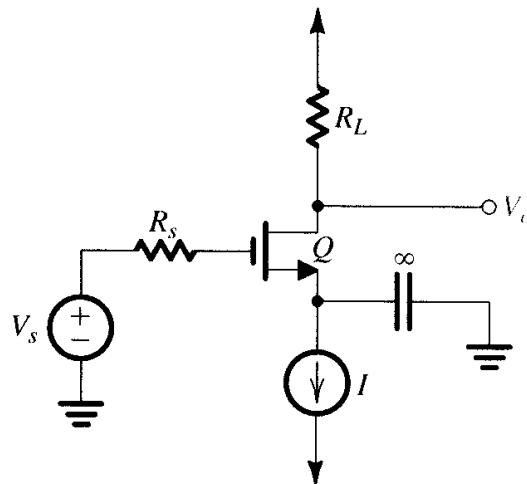
$$\omega_M = \frac{1}{R_S C_{in}}, \quad \omega_N = \frac{1}{R_1 C_N}, \quad \omega_P = \frac{1}{R_2 C_P}$$

interaction between nodes

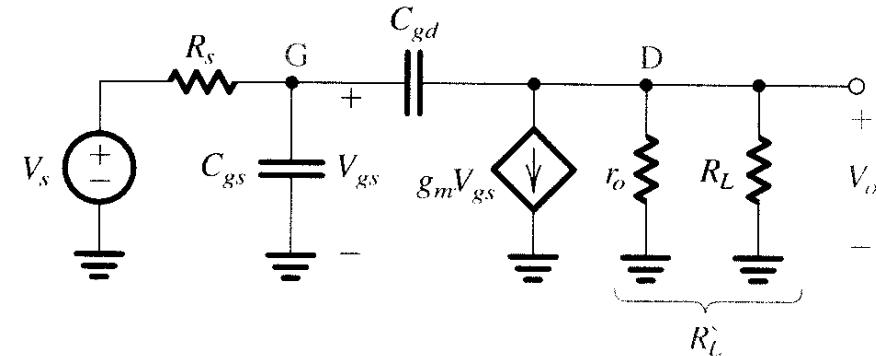
difficult to calculate poles



## High-frequency response of the CS amplifier



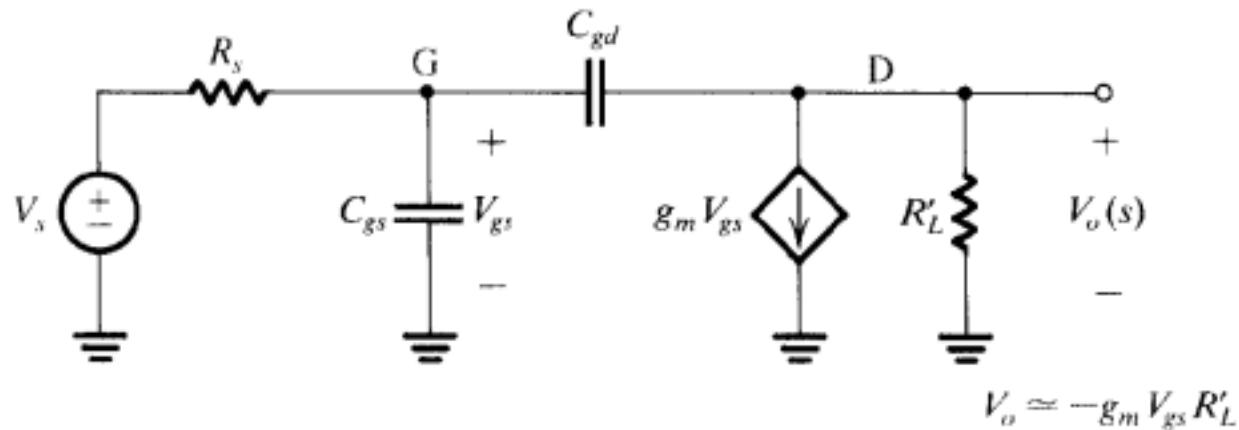
(a)



(a)

- three methods to determine the upper 3-dB freq.
- method of open-circuit time constants
- approximate method using Miller's theorem
- method using the exact high-freq. transfer function

## Method of open-circuit time constants



$$R_{gs} = R_s$$

$$R_{gd} = R_s + R_L' + g_m R_s R_L'$$

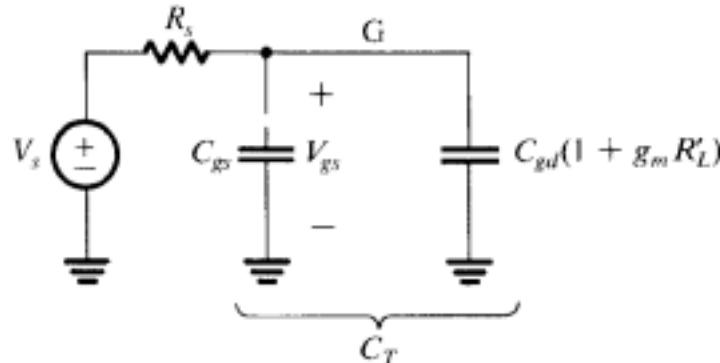
$$\begin{aligned}\omega_H &= \frac{1}{C_{gs} R_{gs} + C_{gd} R_{gd}} \\ &= \frac{1}{C_{gs} R_s + C_{gd} (R_s + R_L' + g_m R_s R_L')}\end{aligned}$$

## Approximate method using Miller's theorem

neglecting the current through  $C_{gd}$  in determining  $V_o$

$$V_o \approx -g_m V_{gs} R_L' \quad \therefore K = V_o / V_{gs} = -g_m V_{gs}$$

3-dB freq. & gain



$$\begin{aligned} C_T &= C_{gs} + C_{gd}(1 + g_m R_L') \\ \omega_H &= \frac{1}{C_T R_s} = \frac{1}{C_{gs} R_s + C_{gd}(R_s + g_m R_s R_L')} \\ A_H &= \frac{A_M}{1 + s/\omega_H}, \quad A_M = -g_m R_L' \end{aligned}$$

the small feedback capacitance  $C_{gd}$

plays an important role in determining the high-freq. resp.

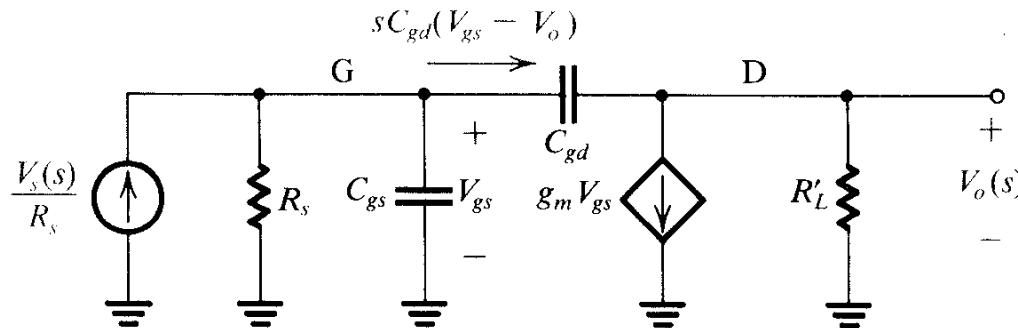
gives rise to a large input cap.  $C_{gd}(1 + g_m R_L')$  **Miller Effect**

To increase the upper 3-dB frequency

$g_m R_L' \downarrow$  reduce the midband gain

$R_s \downarrow$  might not always be possible  
use cascode configuration

## Method using the exact high-freq. Transfer function



transfer function

$$\frac{V_o(s)}{V_s(s)} = -A_M \frac{1 - \frac{s}{g_m / C_{gd}}}{1 + s[C_{gs}R_s + C_{gd}(R_s + R'_L + g_m R_s R'_L)] + s^2 C_{gs} C_{gd} R_s R'_L}$$

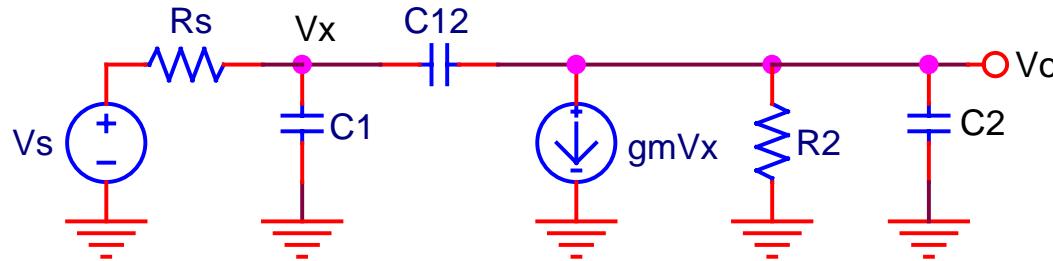
if the two poles are widely separated

$$D(s) = \left(1 + \frac{s}{\omega_{P1}}\right) \left(1 + \frac{s}{\omega_{P2}}\right) = 1 + s \left(\frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}}\right) + \frac{s^2}{\omega_{P1} \omega_{P2}} \approx 1 + \frac{s}{\omega_{P1}} + \frac{s^2}{\omega_{P1} \omega_{P2}}$$

$$\omega_{P1} = \frac{1}{C_{gs} R_s + C_{gd} (R_s + R'_L + g_m R_s R'_L)}$$

$$\omega_{P2} = \frac{C_{gs} + C_{gd} (1 + g_m R'_L + R'_L / R_s)}{C_{gs} C_{gd} R'_L} \approx \frac{g_m}{C_{gs}} \quad \text{for } g_m R'_L \gg 1 \text{ and } R'_L < R_s$$

## General high-frequency analysis of CS amplifiers



$$A_v(s) = \frac{V_o(s)}{V_s(s)} = \frac{\left( -\frac{g_m}{G_2} \right) \left( 1 - s \frac{C_{12}}{g_m} \right)}{1 + s \left( \frac{C_2 + C_{12}}{G_2} + \frac{C_1 + C_{12}}{G_s} + \frac{g_m C_{12}}{G_s G_2} \right) + s^2 \frac{C_1 C_2 + C_1 C_{12} + C_2 C_{12}}{G_s G_2}}$$

If  $\omega_{P1} \ll \omega_{P2}$

$$D(s) = \left( 1 + \frac{s}{\omega_{P1}} \right) \left( 1 + \frac{s}{\omega_{P2}} \right) = 1 + s \left( \frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}} \right) + \frac{s^2}{\omega_{P1} \omega_{P2}} \approx 1 + \frac{s}{\omega_{P1}} + \frac{s^2}{\omega_{P1} \omega_{P2}}$$

$$\begin{aligned}\omega_{P1} &\approx \left( \frac{C_2 + C_{12}}{G_2} + \frac{C_1 + C_{12}}{G_s} + \frac{g_m C_{12}}{G_s G_2} \right)^{-1} \\ \omega_{P2} &\approx \left( \frac{G_s G_2}{C_1 C_2 + C_1 C_{12} + C_2 C_{12}} \right) \frac{1}{\omega_{P1}} \\ &= \left( \frac{G_s G_2}{C_1 C_2 + C_1 C_{12} + C_2 C_{12}} \right) \left( \frac{C_2 + C_{12}}{G_2} + \frac{C_1 + C_{12}}{G_s} + \frac{g_m C_{12}}{G_s G_2} \right)\end{aligned}$$

$$\omega_z = \frac{g_m}{C_{12}}$$

$$A_v(s) = A_M \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{P1}}\right) \left(1 + \frac{s}{\omega_{P2}}\right)}, \quad A_M = -\frac{g_m}{G_2} = -g_m R_2$$

$$\omega_{P1} \ll \omega_{P2} \quad \& \quad g_m R_2 \gg 1$$

$$\omega_{P1} \approx \left( \frac{g_m C_{12}}{G_s G_2} \right)^{-1} = \frac{G_s}{(g_m R_2) C_{12}} = \frac{G_s}{|A_M| C_{12}}$$

$$\omega_{P2} \approx \frac{g_m C_{12}}{C_1 C_2 + C_1 C_{12} + C_2 C_{12}}$$

$$\omega_{P1} \ll \omega_{P2} \quad \& \quad g_m R_2 \gg 1 \quad \& \quad C_2 \approx 0$$

$$\omega_{P1} \approx \frac{G_s}{(g_m R_2) C_{12}} = \frac{G_s}{|A_M| C_{12}}$$

$$\omega_{P2} \approx \frac{g_m}{C_1}$$

$$\omega_{P1} \ll \omega_{P2} \quad \& \quad C_1 \approx C_2 \quad \& \quad R_2 \ll R_s \rightarrow g_m R_2 \approx 1$$

$$\omega_{P1} \approx \left( \frac{C_1 + C_{12}}{G_s} + \frac{C_{12}}{G_s} \right)^{-1} = \frac{G_s}{C_1 + 2C_{12}}$$

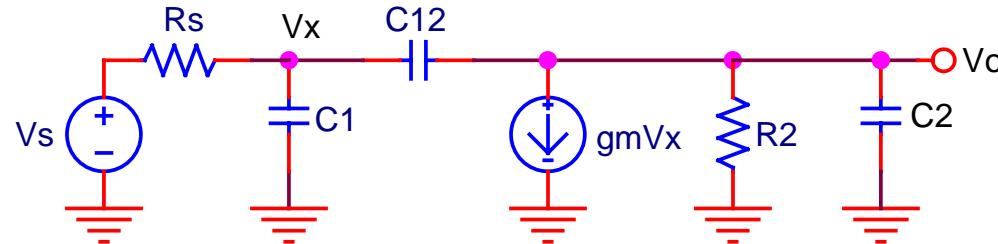
$$\omega_{P2} \approx \frac{G_s G_2}{C_2 (C_1 + 2C_{12})} \frac{1}{\omega_{P1}} = \frac{G_2}{C_2}$$

$$R_s \approx 0$$

$$A_v(s) = \frac{\left( -\frac{g_m}{G_2} \right) \left( 1 - s \frac{C_{12}}{g_m} \right)}{1 + s \left( \frac{C_2 + C_{12}}{G_2} \right)} = A_M \frac{\left( 1 - \frac{s}{\omega_z} \right)}{\left( 1 + \frac{s}{\omega_{P1}} \right)}$$

## Zero in the CS amplifier

zero arises from direct coupling of the input to the output through  $C_{12}$   
 $C_{12}$  provides a feedforward path at high frequencies



$$V_o(s) = Z_L [sC_{12}(V_x - V_o) - g_m V_x]$$

$$= \frac{Z_L(sC_{12} - g_m)V_x}{1 + Z_L sC_{12}}, \quad Z_L = R_2 // \frac{1}{sC_2}$$

$$V_o(s_z) = 0 \rightarrow \begin{cases} Z_L(s_z) = \frac{R_2}{1 + s_z C_2 R_2} = 0 \rightarrow s_z = \infty \\ s_z C_{12} - g_m = 0 \rightarrow s_z = \frac{g_m}{C_{12}} \end{cases}$$

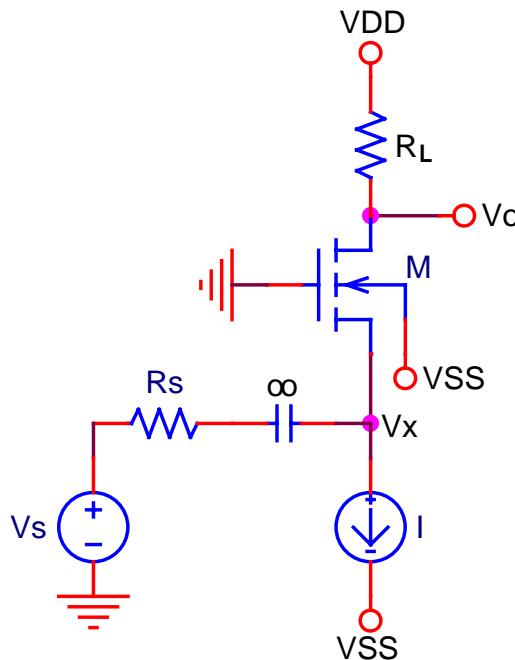
## High-frequency response of the CG amplifier

the high-freq. Response of the CS amplifier is limited by the Miller effect caused by the feedback capacitor  $C_{gd}$

CG amplifier

no feedback capacitor between the input and output

no Miller effect → improved bandwidth

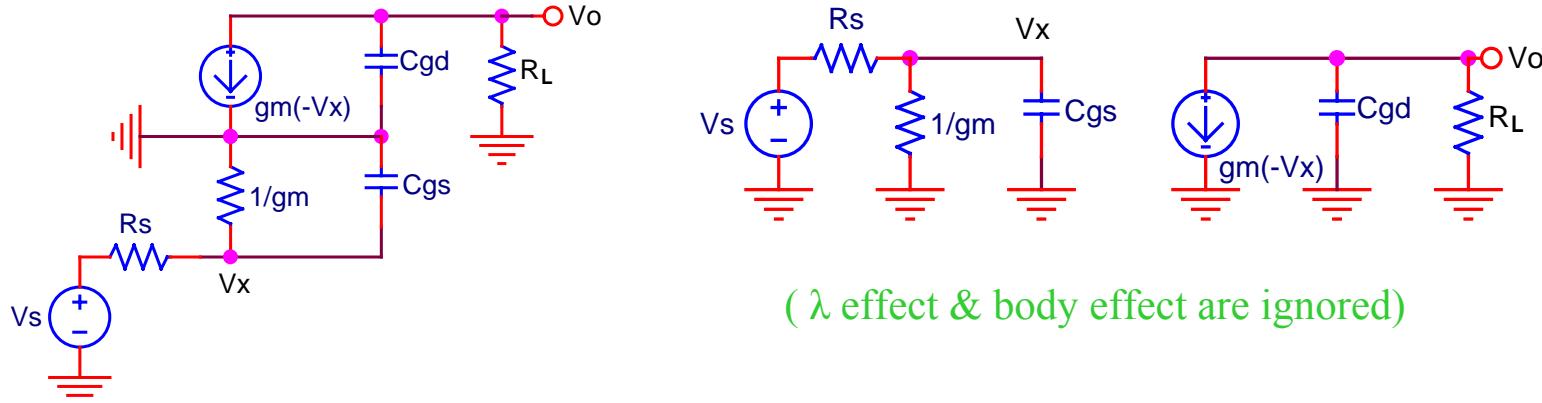


Analysis methods

using the T equivalent circuit

→ STC(Single Time Constant) networks  
directly from the circuit

## Using the T equivalent circuit



1<sup>st</sup> STC network

low-frequency or midband gain :  $A_{M1} = \frac{V_x}{V_s} = \frac{1/g_m}{R_s + 1/g_m} = \frac{1}{1 + g_m R_s}$

time constant :  $\tau_1 = C_{gs} \left( R_s // \frac{1}{g_m} \right)$

transfer function :  $A_l(s) = \frac{V_x(s)}{V_s(s)} = A_{M1} \frac{1}{1 + s/\omega_{P1}}, \quad \omega_{P1} = \frac{1}{\tau_1}$

## 2<sup>nd</sup> STC network

low-frequency or midband gain :  $A_{M2} = \frac{V_o}{V_x} = g_m R_L$

time constant :  $\tau_2 = C_{gd} R_L$

transfer function :  $A_2(s) = \frac{V_o(s)}{V_x(s)} = A_{M2} \frac{1}{1 + s/\omega_{P2}}, \quad \omega_{P2} = \frac{1}{\tau_2}$

overall transfer function

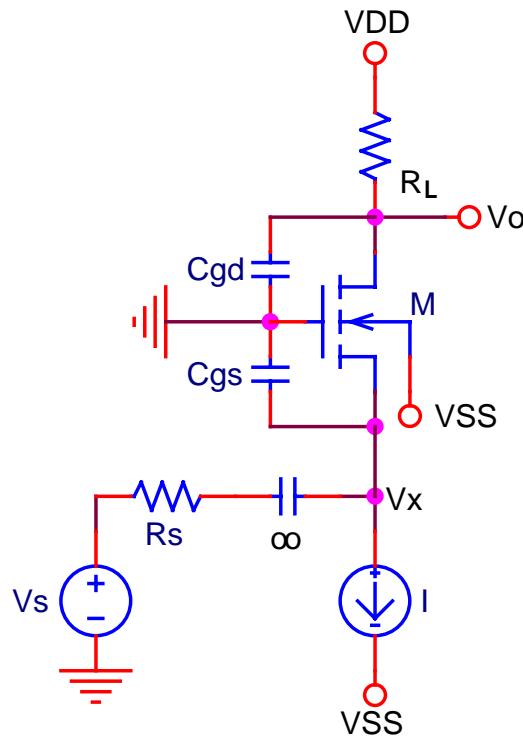
$$A(s) = \frac{V_o(s)}{V_s(s)} = A_1(s) A_2(s) = A_M \left( \frac{1}{1 + s/\omega_{P1}} \right) \left( \frac{1}{1 + s/\omega_{P2}} \right)$$

$$A_M = A_{M1} A_{M2} = \frac{g_m R_L}{1 + g_m R_s}$$

$$\omega_{P1} = \frac{1}{C_{gs} \left( R_s // \frac{1}{g_m} \right)} \quad \omega_{P2} = \frac{1}{C_{gd} R_L}$$

which one is dominant?  $\rightarrow$  depends on  $R_s$  &  $R_L$  &  $C_L$

Directly from the circuit



midband gain

$$\frac{V_x}{V_s} = \frac{1/g_m}{R_s + 1/g_m} = \frac{1}{1 + g_m R_s} \quad \frac{V_o}{V_x} = g_m R_L$$

$$\therefore A_M = \frac{V_o}{V_s} = \frac{g_m R_L}{1 + g_m R_s}$$

time constants

$$@ V_x : \tau_1 = C_{gs} \left( R_s // \frac{1}{g_m} \right)$$

$$@ V_o : \tau_2 = C_{gd} R_L$$

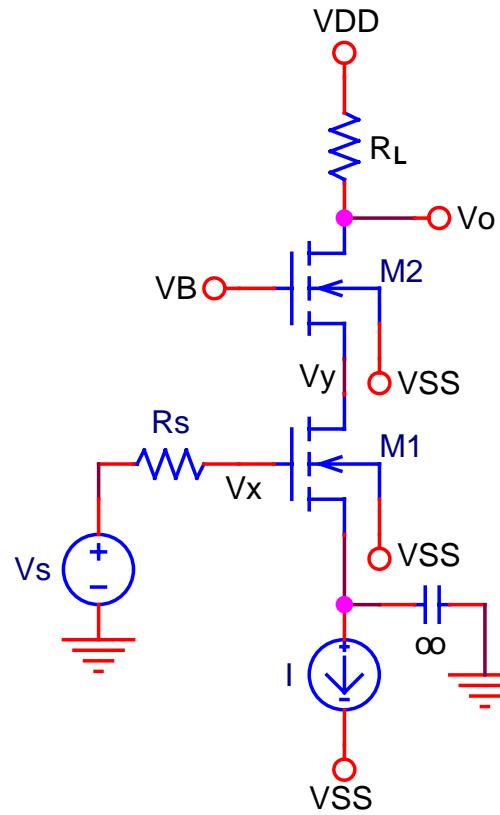
transfer function

$$\frac{V_o(s)}{V_s(s)} = A_M \left( \frac{1}{1 + s/\omega_{P1}} \right) \left( \frac{1}{1 + s/\omega_{P2}} \right)$$

$$\omega_{P1} = \frac{1}{\tau_1} \quad \omega_{P2} = \frac{1}{\tau_2}$$

## The cascode configuration

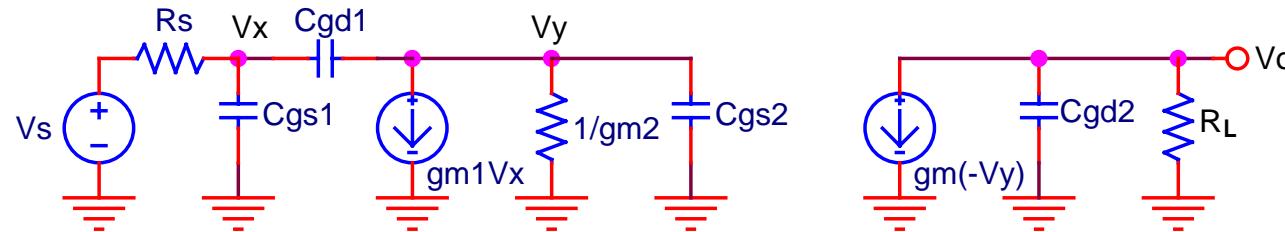
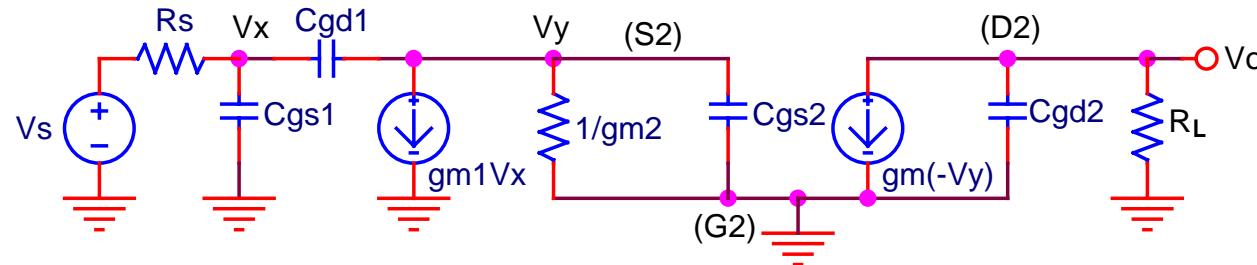
combines the advantages of the CS and CG circuits



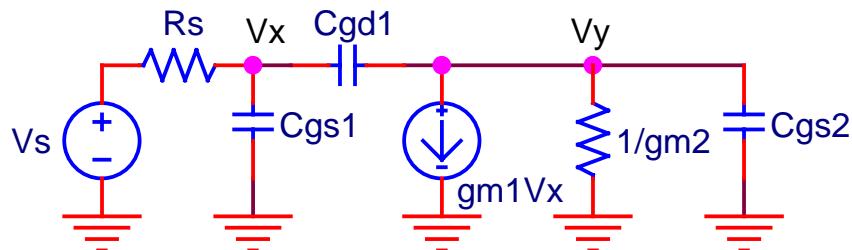
Analysis methods  
using the equivalent circuits  
directly from the circuit

Using the equivalent circuits

( $\lambda$  effect & body effect are ignored)



left circuit : CS amplifier



$$C_1 = C_{gs1}, \quad C_{12} = C_{gd1}, \quad C_2 = C_{gs2}$$

$$g_m = g_{m1}, \quad R_2 = 1/g_{m2}$$



$$\omega_{P1} \ll \omega_{P2} \quad \& \quad C_1 \approx C_2 \quad \& \quad R_2 \ll R_s \rightarrow g_m R_2 \approx 1$$

$$\omega_{P1} \approx \frac{G_s}{C_1 + 2C_{12}} \quad \omega_{P2} \approx \frac{G_2}{C_2} \quad \omega_Z = \frac{g_m}{C_{12}}$$

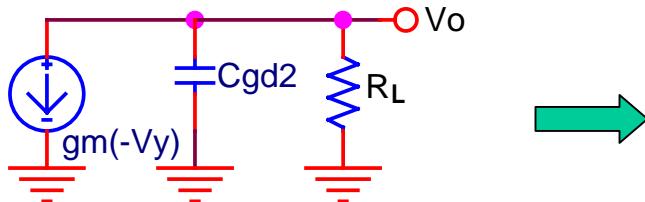
$$K_1 = \frac{V_y}{V_s} = -g_m R_2 = -g_{m1} \frac{1}{g_{m2}} \approx -1$$

$$\omega_{P1} \approx \frac{1}{R_s(C_{gs1} + 2C_{gd1})}$$

$$\omega_{P2} \approx \frac{g_{m2}}{C_{gs2}} \quad \omega_Z = \frac{g_{m1}}{C_{gd1}}$$



right circuit : STC network



$$K_2 = \frac{V_o}{V_y} = g_{m2} R_L \quad \omega_{P3} = \frac{1}{C_{gd2} R_L}$$

overall gain

$$\frac{V_o(s)}{V_s(s)} = A_M \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)\left(1 + \frac{s}{\omega_{P3}}\right)}, \quad A_M = K_1 K_2 = -g_{m2} R_L$$

which pole is dominant? → depends on  $R_s$  &  $R_L$  &  $C_L$

in integrated amplifiers

$R_s$  is small

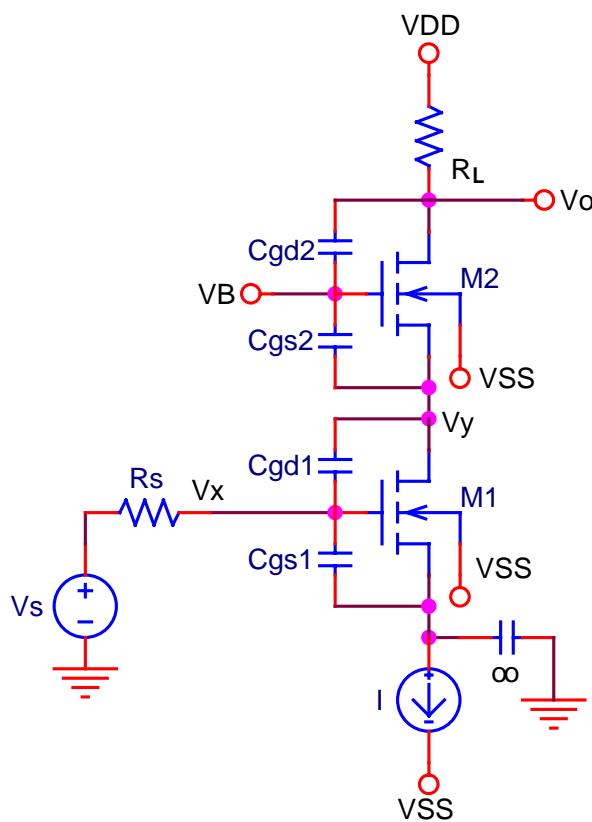
active loads are used →  $R_L$  is large

load capacitor  $C_L$  exists

$$\omega_{P3} = \frac{1}{(C_{gd2} + C_L)R_L}$$

→ dominant

Directly from the circuit



midband gain

$$\frac{V_x}{V_s} = 1, \quad \frac{V_y}{V_x} = -g_{m1} \frac{1}{g_{m2}} = -1, \quad \frac{V_o}{V_y} = g_{m2} R_L$$

$$\therefore A_M = \frac{V_o}{V_s} = \frac{V_x}{V_s} \frac{V_y}{V_x} \frac{V_o}{V_y} = -g_{m2} R_L$$

poles

Miller effect can be ignored      small gain  
 $C_{gd1}$  is usually small

$$@ V_x : \omega_{P1} \approx \frac{1}{R_s C_{gs1}}$$

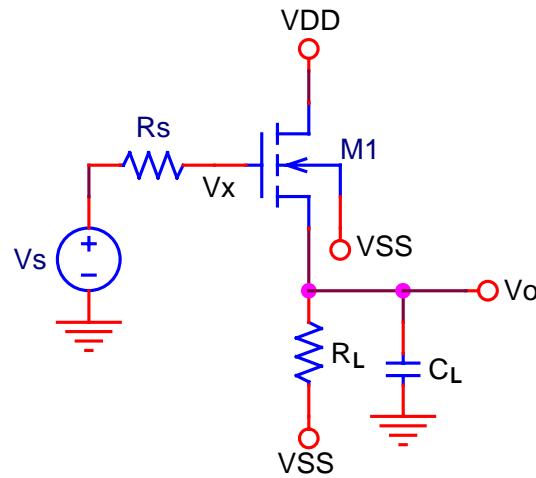
$$@ V_y : \omega_{P2} \approx \frac{1}{C_{gs2}(1/g_{m2})} = \frac{g_{m2}}{C_{gs2}}$$

$$@ V_o : \omega_{P3} = \frac{1}{C_{gd2} R_L}$$

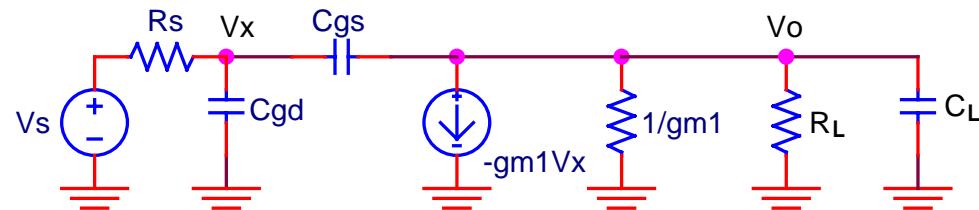
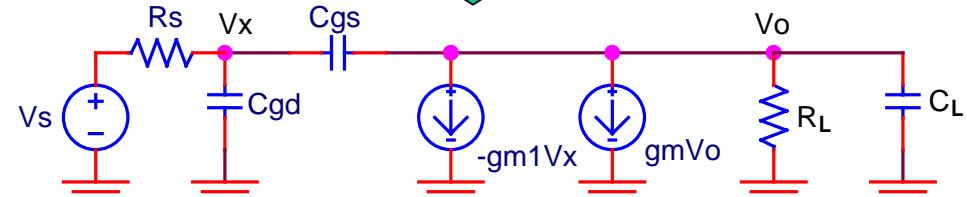
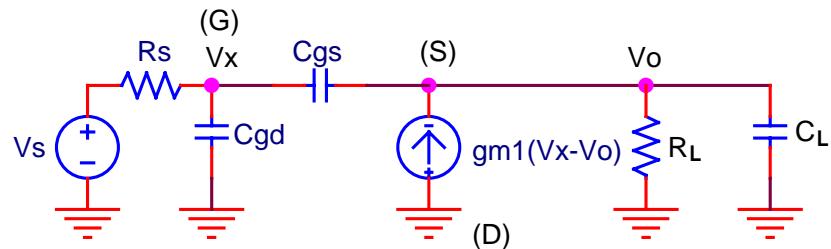
zero

$$\omega_z = \frac{g_{m1}}{C_{gd1}}$$

## High-frequency response of the CD amplifier

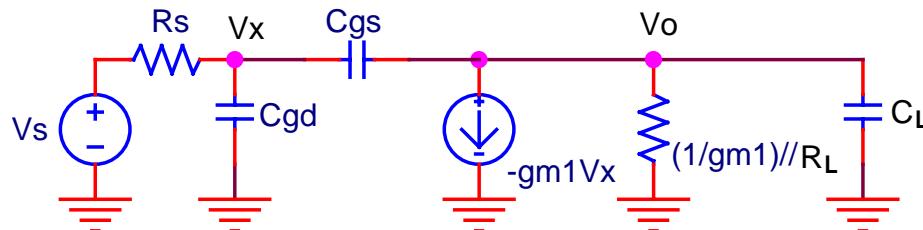


( $\lambda$  effect & body effect are ignored)



(same architecture as that of CS amplifiers)

equivalent circuit



$$\begin{aligned} C_1 &= C_{gd}, \quad C_{12} = C_{gs}, \quad C_2 = C_L \\ g_m &= -g_{m1}, \quad R_2 = (1/g_{m1})/\text{parallel } R_L \end{aligned}$$

transfer function

$$A_v(s) = A_M \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)}$$

midband gain

$$A_M = -g_m R_2 = -(-g_{m1}) R_2 = \frac{g_{m1} R_L}{1 + g_{m1} R_L}$$

zero

$$\omega_z = \frac{g_m}{C_{12}} = \frac{-g_{m1}}{C_{gs}}$$

poles

$R_s$  : large &  $C_L$  : small

$$\begin{aligned}\omega_{P1} &\approx \left( \frac{C_2 + C_{12}}{G_2} + \frac{C_1 + C_{12}}{G_s} + \frac{g_m C_{12}}{G_s G_2} \right)^{-1} \\ &\approx \left( \frac{C_1 + C_{12}}{G_s} + \frac{g_m C_{12}}{G_s G_2} \right)^{-1} = \left( \frac{C_{gd} + C_{gs}}{G_s} - \frac{g_{m1} C_{gs}}{G_s G_2} \right)^{-1} \\ &= \frac{1}{R_s \left( C_{gd} + \frac{C_{gs}}{1 + g_{m1} R_L} \right)} \approx \frac{1}{R_s C_{gd}}\end{aligned}$$

$$R_L > \frac{1}{g_{m1}} \rightarrow g_{m1} R_L > 1$$

$$\rightarrow R_2 = R_L // \frac{1}{g_{m1}} \approx \frac{1}{g_{m1}}$$

$$\rightarrow g_{m1} R_2 \approx 1$$

$$\omega_{P2} \approx \left( \frac{G_s G_2}{C_1 C_2 + C_1 C_{12} + C_2 C_{12}} \right) \frac{1}{\omega_{P1}} \approx \left( \frac{G_s G_2}{C_1 C_{12}} \right) \frac{1}{\omega_{P1}} = \frac{G_2}{C_{12}} = \frac{g_{m1}}{C_{gs}}$$

$R_s$  : small &  $C_L$  : large

$$\omega_{P1} \approx \left( \frac{C_2 + C_{12}}{G_2} + \frac{C_1 + C_{12}}{G_s} + \frac{g_m C_{12}}{G_s G_2} \right)^{-1}$$

$$\approx \left( \frac{C_2 + C_{12}}{G_2} \right)^{-1} = \frac{G_2}{C_{gs} + C_L} \approx \frac{g_{m1}}{C_L}$$

$$\omega_{P2} \approx \left( \frac{G_s G_2}{C_1 C_2 + C_1 C_{12} + C_2 C_{12}} \right) \frac{1}{\omega_{P1}} \approx \frac{G_s G_2}{(C_1 + C_{12}) C_2} \frac{C_2 + C_{12}}{G_2} \approx \frac{1}{R_s (C_{gs} + C_{gd})}$$

- 
1. A. S. Sedra and K. C. Smith, “Microelectronic Circuits,” Fourth Edition, Oxford University Press, 1988.
  2. Behzad Razavi, “Design of Analog CMOS Integrated Circuits,” McGraw-Hill, 2001.
  3. R. L. Geiger, “VLSI Design Techniques for Analog and Digital Circuits,” McGraw-Hill, 1990.