

SENSORLESS SPEED CONTROL OF INDUCTION MOTOR USING VHDL/FPGA

Abstract

This project presents the mathematical principles and algorithm underlying a new sensorless control strategy for a three phase squirrel cage induction motor. The control method comprises of two elements: the stator current control strategy and sensorless speed control strategy both of them based on equivalent 3- ϕ R-L-e circuit where parameter are designed from space vector model of the induction motor. The neural network algorithm is used to generate 3-bit code associated with the output voltage generated by a PWM inverter. The neural network size and architecture is determined based on the necessary calculations precision and available hardware resemblance.. The motor controller structure developed in this project uses FPGA (field programmable gate array) implemented neural network alongside pipelined digital structure to carryout the computationally intensive task of controlling the stator current. The use of specialized digital structure ensures fast operation and allows speed control and current control algorithm to be performed in parallel. The VHDL design for the three phase induction motor and FPGA implementation are presented including simulation result.

Chapter 1

Introduction

1.1 Historical Review

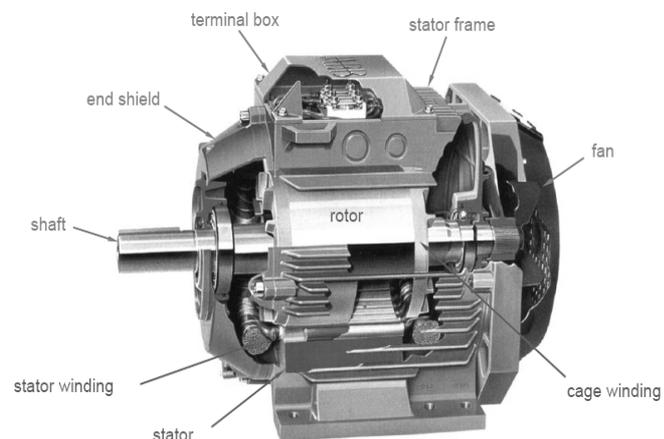
The history of electrical motors goes back as far as 1820, when Hans Christian Oersted discovered the magnetic effect of an electric current. One year later, Michael Faraday discovered the electromagnetic rotation and built the first primitive D.C. motor. Faraday went on to discover electromagnetic induction in 1831, but it was not until 1883 that Tesla invented the A.C asynchronous motor. In 1882, Nikola Tesla identified the rotating magnetic field principle, and pioneered the use of a rotary field of force to operate machines. He exploited the principle to design a unique two-phase induction motor in 1883. In 1885, Galileo Ferraris independently researched the concept. In 1888, Ferraris published his research in a paper to the Royal Academy of Sciences in Turin. Introduction of Tesla's motor from 1888 onwards initiated what is known as the Second Industrial Revolution, making possible the efficient generation and long distance distribution of electrical energy using the alternating current transmission system, also of Tesla's invention (1888). Before the invention of the rotating magnetic field, motors operated by continually passing a conductor through a stationary magnetic field (as in homopolar motors). Tesla had suggested that the commutators from a machine could be removed and the device could operate on a rotary field of force. Professor Poeschel, his teacher, stated that would be akin to building a perpetual motion machine. This classic alternating current electro-magnetic motor was an induction motor. In the *induction motor*, the field and armature were ideally of equal field strengths and the field and armature cores were of equal sizes. The total energy supplied to

operate the device equaled the sum of the energy expended in the armature and field coils. The power developed in operation of the device equaled the product of the energy expended in the armature and field coils. The main advantage is that induction motors do not require an electrical connection between stationary and rotating parts of the motor. Therefore, they do not need any mechanical commutator (brushes), leading to the fact that they are maintenance free motors. Induction motors also have low weight and inertia, high efficiency and a high overload capability. Therefore, they are cheaper and more robust, and less prone to any failure at high speeds. Furthermore, the motor can work in explosive environments because no sparks are produced.

1.2 Basic principle of working of induction motor

As a general rule, conversion of electrical power into mechanical power takes place in the rotating part of an electric motor. In d.c. motors, the electrical power is conducted directly to the armature (i.e. rotating part) through brushes and commutator. Hence in this sense, a d.c. motor can be called a conduction motor.

However, in a.c. motors the rotor does not receive electric power by conduction but by induction in exactly the same way as the secondary of a 2-winding transformer receives its power from the



1.2.1 Layout of Induction motor

primary. That is why such motors are known as induction motors. In fact, an induction motor can be treated as a rotating transformer i.e. one in which primary winding is stationary but the secondary is free to rotate. Induction motor is the most common type of AC motor used in the world. It can be single phase or three phase. They are widely used for different applications ranging from small induction motors in washing machines, household fans etc to vary large induction motors which are capable of tens of thousands of kW in output, for pipeline compressors, wind-tunnel drives and overland conveyer systems. Through electromagnetic induction, the rotating magnetic field induces a current in the conductors in the rotor, which in turn sets up a counterbalancing magnetic field that causes the rotor to turn in the direction the field is rotating. The rotor must always rotate slower than the rotating magnetic field produced by the polyphase electrical supply; otherwise, no counterbalancing field will be produced in the rotor. Induction motors are the workhorses of industry and motors up to about 500 kW (670 horsepower) in output are produced in highly standardized frame sizes, making them nearly completely interchangeable between manufacturers.

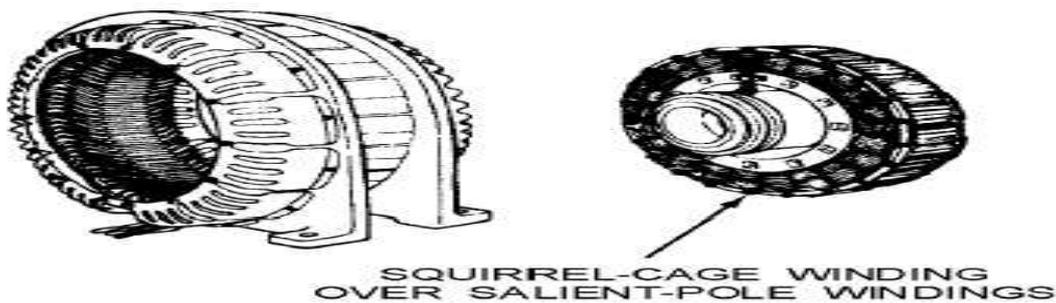
1.3 Type of rotors used in induction motor

There are two types of rotors used in induction motors.

1.3.1 Squirrel Cage rotors:

Most common AC motors use the squirrel cage rotor, which will be found in virtually all domestic and light industrial alternating current motors. The squirrel cage takes its name from its shape - a ring at either end of the rotor, with bars connecting the rings running the length of the rotor. It is typically cast aluminum or copper poured between the iron laminates of the

rotor, and usually only the end rings will be visible. The vast majority of the rotor currents will flow through the bars rather than the higher-resistance and usually varnished laminates. Very low voltages at very high currents are typical in the bars and end rings; high efficiency motors will often use cast copper in order to reduce the resistance in the rotor. In operation, the squirrel cage motor may be viewed as a transformer with a rotating secondary - when the rotor is not rotating in sync with the magnetic field, large rotor currents are induced; the large rotor currents magnetize the rotor and interact with the stator's magnetic fields to bring the rotor into synchronization with the stator's field. An unloaded squirrel cage motor at synchronous speed will consume electrical power only to maintain rotor speed against friction and



1.3.1.1 Squirrel-cage rotor

resistance losses; as the mechanical load increases, so will the electrical load - the electrical load is inherently related to the mechanical load. This is similar to a transformer, where the primary's electrical load is related to the secondary electrical load. This is why, as an example, a squirrel cage blower motor may cause the lights in a home to dim as it starts, but doesn't dim the lights when its fan belt (and therefore mechanical load) is removed. Furthermore, a stalled squirrel cage motor (overloaded or with a jammed shaft) will consume current limited only by circuit resistance as it attempts

to start. Unless something else limits the current (or cuts it off completely) overheating and destruction of the winding insulation is the likely outcome. Virtually every washing machine, dishwasher, standalone fan, record player, etc. uses some variant of a squirrel cage motor.

1.3.2 Wound Rotor:

An alternate design, called the wound rotor, is used when variable speed is required. In this case, the rotor has the same number of poles as the stator and the windings are made of wire, connected to slip rings on the shaft. Carbon brushes connect the slip rings to an external controller such as a variable resistor that allows changing the motor's slip rate. In certain high-power variable speed wound-rotor drives, the slip-frequency energy is captured, rectified and returned to the power supply through an inverter. Compared to squirrel cage rotors, wound rotor motors are expensive and require maintenance of the slip rings and brushes, but they were the standard form for variable speed control before the advent of compact power electronic devices. Transistorized inverters with variable-frequency drive can now be used for speed control, and wound rotor motors are becoming less common. (Transistorized inverter drives also allow the more-efficient three-phase motors to be used when only single-phase mains current is available, but this is never used in household appliances, because it can cause electrical interference and because of high power requirements.) Several methods of starting a polyphase motor are used. Where the large inrush current and high starting torque can be permitted, the motor can be started across the line, by applying full line voltage to the terminals (Direct-on-line, DOL). Where it is necessary to limit the starting inrush current (where the motor is large compared with the short-circuit capacity of the

supply), reduced voltage starting using a series inductors, an autotransformer, thyristors, or other devices are used. A technique sometimes used is star-delta starting, where the motor coils are initially connected in wye for acceleration of the load, then switched to delta when the load is up to speed. This technique is more common in Europe than in North America. Transistorized drives can directly vary the applied voltage as required by the starting characteristics of the motor and load. This type of motor is becoming more common in traction applications such as locomotives, where it is known as the asynchronous traction motor.

1.4 Speed and slip

The speed of the AC motor is determined primarily by the frequency of the AC supply and the number of poles in the stator winding, according to the relation:

$$N_s = 120F / p$$

Where

N_s = Synchronous speed, in revolutions per minute

F = AC power frequency

p = Number of poles per phase winding

Actual RPM for an induction motor will be less than this calculated synchronous speed by an amount known as *slip*, that increases with the torque produced. With no load, the speed will be very close to synchronous. When loaded, standard motors have between 2-3% slip, special motors may have up to 7% slip, and a class of motors known as *torque motors* are rated to operate at 100% slip (0 RPM/full stall).

The slip of the AC motor is calculated by:

$$S = (N_s - Nr) / N_s$$

$$\text{Percentage slip} = (N_s - N_r) / N_s * 100$$

Where

N_r = Rotational speed, in revolutions per minute.

S = Normalized Slip, 0 to 1.

For motoring action $0 < \text{slip} < 1$

For generator action $\text{slip} > 1$

For braking action $\text{slip} < 0$

As an example, a typical four-pole motor running on 60 Hz might have a nameplate rating of 1725 RPM at full load, while its calculated speed is 1800 RPM. The speed in this type of motor has traditionally been altered by having additional sets of coils or poles in the motor that can be switched on and off to change the speed of magnetic field rotation. However, developments in power electronics mean that the frequency of the power supply can also now be varied to provide a smoother control of the motor speed.

A 3- ϕ induction motor is practically a constant speed machine, more or less like a dc shunt motor. The speed regulation of an induction motor (having a low resistance) is usually less than 5 % at full load. In the case of induction motor when the load is increased then speed will decrease and the speed reduction is accompanied by a corresponding loss of efficiency and good speed regulation. So it is quite difficult to build a good adjustable speed induction motor. Different methods by which speed control of induction motor is achieved are as follows

Control from the stator side-

- ✓ By changing the applied voltage
- ✓ By changing the applied frequency

- ✓ By changing the number of stator poles

Control from the rotorside-

- ✓ Rotor rheostat control
- ✓ By operating two motor in concatenation or cascade
- ✓ By injecting an emf in the rotor circuit

1.5 Sensorless Speed Control of Induction Motors:

In the past, DC motors were used extensively in areas where variable-speed operations were required. DC motors have certain disadvantages, however, which are due to the existence of the commutator and the brushes. They require periodic maintenance and thus they cannot be used in explosive or corrosive environments. They have limited commutator capability under high-speed, high-voltage operational conditions. These problems could be overcome by application of AC motors. AC motors have simpler and more rugged structure, higher maintainability and economy than DC motors. They are also robust and immune to heavy loading. Their small dimension compared with DC motors allows AC motors to be designed with substantially higher output ratings for small load, low speed operations. Progress in the field of power electronics have made it possible to overcome difficulty of control of AC motors and to apply AC drives for high-performance applications where traditionally only DC drives were applied. In recent years, so-called sensorless control scheme for AC drives has been one of the most popular research topics in this area. For direct control of AC motors, information about its rotational speed (or rotational position) is crucial and in general shaft-mounted tacho-generators and resolvers are used to measure them. The elimination of those transducers has long been an attractive prospect, since the shaft transducers and the associated signal

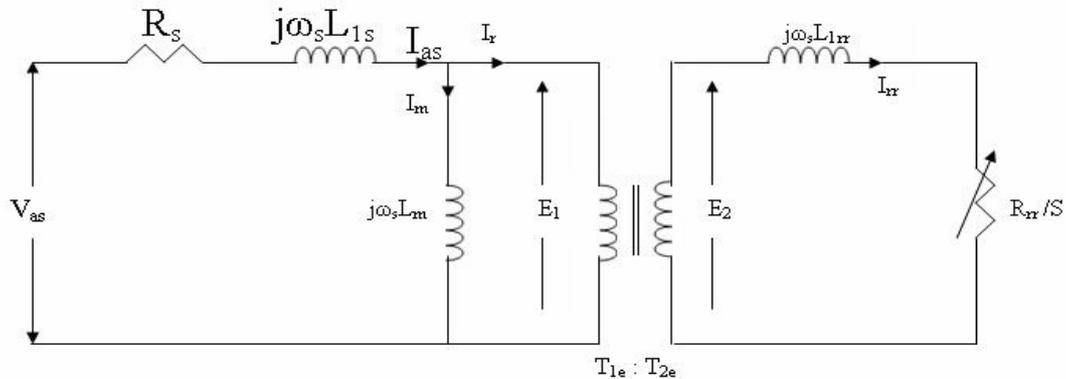
wiring are a significant source of failure, additional cost, and additional weight. Numerous approaches have been proposed to estimate the rotor velocity and/or position from the machine terminal properties, such as the stator current or voltage. Sensorless speed control drives have reached the status of a maturing technology in a broad range of applications ranging from low-cost to high-performance systems. Eliminating the speed sensor on the motor shaft represents a cost advantage, which combines favorably with increased reliability due to the absence of this mechanical component and its sensor cable. This project presents the mathematical principles and algorithm underlying a new sensorless control strategy for a three phase squirrel cage induction motor. The control method comprises of two elements: the stator current control strategy and sensorless speed control strategy both of them based on equivalent 3- ϕ R-L-e circuit where parameter are designed from space vector model of the induction motor. The neural network algorithm is used to generate 3-bit code associated with the output voltage generated by a PWM inverter. The neural network size and architecture is determined based on the necessary calculations precision and available hardware resemblance. A new sensorless speed control method is formulated and tested by simulation. The motor controller structure developed in this project uses FPGA (field programmable gate array) implemented neural network alongside pipelined digital structure to carryout the computationally intensive task of controlling the stator current. The use of specialized digital structure ensures fast operation and allows speed control and current control algorithm to be performed in parallel. The VHDL design and FPGA implementation are presented including simulation result.

Chapter 2

Mathematical Equations and algorithms

2.1 Equivalent Circuit of Induction motor

The induction motor can be treated essentially as a transformer for analysis. The equivalent circuit of the induction motor is very similar to that for a transformer. Although the rotor currents are at slip frequency, the rotor is incorporated into the circuit in a simple way. Recognizing the fact that the



2.1.1 Single phase equivalent circuit of induction motor

stator and rotor windings have resistances and leakage inductances and that the mutual inductance for modeling the mutual flux links the stator and rotor winding. The parameters are stator resistance per phase, R_s rotor resistance per phase, R_{rr} mutual inductance, L_m stator leakage inductance per phase L_{1s} , rotor leakage reactance per phase, L_{1r} stator turns per phase T_1 , rotor turns per phase T_2 , induced emf in the stator per phase, E_1 , and induced emf in the rotor phase, E_2 , Effective stator or rotor turns per phase is equal to the product of the number of turns per phase and the winding factor of the stator and rotor and respectively, is denoted as K_{w1} and K_{w2} . The winding factor is the product of pitch and distribution factors that account for the specific winding characteristics. The transformer in the centre of the equivalent

circuit can be eliminated by adjusting the values of the rotor components in accordance with the effective turn's ratio of the transformer. From the equivalent circuit and a basic knowledge of the operation of the induction motor, it can be seen that the magnetizing current component and the iron loss of the motor are voltage dependant, and not load dependant. Additionally, the full voltage starting current of a particular motor is voltage and speed dependant, but not load dependant. The magnetizing current varies depending on the design of the motor. For small motors, the magnetizing current may be as high as 60%, but for large two pole motors, the magnetizing current is more typically 20 - 25%. At the design voltage, the iron is typically near saturation, so the iron loss and magnetizing current do not vary linearly with voltage with small increases in voltage resulting in a high increase in magnetizing current and iron loss. The relationship between the induced emfs is

$$\frac{E_2}{E_1} = s \frac{T_{2e}}{T_{1e}} = \frac{s}{a} \quad \dots\dots(2.1)$$

$$a = \frac{T_{1e}}{T_{2e}} = \frac{k_{w1}T_1}{k_{w2}T_2} = \text{turns ratio} \quad (2.2)$$

Where T_{1e} and T_{2e} are the effective stator and rotor turns per phase, and the turns ratio, a , between the stator to rotor effective turns per phase is given as:

The rotor current I_{rr} , then is

$$I_{rr} = \frac{E_2}{R_{rr} + j\omega_{sl}L_{lrr}} = \frac{E_2}{R_{rr} + js\omega_s L_{lrr}} \quad (2.3)$$

Substituting for E_2 from equation 2.1 into equation 2.3, the rotor current is

$$I_{rr} = \frac{E_1}{\frac{aR_{rr}}{s} + j\omega_s(aL_{lrr})} = \frac{E_1/a}{\frac{R_{rr}}{s} + j\omega_s L_{lrr}} \quad (2.4)$$

Equation is incorporated into the equivalent circuit as shown in Fig 2.2. Note that both the rotor and stator uniformly have the same frequency, which is that of the stator in equation (2.4). The rotor current reflected into the stator is denoted as I_r in figure 2.2 and is given in terms of the rotor current I_{rr} as

$$I_r = \frac{I_{rr}}{a} \quad \dots (2.5)$$

Note that the slip does not enter in this the ratio between the stator and rotor induced emfs viewed at stator frequency does not contain it and rotor independence absorbs this slip. Substitution of equation (2.4) into equation yields.

$$I_r = \frac{E_1}{\frac{(a^2 R_{rr})}{s} + j\omega_s(a^2 L_{lrr})} = \frac{E_1}{\frac{R_r}{s} + j\omega_s L_{lr}} \quad (2.6)$$

Where R_r and L_{lr} are the stator referred rotor resistance and leakage inductance respectively, given as

$$\begin{aligned} R_r &= a^2 R_{rr} \\ L_{lr} &= a^2 L_{lrr} \end{aligned} \quad (2.7)$$

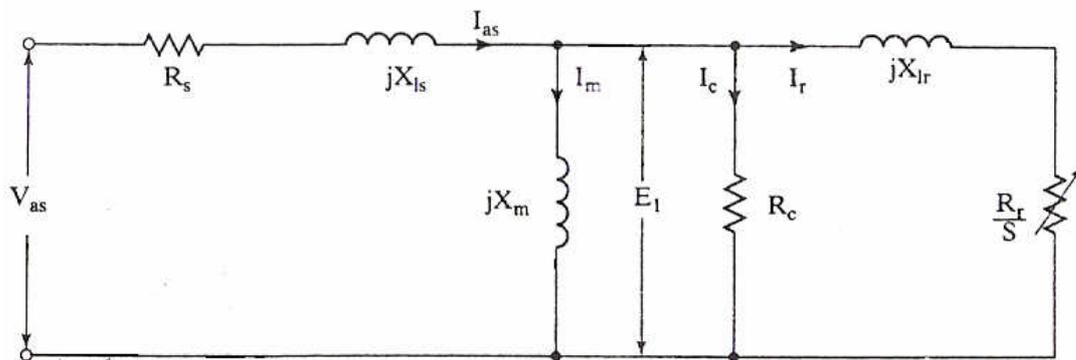
As the fictitious rotor and stator at the air gap have the same induced emf, E_1 the physical isolation can be removed to get a connected circuit. The final equivalent circuit, referred to the stator. Magnetization is accounted for by the magnetizing branch of the equivalent circuit, consisting of the magnetizing inductance can represented the core losses. This core – less

resistance is in parallel to the magnetizing inductance, because the core losses are dependant on the flux and hence proportional to the flux linkages and the resulting air gap voltage E_1 . The self – inductances of the stator phase winding, L_s and of the rotor phase winding referred to the stator, L_r are obtained as the sum of the magnetizing inductance and respective leakage inductances.

$$L_s = L_m + L_{ls}$$

$$L_r = L_m + L_{lr} \dots (2.8)(2.9)$$

The corresponding reactances are obtained by multiplying the inductances by the stator angular frequency. A simple phasor diagram of induction motor is drawn by using the equivalent circuit given in Figure 2.2. The basis is provided by the following relationships.



2.1.2 Equivalent circuit with the rotor at stator frequency

When the machine is energized with no load on the rotor, then the rotor circuit is open – circuited because the slip is zero. The stator current drawn during this condition, known as the no-load current, contributes to the magnetization of the machine and resulting core losses. Then this no – load current is viewed as the sum of the magnetizing and core – loss components of the current and accordingly is written as

$$I_o = I_m + I_c \quad (2.10)$$

Where I_r is the no – load phase current I_m is the magnetizing current, and I_c is the core – loss current. The magnetizing current in terms of the air gap voltage and the magnetizing reactance is written as

$$I_m = \frac{E_1}{jX_m} \quad (2.11)$$

Where E_1 is the air gap voltage and X_m is the magnetizing reactance, similarly, the core – loss component of the stator current is written as

$$I_c = \frac{E_1}{R_c} \quad (2.12)$$

Where R_c is the resistance to account for the core losses. The rotor phase current is given by

$$I_r = \frac{E_1}{\frac{R_r}{s} + jX_{lr}} \quad (2.13)$$

Where I_r is the rotor phase current. The stator phase current then is

$$I_{as} = I_r + I_o \quad (2.14)$$

In terms of the induced emf stator current and stator parameters, the applied stator voltage is expressed as the sum of the induced emf and stator impedance voltage drop and is given by

$$V_{as} = E_1 + (R_s + jX_{ls})I_{as} \quad (2.15)$$

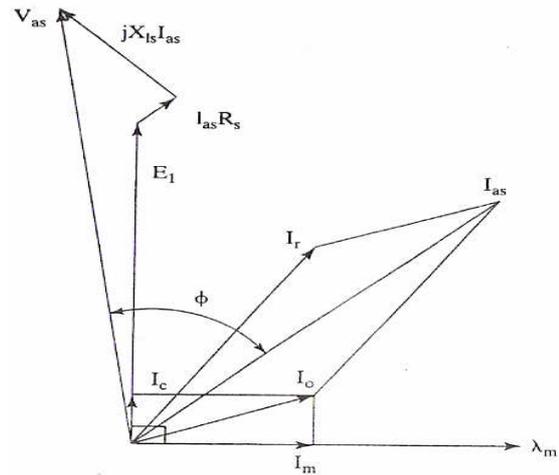
Where V_{as} is the stator phase voltage

The phasor diagram is shown in figure 2.3. The angle ϕ between the stator input voltage and the stator current is the power factor angle of the

stator and λ_m is the mutual flux. The steady – state performance is taken up next by using the equivalent circuit.

2.2 Steady state performance Equations of induction motor:

The key variables in the machine are the air gap power, mechanical and shaft output power, and electromagnetic torque. These are derived from the equivalent circuit of the induction machine as follows. The real power transmitted from the stator, P_i , to the air gap, P_a , is the difference between total input power to the stator windings and copper losses in the stator and is given as



2.2.1 Phasor diagram of induction motor

$$P_a = P_i - 3I_s^2 R_s \quad (\text{W}) \quad \dots(2.16)$$

Neglecting the core losses, the air gap power is equal to the total power dissipated in R_1/s in the three phases of the machine; there is no other element to consume power in the rotor equivalent circuit. It is given as

$$P_a = 3I_r^2 \frac{R_r}{s} \quad \dots (2.17)$$

Which could be written alternatively as

$$P_a = 3I_r^2 R_r + 3I_r^2 R_r \frac{(1-s)}{s} \quad \dots (2.18)$$

The $I_r^2 R_r$ term is recognized as rotor copper loss, and hence the remainder, $I_r^2 R_r (1 - S) / S$, gives the power converted into mechanical form. The rotor copper losses are equal to the product of slip and air gap power from equation (2.17) and this is referred to as slip power. The common term of three in equations (2.17) and (2.18) accounts for the number of phases in the machines, which throughout taken as three. The mechanical power output P_m is obtained as

$$P_m = 3I_r^2 R_r \frac{(1 - s)}{s} \quad (W) \quad \dots (2.19)$$

Alternately, in terms of the electromagnetic torque and rotor speed, the mechanical power output is equal to their product:

$$P_i = \frac{3}{2} (v_{qs} i_{qs} + v_{ds} i_{ds}) \quad \dots (2.21)$$

Where T_e is the internal or electromagnetic torque, derived from equations

$$T_e = \frac{3I_r^2 R_r (1 - s)}{s\omega_m} \quad \dots (2.22)$$

Substituting for the rotor speed in terms of the slip and stator frequency, given by

$$\omega_m = \frac{\omega_r}{P/2} = \frac{\omega_s (1 - s)}{P/2} \quad \dots (2.23)$$

Into, the electromagnetic or air gap torque is obtained as

$$T_e = 3 \left(\frac{P}{2} \right) \frac{I_r^2 R_r}{s\omega_s} \quad \dots (2.24)$$

To obtain the shaft output power of the machine, P_s , the windings and friction losses of the rotor, denoted as P_{fw} have to be subtracted from the mechanical output power of the machine, symbolically given as follows:

$$P_s = P_m - P_{fw} \dots (2.25)$$

2.3 Dynamic Modeling of Induction Machine:

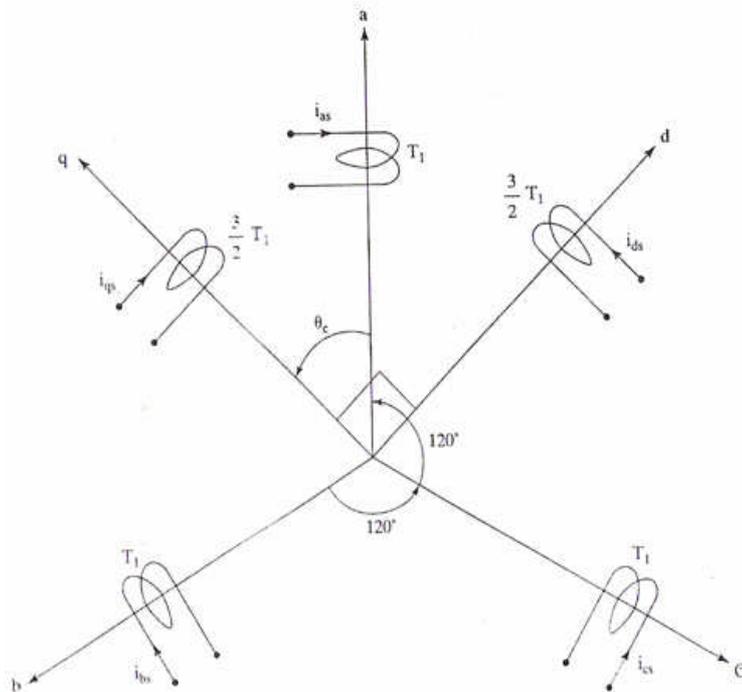
The steady state model and equivalent circuit developed in earlier sections are useful for studying the performance of the machine in steady state. This implies that all electrical transients are neglected during load changes and stator frequency variations. Such variations arise in applications involving variable speed drives. The variable speed drives are converter fed from finite sources unlike the utility sources due to the limitations of the switch ratings and filter sizes. This results in their incapability to supply large transient power. Hence we need to evaluate the dynamics of converter fed variable speed drives to assess the adequacy of the converter switches and the converters for a given motor and their interaction to determine the excursions of currents and torque in the converter and motor. The dynamic model considers the instantaneous effects of varying voltages / current stator frequency, and torque disturbance. The dynamic model of the induction motor is derived because of the conceptual simplicity obtained with two sets of windings, one on the stator and the other on the rotor. The equivalence between the three phase and two phase machine model is derived from simple observation, and this approach is suitable for extending it to model an n – phase machine by means of a two phase machine. The concept of power invariance is introduced. The power must be equal in the three phase machine and its equivalent two phase model. The required transformation in voltage currents or flux linkages is derived in a generalized way. The reference frames are chosen to be arbitrary, and particular cases such as stationary, rotor and synchronous reference frames, are simple instances of

the general case. Derivations for electromagnetic torque involving the currents and flux linkages are given. The differential equations describing the induction motors are nonlinear. For stability and controller design studies, it is important to linearize the machine equation around a steady – state operating point to obtain small signal equations. The space model powerfully evokes the similarity and equivalence between the induction machines and de machines from the modeling and control points of view.

2.3.1 Three-Phase to Two-Phase Transformation:

A dynamic model for the three-phase induction machine can be derived from the two-phase machine if the equivalence between three and two phases is established.

The equivalence is based on the equality of the mmf produced in the two-phase and three-phase windings and equal current magnitudes. Figure 2.4 shows the three-phase and two-phase windings. Assuming that each of the three-phase windings has T_1 turns per phase and equal current



2.3.1.1 Two phase and three phase stator windings

magnitudes, the two-phase windings will have $3T_1/2$ turns per phase for

mmf equality. The d and q axes mmfs are found by resolving the mmfs of the three phases along the d and q axes.

The common term, the number of turns in the winding, is canceled on either side of the equations, leaving the current equalities. The q axis is assumed to be lagging the a axis by θ_c . The relationship between i_{qdo} and abc currents is as follows:

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta_c & \cos\left(\theta_c - \frac{2\pi}{3}\right) & \cos\left(\theta_c + \frac{2\pi}{3}\right) \\ \sin \theta_c & \sin\left(\theta_c - \frac{2\pi}{3}\right) & \sin\left(\theta_c + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

The current i_o represents the imbalances in the a, b, and c phase currents and can be recognized as the zero-sequence component of the current.

$$i_{qdo} = [T_{abc}] i_{abc}$$

Where

$$i_{qdo} = [i_{qs} \quad i_{ds} \quad i_o]^t$$

$$i_{abc} = [i_{as} \quad i_{bs} \quad i_{cs}]^t$$

And the transformation from

$$[T_{abc}] = \frac{2}{3} \begin{bmatrix} \cos \theta_c & \cos\left(\theta_c - \frac{2\pi}{3}\right) & \cos\left(\theta_c + \frac{2\pi}{3}\right) \\ \sin \theta_c & \sin\left(\theta_c - \frac{2\pi}{3}\right) & \sin\left(\theta_c + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The zero – sequence current does not produce a resultant magnetic field. The transformation from two-phase currents to three-phase currents can be obtained as

$$i_{abc} = [T_{abc}]^{-1} i_{qdo}$$

Where

$$[T_{abc}]^{-1} = \begin{bmatrix} \cos \theta_c & \sin \theta_c & 1 \\ \cos(\theta_c - 2\pi/3) & \sin(\theta_c - 2\pi/3) & 1 \\ \cos(\theta_c + 2\pi/3) & \sin(\theta_c + 2\pi/3) & 1 \end{bmatrix}$$

This transformation could all be thought of as a transformation from three (abc) axes to three new (dqo) axes; for uniqueness of the transformation from one set of axes to another set of axes, including unbalances in the abc variables requires three variables such as the dqo. The reason for this is that it is easy to convert from three abc variables to two qd variables if the abc variables have an inherent relationship among themselves, such as the equal-phase displacement and magnitude. Therefore, in such a case, there are only two independent variables in abc; the third is a dependent variable, obtained as the negative sum of the other two variables. Hence a qd-to-abc transformation is unique under that circumstance. When the abc variables have no such inherent relationship, then there are three independent variables; hence, the third variable cannot be recovered from the knowledge of the other two variables only. It also means that they are not recoverable from two variables qd but require another variable, such as the zero sequence component, to recover the abc variables from the dqo variable.

2.4 Power Equivalence

The power input to the three-phase motor has to be equal to the power input to II" two-phase machine to have meaningful interpretation in the

modeling, analysis and simulation. Such an identity is derived in this section. The three-phase instantaneous power input is

$$P_i = \mathbf{v}_{abc}^t \mathbf{i}_{abc} = v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs}$$

From above equation, the abc phase currents and voltages are transformed into their equivalent qd-axes currents and voltages as

$$\begin{aligned} \mathbf{i}_{abc} &= [\mathbf{T}_{abc}]^{-1} \mathbf{i}_{qdo} \\ \mathbf{v}_{abc} &= [\mathbf{T}_{abc}]^{-1} \mathbf{v}_{qdo} \end{aligned}$$

Substituting equations and into equation gives the power input as

$$P_i = \mathbf{v}_{qdo}^t ([\mathbf{T}_{abc}]^{-1})^t [\mathbf{T}_{abc}]^{-1} \mathbf{i}_{qdo}$$

Expanding the right-hand side of the equation gives the power input in d_{qo} variables:

$$P_i = \frac{3}{2} ((v_{qs} i_{qs} + v_{ds} i_{ds}) + 2v_o i_o)$$

For a balanced three-phase machine. the zero-sequence current does not exist; hence, the power input is compactly represented by

$$P_i = \frac{3}{2} (v_{qs} i_{qs} + v_{ds} i_{ds})$$

The model development has so far kept the d and q axes stationary with respect to the stator. These axes or frames are known as reference frames. A generalized treatment where the speed of the reference frames is arbitrary is derived in the next section. The input power given by equation (5.98)

remains valid for all occasions, provided that the voltages and currents correspond to the frames under consideration.

2.5 Generalized Model in Arbitrary Reference Frames

Reference frames are very much like observer platforms. In that each of the platforms gives a unique view of the system at hand as well as a dramatic simplification of the system equations. For example, consider that, for the purposes of control, it is desirable to have the system variables as dc quantities, although the actual variables are sinusoidal. This could be accomplished by having a reference frame revolving at the same angular speed as that of the sinusoidal variable. As the reference frames are moving at an angular speed equal to the angular frequency of the sinusoidal supply, say, then the differential speed between them is reduced to zero, resulting in the sinusoid being perceived as a dc signal from the reference frames. Then, by moving to that plane, it becomes easier to develop a small-signal equation out of a nonlinear equation, as the quiescent or operating point is described only by dc values; this then leads to the linearized system around an operating point. Now, it is easier to synthesize a compensator for the system by using standard linear control-system techniques. Likewise, the independent rotor field position determines the induced emf and affects the dynamic system equations of both the wound-rotor and permanent-magnet synchronous machines. Therefore, looking at the entire system from the rotor, i.e., rotating reference frames, the system inductance matrix becomes independent of rotor position, thus leading to the simplification and compactness of the system equations. Such advantages are many from using reference frames. Instead of deriving the transformations for each and every

particular reference frame, it is advantageous to derive the general transformation for an arbitrary rotating reference frame. Then any particular reference frame model can be derived by substituting the appropriate frame speed and position in the generalized reference model.

Reference frames rotating at an arbitrary speed are hereafter called arbitrary reference frames. Other reference frames are particular cases of these arbitrary reference frames. The relationship between the stationary reference frames denoted by d and q axes and the arbitrary reference frames denoted by d^c and q^c axes are shown in Figure 2.5.1. From now on, the three-phase machine is assumed to have balanced windings and balanced inputs, thus making the zero-sequence components be zero and eliminating the zero-sequence equations from further consideration. Note that the zero-sequence equations have to be included only for unbalanced operation of the motor, a situation common with a fault in the machine or converter.

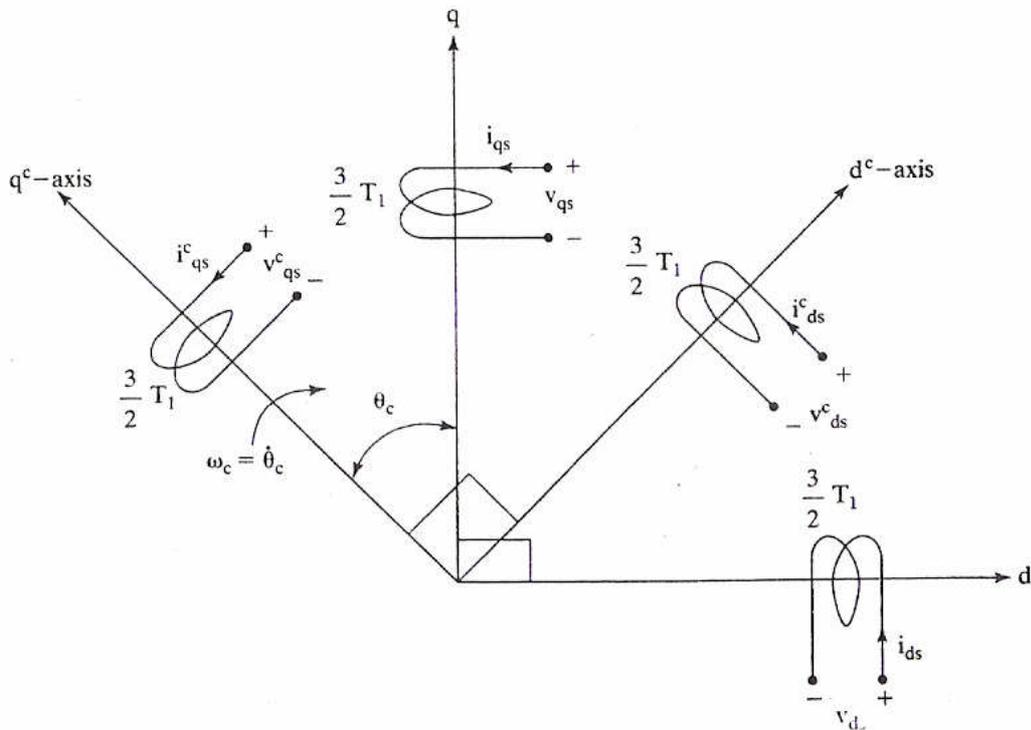


Fig 2.5.1 stationary and arbitrary reference frame

Assuming that the windings have equal number of turns on both of the reference frames, the arbitrary reference frame currents are resolved on the d and q axes to find the currents in the stationary reference frames. The relationships between the currents are written as

$$\mathbf{i}_{qds} = [\mathbf{T}^c] \mathbf{i}_{qds}^c$$

Where

$$\mathbf{i}_{qds} = [i_{qs} \quad i_{ds}]^t$$

$$\mathbf{i}_{qds}^c = [i_{qs}^c \quad i_{ds}^c]^t$$

$$\mathbf{T}^c = \begin{bmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{bmatrix}$$

and

The speed of the arbitrary reference frames is

$$\theta_c = \omega_c$$

Similarly, the fictitious rotor currents are transformed into arbitrary frames by using t^c , and they are written as

$$\mathbf{i}_{qdr} = [\mathbf{T}^c] \mathbf{i}_{qdr}^c$$

where

$$\mathbf{i}_{qdr} = [i_{qr} \quad i_{dr}]^t$$

$$\mathbf{i}_{qdr}^c = [i_{qr}^c \quad i_{dr}^c]^t$$

Likewise, the voltage relationship is

$$\mathbf{v}_{qds} = [\mathbf{T}^c] \mathbf{v}_{qds}^c$$

$$\mathbf{v}_{qdr} = [\mathbf{T}^c] \mathbf{v}_{qdr}^c$$

Where

$$\begin{aligned} \mathbf{v}_{qds} &= [\mathbf{v}_{qs} \quad \mathbf{v}_{ds}]^t \\ \mathbf{v}_{qds}^c &= [\mathbf{v}_{qs}^c \quad \mathbf{v}_{ds}^c]^t \\ \mathbf{v}_{qdr} &= [\mathbf{v}_{qr} \quad \mathbf{v}_{dr}]^t \\ \mathbf{v}_{qdr}^c &= [\mathbf{v}_{qr}^c \quad \mathbf{v}_{dr}^c]^t \end{aligned}$$

By substituting equations given above, the induction-motor model in arbitrary reference frames is obtained. It is given below for use in subsequent sections.

$$\begin{bmatrix} \mathbf{v}_{qs}^c \\ \mathbf{v}_{ds}^c \\ \mathbf{v}_{qr}^c \\ \mathbf{v}_{dr}^c \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_c - \omega_r) L_m & R_r + L_r p & (\omega_c - \omega_r) L_r \\ -(\omega_c - \omega_r) L_m & L_m p & -(\omega_c - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^c \\ i_{ds}^c \\ i_{qr}^c \\ i_{dr}^c \end{bmatrix}$$

Where

$$\omega_r = \theta_r$$

ω_r is the rotor speed in electrical radians/sec. The relationship between the arbitrary reference frame variables and the a, b, and c variables is derived by using

$$i_{qds}^c = [T^c]^{-1} i_{qds}$$

By substituting from equation for i_{qds} in terms of a, b, and c phase currents in the stator reference frames, the qdo currents in the arbitrary reference frames are obtained as

$$i_{qdo}^c = \begin{bmatrix} [T^c]^{-1} & 0 \\ 0 & 1 \end{bmatrix} [T_{abc}^s] i_{abc} = [T_{abc}] i_{abc}$$

where $[0]$ is a 1×2 null vector. Note that the zero-sequence currents remain unchanged in the arbitrary reference frames. This transformation is valid for currents, voltages, and flux-linkages for both the stator and the rotor. Particular cases of the reference frames are derived in a later section. The next section contains the derivation of the electromagnetic torque in terms of the current variables in the arbitrary reference frames.

2.6 Electromagnetic Torque

The electromagnetic torque is an important output variable that determines such mechanical dynamics of the machine as the rotor position and speed. Therefore, its importance cannot be overstated in any of the simulation studies. It is derived from the machine matrix equation by looking at the input power and its various components, such as 'resistive losses, mechanical power, rate of change of stored magnetic energy, and reference-frame power. Elementary reasoning leads to the fact that there cannot be a power component due to the introduction of reference frames. Similarly, the rate of change of stored magnetic energy must be zero in steady state. Hence, the output power is the difference between the input power and the resistive losses in steady state. Dynamically, the rate of change of stored magnetic energy need not be zero. Based on these observations, the derivation of the electromagnetic torque is made as follows.

$$\mathbf{V} = [\mathbf{R}]\mathbf{i} + [\mathbf{L}]\dot{\mathbf{p}}\mathbf{i} + [\mathbf{G}]\omega_r\mathbf{i} + [\mathbf{F}]\omega_c\mathbf{i}$$

where the vectors and matrices are identified by observation. Premultiplying the equation by the transpose of the current vector gives the instantaneous input power as,

$$p_i = i^t V = i^t [R] i + i^t [L] \dot{p} i + i^t [G] \omega_r i + i^t [F] \omega_c i$$

where the [R] matrix consists of resistive elements, the [L] matrix consists of the coefficients of the derivative operator \dot{p} , the [G] matrix has elements that are the coefficients of the electrical rotor speed ω_r and [F] is the frame matrix in terms of the coefficients of the reference frame speed, ω_c . The term $i^t [R] i$ gives stator and rotor resistive losses. The term $i^t [F] \omega_c i$ is the reference frame power, and upon expansion is found to be identically equal to zero, as it should be, because there cannot be a power associated with a fictitious element introduced for the sake of simplifying the model and analysis. The term $i^t [L] \dot{p} i$ denotes the rate of change of stored magnetic energy. Therefore, what is left of the power component must be equal to the air gap power, given by the term $i^t [G] \omega_r i$. From the fundamentals, it is known that the air gap power has to be associated with the rotor speed. The air gap power is the product of the mechanical rotor speed and air gap or electromagnetic torque. Hence air gap torque, T_e , is derived from the terms involving the rotor speed, ω_m in mechanical rad/sec, as

$$\omega_m T_e = P_a = i^t [G] i \times \omega_r$$

Substituting for ω_r in terms of ω_m leads to electromagnetic torque as

$$T_e = \frac{P_a}{\omega_m} = \frac{P_a}{\omega_m} i^t [G] i$$

By substituting for [G] in equation above by observation from the equation, the electromagnetic torque is obtained as

$$T_e = \frac{3}{2} \frac{P}{2} L_m (i_{qs}^c i_{dr}^c - i_{ds}^c i_{qr}^c)$$

The factor 3/2 is introduced into the right-hand side of equation from the power-equivalence condition between the three-phase and two-phase induction motors. The next section considers the frequently used models in various reference frames and their derivation from the generalized induction-motor model in arbitrary reference frames.

2.7 Equations in Flux Linkages

The dynamic equations of the induction motor in arbitrary reference frames can be represented by using flux linkages as variables. This involves the reduction of a number of variables in the dynamic equations, which greatly facilitates their solution by using analog and hybrid computers. Even when the voltages and currents are discontinuous, the flux linkages are continuous. This gives the advantage of differentiating these variables with numerical stability. In addition, the flux-linkages representation is used in motor drives to highlight the process of the decoupling the flux and torque channels in the induction and synchronous machines.

The stator and rotor flux linkages in the arbitrary reference frames are defined as

$$\left. \begin{aligned} \lambda_{qs}^c &= L_s i_{qs}^c + L_m i_{qr}^c \\ \lambda_{ds}^c &= L_s i_{ds}^c + L_m i_{dr}^c \\ \lambda_{qr}^c &= L_r i_{qr}^c + L_m i_{qs}^c \\ \lambda_{dr}^c &= L_r i_{dr}^c + L_m i_{ds}^c \end{aligned} \right\}$$

The zero-sequence flux linkages are

$$\left. \begin{aligned} \lambda_{os} &= L_{ls} i_{os} \\ \lambda_{or} &= L_{lr} i_{or} \end{aligned} \right\}$$

The q axis stator voltage in the arbitrary reference frame is

$$v_{qs}^c = R_s i_{qs}^c + \omega_c (L_s i_{ds}^c + L_m i_{dr}^c) + L_m p i_{qr}^c + L_s p i_{qs}^c$$

Substituting from the defined flux-linkages into the voltage equation yields

$$v_{qs}^c = R_s i_{qs}^c + \omega_c \lambda_{ds}^c + p \lambda_{qs}^c$$

Similarly, the stator d axis voltage, the d and q axes rotor voltages, and the zero-sequence voltage equations are derived as

$$\begin{aligned} v_{ds}^c &= R_s i_{ds}^c - \omega_c \lambda_{qs}^c + p \lambda_{ds}^c \\ v_{os} &= R_s i_{os} + p \lambda_{os} \\ v_{qr}^c &= R_r i_{qr}^c + (\omega_c - \omega_r) \lambda_{dr}^c + p \lambda_{qr}^c \\ v_{dr}^c &= R_r i_{dr}^c - (\omega_c - \omega_r) \lambda_{qr}^c + p \lambda_{dr}^c \\ v_{or} &= R_r i_{or} + p \lambda_{or} \end{aligned}$$

These equations can be represented in equivalent circuits.

Power-system engineers and design engineers use normalized or per unit (p.u) values for the variables. The normalization of the variables is made via reactances rather than inductances. To facilitate such a step, a modified flux linkage is defined whose unit in volts is

$$\psi_{qs}^c = \omega_b \lambda_{qs}^c = \omega_b (L_s i_{qs}^c + L_m i_{qr}^c) = X_s i_{qs}^c + X_m i_{qr}^c$$

Where ω_b is the base frequency in rad/sec. Similarly the other modified flux linkages are written as

$$\begin{aligned}\psi_{ds}^c &= X_s i_{ds}^c + X_m i_{dr}^c \\ \psi_{qr}^c &= X_r i_{qr}^c + X_m i_{qs}^c \\ \psi_{dr}^c &= X_r i_{dr}^c + X_m i_{ds}^c \\ \psi_{os} &= X_s i_{os} \\ \psi_{or} &= X_r i_{or}\end{aligned}$$

Substituting the flux linkages in terms of the modified flux linkages yields

$$\lambda_{qs}^c = \frac{\psi_{qs}^c}{\omega_b}, \lambda_{ds}^c = \frac{\psi_{ds}^c}{\omega_b}, \lambda_{os}^c = \frac{\psi_{os}^c}{\omega_b}, \lambda_{qr}^c = \frac{\psi_{qr}^c}{\omega_b}, \lambda_{dr}^c = \frac{\psi_{dr}^c}{\omega_b}, \lambda_{or}^c = \frac{\psi_{or}^c}{\omega_b}$$

and, by substituting, the resulting equations in modified flux linkages are

$$\begin{aligned}v_{qs}^c &= R_s i_{qs}^c + \frac{\omega_c}{\omega_b} \psi_{ds}^c + \frac{P}{\omega_b} \psi_{qs}^c, v_{ds}^c = R_s i_{ds}^c - \frac{\omega_c}{\omega_b} \psi_{qs}^c + \frac{P}{\omega_b} \psi_{ds}^c, v_{os} = R_s i_{os} + \frac{P}{\omega_b} \psi_{os}, \\ v_{qr}^c &= R_r i_{qr}^c + \frac{(\omega_c - \omega_r)}{\omega_b} \psi_{dr}^c + \frac{P}{\omega_b} \psi_{qr}^c, v_{dr}^c = R_r i_{dr}^c - \frac{(\omega_c - \omega_r)}{\omega_b} \psi_{qr}^c + \frac{P}{\omega_b} \psi_{dr}^c, \\ v_{or} &= R_r i_{or} + \frac{P}{\omega_b} \psi_{or}\end{aligned}$$

The electromagnetic torque in flu linkages and currents is derived as

$$T_e = \frac{3}{2} \frac{P}{2} L_m (i_{qs}^c i_{dr}^c - i_{ds}^c i_{qr}^c) = \frac{3}{2} \frac{P}{2} (i_{qs}^c (L_m i_{dr}^c) - i_{ds}^c (L_m i_{qr}^c))$$

The rotor currents can be substituted in terms of the stator currents and stator flux linkages from the basic definitions of the flux linkages. From equation ,

$$L_m i_{dr}^c + L_s i_{ds}^c = \lambda_{ds}^c$$

Hence,

$$L_m i_{dr}^c = (\lambda_{ds}^c - L_s i_{ds}^c)$$

Similarly,

$$L_m i_{qr}^c = (\lambda_{qs}^c - L_s i_{qs}^c)$$

the electromagnetic torque in stator flux linkages and stator currents is

$$T_e = \frac{3}{2} \frac{P}{2} (i_{qs}^c (\lambda_{ds}^c - L_s i_{ds}^c) - i_{ds}^c (\lambda_{qs}^c - L_s i_{qs}^c)) = \frac{3}{2} \frac{P}{2} (i_{qs}^c \lambda_{ds}^c - i_{ds}^c \lambda_{qs}^c)$$

Alternatively, the electromagnetic torque in terms of modified flux linkages and currents is

$$T_e = \frac{3}{2} \frac{P}{2} \frac{1}{\omega_b} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c)$$

The electromagnetic torque can be expressed in only rotor variables and in many other forms.

2.8 Per-Unit Model

The normalized model of the induction motor is derived by defining the base variables both in the abc and the dqp variables. In the abc frames, let the rms values of the rated phase voltage and current form the base quantities, given as

$$\text{Base Power} = P_b = 3V_{b3}I_{b3}$$

Where V_{b3} and I_{b3} are three-phase base voltage and current respectively. Selecting the base quantities in dq frames denoted by V_b and I_b to be equal to the peak value of the phase voltage and current in abc frames, we get

$$V_b = \sqrt{2}V_{b3}$$

$$I_b = \sqrt{2}I_{b3}$$

Hence, the base power is defined as

$$P_b = 3V_{b3}I_{b3} = 3 \frac{V_b}{\sqrt{2}} \frac{I_b}{\sqrt{2}} = \frac{3}{2} V_b I_b$$

The arbitrary reference frame model is chosen to illustrate the normalization process. Consider the q axis stator voltage to begin with, which is given in the following:

$$v_{qs}^c = R_s i_{qs}^c + \omega_c (L_s i_{ds}^c + L_m i_{dr}^c) + L_m p i_{qr}^c + L_s p i_{qs}^c$$

Writing the inductances in terms of reactances at base frequency of ω_b gives

$$v_{qs}^c = R_s i_{qs}^c + \frac{X_s}{\omega_b} p i_{qs}^c + \frac{\omega_c X_s}{\omega_b} i_{ds}^c + \frac{X_m}{\omega_b} p i_{qr}^c + \frac{\omega_c X_m}{\omega_b} i_{dr}^c$$

It is normalized by dividing by the base voltage, V_b :

$$v_{qsn}^c = \frac{v_{qs}^c}{V_b} = \frac{R_s}{V_b} i_{qs}^c + \frac{X_s}{\omega_b V_b} p i_{qs}^c + \frac{\omega_c X_s}{\omega_b V_b} i_{ds}^c + \frac{X_m}{\omega_b V_b} i_{qr}^c + \frac{\omega_c X_m}{\omega_b V_b} i_{dr}^c$$

Substituting for base voltage in terms of base current, I_b , and base impedance, Z_b , as follows,

$$V_b = I_b Z_b$$

$$v_{qsn}^c = \left(\frac{R_s}{Z_b} \right) \left(\frac{i_{qs}^c}{I_b} \right) + \frac{1}{\omega_b} \left(\frac{X_s}{Z_b} \right) p \left(\frac{i_{qs}^c}{I_b} \right) + \left(\frac{\omega_c}{\omega_b} \right) \left(\frac{X_s}{Z_b} \right) \left(\frac{i_{ds}^c}{I_b} \right) + \frac{1}{\omega_b} \left(\frac{X_m}{Z_b} \right) p \left(\frac{i_{qr}^c}{I_b} \right) + \left(\frac{\omega_c}{\omega_b} \right) \left(\frac{X_m}{Z_b} \right) \left(\frac{i_{dr}^c}{I_b} \right)$$

By defining the normalized parameters and variables in the following manner,

$$\begin{aligned} R_{sn} &= \frac{R_s}{Z_b} \text{ p.u.}, X_{sn} = \frac{X_s}{Z_b} \text{ p.u.}, \omega_{mn} = \frac{\omega_r}{\omega_b} \text{ p.u.}, i_{dsn}^c = \frac{i_{ds}^c}{I_b} \text{ p.u.}, v_{dsn}^c = \frac{v_{ds}^c}{V_b} \text{ p.u.} \\ R_{rn} &= \frac{R_r}{Z_b} \text{ p.u.}, X_{rn} = \frac{X_r}{Z_b} \text{ p.u.}, i_{qsn}^c = \frac{i_{qs}^c}{I_b} \text{ p.u.}, i_{drn}^c = \frac{i_{dr}^c}{I_b} \text{ p.u.}, v_{qrn}^c = \frac{v_{qr}^c}{V_b} \text{ p.u.} \\ X_{mn} &= \frac{X_m}{Z_b} \text{ p.u.}, \omega_{cn} = \frac{\omega_c}{\omega_b} \text{ p.u.}, i_{qrn}^c = \frac{i_{qr}^c}{I_b} \text{ p.u.}, v_{qsn}^c = \frac{v_{qs}^c}{V_b} \text{ p.u.}, v_{drn}^c = \frac{v_{dr}^c}{V_b} \text{ p.u.} \end{aligned}$$

and by substituting equations into appropriate places, V_{qsn}^c is obtained as,

$$v_{qsn}^c = \left(R_{sn} + \frac{X_{sn}}{\omega_b} p \right) i_{qsn}^c + \omega_{cn} X_{sn} i_{dsn}^c + \frac{X_{mn}}{\omega_b} p i_{qrn}^c + \omega_{cn} X_{mn} i_{drn}^c$$

Similarly, the other equations are normalized and given below:

$$\begin{bmatrix} v_{qsn}^c \\ v_{dsn}^c \\ v_{qrn}^c \\ v_{drn}^c \end{bmatrix} = \begin{bmatrix} R_{sn} + \frac{X_{sn}}{\omega_b} p & \omega_{cn} X_{sn} & \frac{X_{mn}}{\omega_b} p & \omega_{cn} X_{mn} \\ -\omega_{cn} X_{sn} & R_{sn} + \frac{X_{sn}}{\omega_b} p & -\omega_{cn} X_{mn} & \frac{X_{mn}}{\omega_b} p \\ \frac{X_{mn}}{\omega_b} p & (\omega_{cn} - \omega_{rn}) X_{mn} & R_{rn} + \frac{X_{rn}}{\omega_b} p & (\omega_{cn} - \omega_{rn}) X_{mn} \\ -(\omega_{cn} - \omega_{rn}) X_{mn} & \frac{X_{mn}}{\omega_b} p & -(\omega_{cn} - \omega_{rn}) X_{rn} & R_{rn} + \frac{X_{rn}}{\omega_b} p \end{bmatrix} \begin{bmatrix} i_{qsn}^c \\ i_{dsn}^c \\ i_{qrn}^c \\ i_{drn}^c \end{bmatrix}$$

The electromagnetic torque is,

$$T_e = \frac{3}{2} \frac{P}{2} \frac{1}{\omega_b} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c)$$

but the base torque is,

$$T_b = \frac{P_b}{\omega_b} = \frac{P}{2} \frac{P_b}{\omega_b}$$

from which the normalized electromagnetic torque is obtained as

$$T_{en} = \frac{T_e}{T_b} = \frac{\frac{3}{2} \frac{P}{2} \frac{1}{\omega_b}}{\frac{P}{2} \frac{P_b}{2 \omega_b}} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c) = \frac{3/2}{P_b} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c) = \frac{3/2}{(3/2) V_b I_b} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c)$$

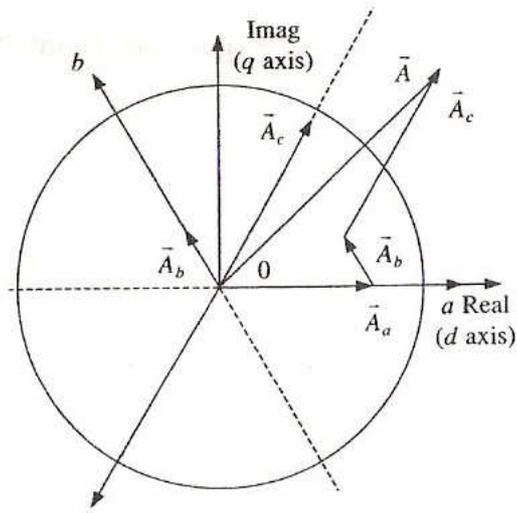
$$= \left(\frac{i_{qs}^c}{I_b} \right) \left(\frac{\psi_{ds}^c}{V_b} \right) - \left(\frac{i_{ds}^c}{I_b} \right) \left(\frac{\psi_{qs}^c}{V_b} \right) = (i_{qsn}^c \psi_{dsn}^c - i_{dsn}^c \psi_{qsn}^c) \text{ p.u.}$$

where

$$\left. \begin{aligned} \psi_{dsn}^c &= \frac{\psi_{ds}^c}{V_b} \text{ p.u.} \\ \psi_{qsn}^c &= \frac{\psi_{qs}^c}{V_b} \text{ p.u.} \end{aligned} \right\}$$

2.9 The space vector in electrical system:

The basic principle is to transform the scalar electromagnetic quantities describing the system (current, voltages, and magnetic fluxes) into two dimensional vectors named space vectors. One space vector replaces a set of three scalar quantities of the same type, thereby generating a more compact notation for the mathematical equations. Therefore, space vectors are largely used to analyze the operation of three phase electrical machines.



2.4.1 The relationship between phase quantities and the corresponding space vectors

If 'A' is the electromagnetic quantity then A_a , A_b , A_c are the three values corresponding to the three system phases. They are initially associated with two-dimensional vectors situated on the three directions 120° apart in a plane: A_a , A_b , A_c are illustrated in the fig 2.4.1 Adding the three vectors together a single two-dimensional vector is obtained according

to the equation 2.26. \vec{A} is a space vector associated with scalar quantities A_a , A_b , and A_c . The vector components along the real axis (axis 'd') and on the imaginary axis (axis 'q') are given in equation (2.27)

$$\vec{A} = \vec{A}_a + \vec{A}_b + \vec{A}_c \dots\dots\dots (2.26)$$

$$\begin{cases} A_d = A_a \cdot \cos(0) + A_b \cdot \cos\left(\frac{2\pi}{3}\right) + A_c \cdot \cos\left(\frac{4\pi}{3}\right) = A_a - \frac{1}{2} A_b - \frac{1}{2} A_c \\ A_q = A_a \cdot \sin(0) + A_b \cdot \sin\left(\frac{2\pi}{3}\right) + A_c \cdot \sin\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2} A_b - \frac{\sqrt{3}}{2} A_c \end{cases} \dots\dots\dots (2.27)$$

In practical calculations, the space vectors are represented either by 2*1 matrices or by complex quantities. Using matrix notation equation 2.27 becomes equation 2.28 while equation 2.29 represents the complex approach to space vector model.

$$\begin{pmatrix} A_d \\ A_q \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \cdot \begin{pmatrix} A_a \\ A_b \\ A_c \end{pmatrix} \dots\dots\dots (2.28)$$

$$\begin{cases} \underline{A} = A_a + \varepsilon \cdot A_b + \varepsilon^2 \cdot A_c \\ \varepsilon = \cos\left(\frac{2\pi}{3}\right) + j \cdot \sin\left(\frac{2\pi}{3}\right) \end{cases} \dots\dots\dots (2.29)$$

The transformation of the set of the three scalar variables into a space vector is equivalent to a transformation from a three phase system to a two phase system. The inverse transformation can be calculated based on the property that the algebraic sum of the three scalar values is always null. This property is shared by all the electromagnetic quantities related to individual phases (current, voltages, and magnetic fluxes) if the power supply generates symmetric voltages and the load is symmetric and Y- connected.

$$A_a + A_b + A_c = 0 \quad \dots (2.30)$$

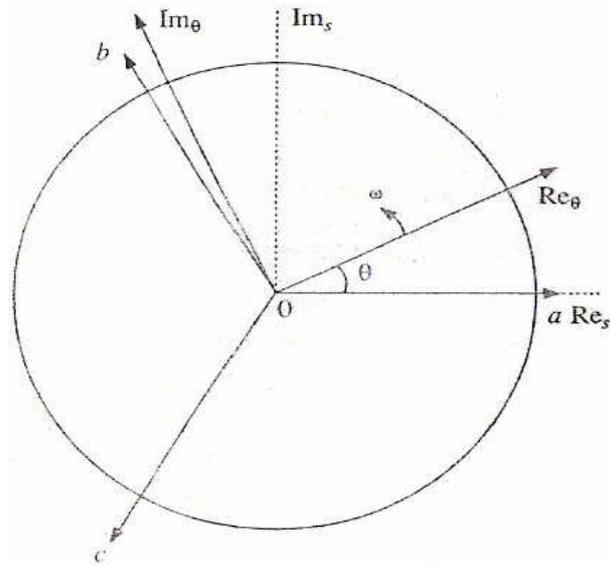
Combining 2.30 with 2.27, the system 2.31 is generated from which 2.32 is derived. The system 2.32 describes the inverse transformation of a space vector into the corresponding set of three scalar phase quantities.

$$\begin{cases} A_d = A_a - \frac{1}{2} A_b - \frac{1}{2} A_c \\ A_q = \frac{\sqrt{3}}{2} A_b - \frac{\sqrt{3}}{2} A_c \\ A_a + A_b + A_c = 0 \end{cases} \quad \dots (2.31)$$

$$\begin{cases} A_a = \frac{2}{3} \cdot A_d \\ A_b = -\frac{1}{3} \cdot A_d + \frac{1}{\sqrt{3}} \cdot A_q \\ A_c = -\frac{1}{3} \cdot A_d - \frac{1}{\sqrt{3}} \cdot A_q \end{cases} \quad \dots (2.32)$$

2.9.1 Space vector model of a three phase induction motor:

The space vector model of the induction motor is the lumped-parameter model with the largest use in the study and design of electrical drive applications. The induction motor space vector model is derived from the basic electrical equations describing each of the stator winding and rotor winding. The stator winding equations are given in



2.9.1.1 the fixed stator reference frame and the general mobile θ reference frame

2.33 while where u_{as} , u_{bs} , and u_{cs} are the phase voltages, i_{as} , i_{bs} , and i_{cs} are the phase currents, while Ψ_{as} , Ψ_{bs} , and Ψ_{cs} are the phase magnetic fluxes.

$$\begin{cases} u_{as} = R_s i_{as} + \frac{d\Psi_{as}}{dt} \\ u_{bs} = R_s i_{bs} + \frac{d\Psi_{bs}}{dt} \\ u_{cs} = R_s i_{cs} + \frac{d\Psi_{cs}}{dt} \end{cases} \dots (2.33)$$

The associated space vectors (expressed as complex numbers) are obtained by multiplying the second equation in 2.33 with e and the third equation by e^2 , after which all the three equations are added together. The conversion of the three scalar quantities into one space equation is given in the equation 2.34 and 2.35.

$$\begin{cases} u_{as} = R_s i_{as} + \frac{d\Psi_{as}}{dt} \\ \varepsilon u_{bs} = R_s \cdot \varepsilon i_{bs} + \varepsilon \frac{d\Psi_{bs}}{dt} \\ \varepsilon^2 u_{cs} = R_s \cdot \varepsilon^2 i_{cs} + \varepsilon^2 \frac{d\Psi_{cs}}{dt} \end{cases} \dots(2.34)$$

$$\begin{cases} \underline{u}_s^s = u_{as} + \varepsilon \cdot u_{bs} + \varepsilon^2 \cdot u_{cs} \\ \underline{i}_s^s = i_{as} + \varepsilon \cdot i_{bs} + \varepsilon^2 \cdot i_{cs} \\ \underline{\Psi}_s^s = \Psi_{as} + \varepsilon \cdot \Psi_{bs} + \varepsilon^2 \cdot \Psi_{cs} \end{cases} \Rightarrow \underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\Psi_s^s}{dt} \dots(2.35)$$

Different reference frames (still or rotating) can be used to calculate the coordinates of the electromagnetic space vector.

Any rotating reference frame is defined by the electrical angle function $\theta(t)$ that indicates the relative position to the still reference frame. Alternatively, it can be defined by the electrical rotation speed $\omega_e(t)$ and the initial electrical angle. For a general rotating frame the equations (2.35) are transformed into (2.36). The fourth equation in 2.35 can be rewritten as 2.37. Equation 2.38 is eventually obtained by dividing 2.36 with $e^{j\theta}$.

$$\begin{cases} \underline{u}_s^\theta = \underline{u}_s \cdot e^{-j\theta} \\ \underline{i}_s^\theta = \underline{i}_s \cdot e^{-j\theta} \\ \underline{\Psi}_s^\theta = \underline{\Psi}_s \cdot e^{-j\theta} \\ \underline{u}_s^\theta \cdot e^{j\theta} = R_s \underline{i}_s^\theta \cdot e^{j\theta} + \frac{d}{dt} (\underline{\Psi}_s^\theta \cdot e^{j\theta}) \end{cases} \dots (2.36)$$

$$\underline{u}_s^\theta \cdot e^{j\theta} = R_s \underline{i}_s^\theta \cdot e^{j\theta} + \frac{d\Psi_s^\theta}{dt} \cdot e^{j\theta} + \underline{\Psi}_s^\theta \cdot e^{j\theta} \cdot j \frac{d\theta}{dt}$$

$$\underline{u}_s^\theta = R_s \underline{i}_s^\theta + \frac{d\Psi_s^\theta}{dt} + j\omega_e \cdot \underline{\Psi}_s^\theta \dots(2.37)$$

A similar complex equation describes the rotor circuit with the difference that the reference frame rotation speed relative to the rotor $\omega_e - \omega_{er}$ instead of ω_e (ω_{er} is the electrical rotor angular speed). Moreover, the rotor voltage is always zero for the squirrel cage induction motor.

$$\underline{u}_r^\theta = R_s \underline{i}_r^\theta + \frac{d\underline{\Psi}_r^\theta}{dt} + j(\omega_e - \omega_{er}) \cdot \underline{\Psi}_r^\theta = 0 \quad \dots(2.38)$$

The individual phase fluxes that are used to calculate the magnetic flux vectors are each composed of six components. The flux components are generated by the electromagnetic interaction between the three rotor windings and the three stator windings.

$$\left\{ \begin{array}{l} \Psi_{sa} = \Psi_{sasa} + \Psi_{sasb} + \Psi_{sasc} + \Psi_{sara} + \Psi_{sarb} + \Psi_{sarc} \\ \Psi_{sb} = \Psi_{sbsa} + \Psi_{sbsb} + \Psi_{sbsc} + \Psi_{sbra} + \Psi_{sbrb} + \Psi_{sbrc} \\ \Psi_{sc} = \Psi_{scsa} + \Psi_{scsb} + \Psi_{scsc} + \Psi_{scra} + \Psi_{scrib} + \Psi_{scrc} \\ \Psi_{ra} = \Psi_{rasa} + \Psi_{rasb} + \Psi_{rase} + \Psi_{rara} + \Psi_{rarb} + \Psi_{rare} \\ \Psi_{rb} = \Psi_{rbsa} + \Psi_{rbsb} + \Psi_{rbsc} + \Psi_{rbra} + \Psi_{rbrb} + \Psi_{rbrc} \\ \Psi_{rc} = \Psi_{rcsa} + \Psi_{rcsb} + \Psi_{rcsc} + \Psi_{rcra} + \Psi_{rcrb} + \Psi_{rcrc} \end{array} \right. \dots(2.39)$$

In the equation above each flux component is identified by four indices: the first two indicates the winding where the magnetic flux is measured while the last two indicates the winding that generates it, for instances Ψ_{sarb} is the flux generated into stator winding 'a' by the rotor winding 'b'.

The induction motor equations formulated for a reference frame defined by the angle $\theta(t)$ and the rotation speed $\omega(t)$, are:

$$\left\{ \begin{array}{l} \underline{u}_s^\theta = R_s \underline{i}_s^\theta + \frac{d\Psi_s^\theta}{dt} + j\omega_e \Psi_s^\theta \\ \underline{u}_r^\theta = R_r \underline{i}_r^\theta + \frac{d\Psi_r^\theta}{dt} + j(\omega_e - \omega_{er}) \Psi_r^\theta \\ \Psi_s^\theta = \left(l_{\sigma s} - m_{\sigma s} + \frac{3}{2} \cdot l_{ms} \right) \cdot \underline{i}_s^\theta + \frac{3}{2} \cdot l_{msr} \cdot \underline{i}_r^\theta \\ \Psi_r^\theta = \left(l_{\sigma r} - m_{\sigma r} + \frac{3}{2} \cdot l_{mr} \right) \cdot \underline{i}_r^\theta + \frac{3}{2} \cdot l_{msr} \cdot \underline{i}_s^\theta \end{array} \right. \dots(2.40)$$

The magnetic flux expressions in the 2.40 are complicated as seven inductances are involved. The mathematical technique of referring the rotor quantities with respect to the stator is usually applied to the equation in order to simplify the flux equations. The basic principle of referring the rotor quantity to the stator is to multiply rotor quantity with constant values in such a manner that the power transfer between stator and rotor is not altered. Thus, if the rotor current is multiplied by constant k then the rotor voltage and rotor flux are multiplied by 1/k. On the other hand, the rotor resistance and the rotor inductance are multiplied by 1/k². the constant k that generates the simplest transformation of system 2.40 is given by 2.41 while the corresponding referred rotor quantities are 2.42. the equation linking all the referred quantities is 2.43.

$$k = \frac{l_{msr}}{l_{ms}} \dots\dots\dots (2.41)$$

$$\left\{ \begin{array}{l} \underline{i}'_r = k \cdot \underline{i}_r = \frac{l_{msr}}{l_{ms}} \cdot \underline{i}_r \\ \underline{u}'_r = \frac{1}{k} \cdot \underline{u}_r = \frac{l_{ms}}{l_{msr}} \cdot \underline{u}_r \\ \underline{\Psi}'_r = \frac{1}{k} \cdot \underline{\Psi}_r = \frac{l_{ms}}{l_{msr}} \cdot \underline{\Psi}_r \\ R'_r = \frac{1}{k^2} \cdot R_r = \frac{l_{ms}^2}{l_{msr}^2} \cdot R_r \end{array} \right. \dots (2.42)$$

$$\underline{u}'_r = R'_r \underline{i}'_r + \frac{d\underline{\Psi}'_r}{dt} + j(\omega_e - \omega_{er}) \underline{\Psi}'_r \dots(2.43)$$

The referred rotor flux can be expressed as a function of the stator current and the referred rotor current vector 2.44

$$\underline{\Psi}'_r = \frac{l_{ms}}{l_{msr}} \underline{\Psi}_r = \frac{l_{ms}^2}{l_{msr}^2} \left(l_{\sigma r} - m_{\sigma r} + \frac{3}{2} l_{mr} \right) \cdot \underline{i}'_r + \frac{3}{2} l_{ms} \cdot \underline{i}_s \dots (2.44)$$

The inductance l_{ms} , l_{mr} , and l_{msr} are always related by equation 2.45. The relationship allows the rewriting the equation 2.44 as 2.46 and 2.47.

$$l_{ms} \cdot l_{mr} = l_{msr}^2 \dots (2.45)$$

$$\underline{\Psi}'_r = \frac{l_{ms}^2}{l_{msr}^2} (l_{\sigma r} - m_{\sigma r}) \cdot \underline{i}'_r + \frac{3}{2} l_{ms} \cdot \underline{i}'_r + \frac{3}{2} l_{ms} \cdot \underline{i}_s \dots(2.46)$$

$$\underline{\Psi}'_r = (L'_{\sigma r} + L_m) \cdot \underline{i}'_r + L_m \cdot \underline{i}_s = L'_r \cdot \underline{i}'_r + L_m \cdot \underline{i}_s \dots(2.47)$$

The significance of the symbols in the previous equations is:

- $L'_{\sigma r} = \frac{l_{ms}^2}{l_{msr}^2} (l_{\sigma r} - m_{\sigma r})$ – the total referred rotor leakage inductance.
- $L_m = \frac{3}{2} l_{ms}$ – the resulting stator–rotor mutual inductance.
- $L'_r = L'_{\sigma r} + L_m$ – the total referred rotor inductance.

The simplified induction motor space vector equations can be written as:

$$\left\{ \begin{array}{l} \underline{u}_s^\theta = R_s \underline{i}_r^\theta + \frac{d\Psi_s^\theta}{dt} + j\omega \Psi_s^\theta \\ \underline{u}_r'^\theta = R_r' \underline{i}_r'^\theta + \frac{d\Psi_r'^\theta}{dt} + j(\omega_e - \omega_{er}) \Psi_r'^\theta = 0 \\ \Psi_s^\theta = L_s \underline{i}_s^\theta + L_m \underline{i}_r'^\theta \\ \Psi_r'^\theta = L_r' \underline{i}_r'^\theta + L_m \underline{i}_s^\theta \end{array} \right. \dots (2.48)$$

This model will be used in the neural induction motor control strategy developed in this project.

2.10 Induction motor control strategies:

In the earlier days the induction motor was known as a constant speed machine, the advent of the power converters made possible the use of the induction motor as a variable speed machine.

Correct control over the motor torque is a prerequisite of all the speed control strategy. The torque equation can be derived from the power-based consideration and can be expressed as a function of the current and the voltage space vector. The total power consumed by the motor has three components:

- The power dissipated by the winding resistance P_R ,
- The power stored in the internal magnetic field P_μ ,
- The mechanical power P_M

The motor torque is proportional to the mechanical power and inversely proportional to the rotor speed 2.48. The total motor power is the power consumed by all six stator and rotor windings. So it can be calculated as in eq 2.49. Elementary algebraic calculations show that the rotor power and the

stator power can be calculated as indicated by 2.50. The calculation can be performed in any reference frame defined by the time function $\theta(t)$.

$$\begin{cases} P = P_R + P_\mu + P_M \\ T = \frac{P_M}{\omega_r} = p \cdot \frac{P_M}{\omega_{er}} \end{cases} \dots (2.49)$$

Therefore,

$$P = P_s + P_r = u_{sa}i_{sa} + u_{sb}i_{sb} + u_{sc}i_{sc} + u_{ra}i_{ra} + u_{rb}i_{rb} + u_{rc}i_{rc} \dots(2.50)$$

Where:

$$\begin{cases} P_s = u_{sa}i_{sa} + u_{sb}i_{sb} + u_{sc}i_{sc} = \frac{2}{3} \operatorname{Re} \{ \underline{u}_s^s \cdot \underline{i}_s^{s*} \} = \frac{2}{3} \operatorname{Re} \{ \underline{u}_s^\theta \cdot \underline{i}_s^{\theta*} \} \\ P_r = u_{ra}i_{ra} + u_{rb}i_{rb} + u_{rc}i_{rc} = \frac{2}{3} \operatorname{Re} \{ \underline{u}_r^r \cdot \underline{i}_r^{r*} \} = \frac{2}{3} \operatorname{Re} \{ \underline{u}_r^\theta \cdot \underline{i}_r^{\theta*} \} \end{cases} \dots(2.51)$$

The equations 2.52 are obtained by substituting general equations 2.48 into 2.50. Thus the three power components are calculated according to 2.53.

$$\begin{cases} P_s = \frac{2}{3} \operatorname{Re} \left\{ \left[R_s \underline{i}_s^\theta + \frac{d\underline{\Psi}_s^\theta}{dt} + j\omega_e \cdot \underline{\Psi}_s^\theta \right] \cdot \underline{i}_s^{\theta*} \right\} \\ P_r = \frac{2}{3} \operatorname{Re} \left\{ \left[R_r \underline{i}_r^\theta + \frac{d\underline{\Psi}_r^\theta}{dt} + j(\omega_e - \omega_{er}) \cdot \underline{\Psi}_r^\theta \right] \cdot \underline{i}_r^{\theta*} \right\} \end{cases} \dots(2.52)$$

$$\begin{cases} P_R = \frac{2}{3} \operatorname{Re} \{ R_s \underline{i}_s^\theta \cdot \underline{i}_s^{\theta*} + R_r \underline{i}_r^\theta \cdot \underline{i}_r^{\theta*} \} \\ P_\mu = \frac{2}{3} \operatorname{Re} \left\{ \frac{d\underline{\Psi}_s^\theta}{dt} \cdot \underline{i}_s^{\theta*} + \frac{d\underline{\Psi}_r^\theta}{dt} \cdot \underline{i}_r^{\theta*} \right\} \\ P_M = \frac{2}{3} \operatorname{Re} \{ j\omega_e \cdot \underline{\Psi}_s^\theta \cdot \underline{i}_s^{\theta*} + j(\omega_e - \omega_{er}) \cdot \underline{\Psi}_r^\theta \cdot \underline{i}_r^{\theta*} \} \end{cases} \dots(2.53)$$

The imaginary number j in the expression of the mechanical power component P_M can be eliminated using the general algebraic property (2.54)

$$\text{Re}\{j \cdot z\} = -\text{Im}\{z\} \quad ..(2.54)$$

The two components of the imaginary part in 2.54 can be rewritten as in 2.55 so that the mechanical power equation becomes 2.56

$$\begin{cases} \text{Im}\{\omega_e \cdot \underline{\Psi}_s^\theta \cdot \underline{i}_s^{\theta*}\} = \text{Im}\{\omega_e \cdot (L_s \underline{i}_s^\theta + L_m \underline{i}_r^\theta) \cdot \underline{i}_s^{\theta*}\} = \omega_e L_m \cdot \text{Im}\{\underline{i}_r^\theta \cdot \underline{i}_s^{\theta*}\} \\ \text{Im}\{(\omega_e - \omega_{er}) \cdot \underline{\Psi}_r^\theta \cdot \underline{i}_r^{\theta*}\} = (\omega_e - \omega_{er}) L_m \cdot \text{Im}\{\underline{i}_s^\theta \cdot \underline{i}_r^{\theta*}\} \end{cases} \quad (2.55)$$

$$P_M = -\frac{2}{3} L_m [\omega_e \cdot \text{Im}\{\underline{i}_r^\theta \cdot \underline{i}_s^{\theta*}\} + (\omega_e - \omega_{er}) \cdot \text{Im}\{\underline{i}_s^\theta \cdot \underline{i}_r^{\theta*}\}] \quad (2.56)$$

Based on the mathematical property 2.57 equation 2.56 is further transformed into 2.58.

$$\text{Im}\{\underline{x} \cdot \underline{y}^*\} + \text{Im}\{\underline{y} \cdot \underline{x}^*\} = 0 \quad (2.57)$$

$$P_M = \frac{2}{3} \omega_{er} L_m \cdot \text{Im}\{\underline{i}_s^\theta \cdot \underline{i}_r^{\theta*}\} = \frac{2}{3} p \omega_r L_m \cdot \text{Im}\{\underline{i}_s^\theta \cdot \underline{i}_r^{\theta*}\} \quad (2.58)$$

Therefore, the motor torque may be expressed by 2.59 it is seen that the motor torque depends only on the rotor current vector and on the stator current vector.

$$T = \frac{2}{3} p L_m \cdot \text{Im}\{\underline{i}_s^\theta \cdot \underline{i}_r^{\theta*}\} \quad .. (2.59)$$

Alternatively, the torque can be expressed as a equivalent function of the stator magnetic flux and or the rotor magnetic flux as shown in 2.54 where δ is the angle between the stator flux and the rotor flux.

$$\begin{cases}
 T = \frac{2}{3} p \cdot \text{Im} \{ \underline{\Psi}_r^\theta \cdot i_r^{\theta*} \} \\
 T = \frac{2}{3} p \cdot \text{Im} \{ i_s^\theta \cdot \underline{\Psi}_s^{\theta*} \} \\
 T = \frac{2}{3} p \cdot \frac{L_m}{L_s L_r - L_m^2} \cdot \text{Im} \{ \underline{\Psi}_s^\theta \cdot \underline{\Psi}_r^{\theta*} \} = \frac{2}{3} p \cdot \frac{L_m}{L_s L_r - L_m^2} \cdot |\underline{\Psi}_s^\theta| \cdot |\underline{\Psi}_r^\theta| \cdot \sin \delta
 \end{cases}
 \quad \dots (2.60)$$

Relations 2.59 and 2.60 directly or indirectly underlie all induction motor control strategies.

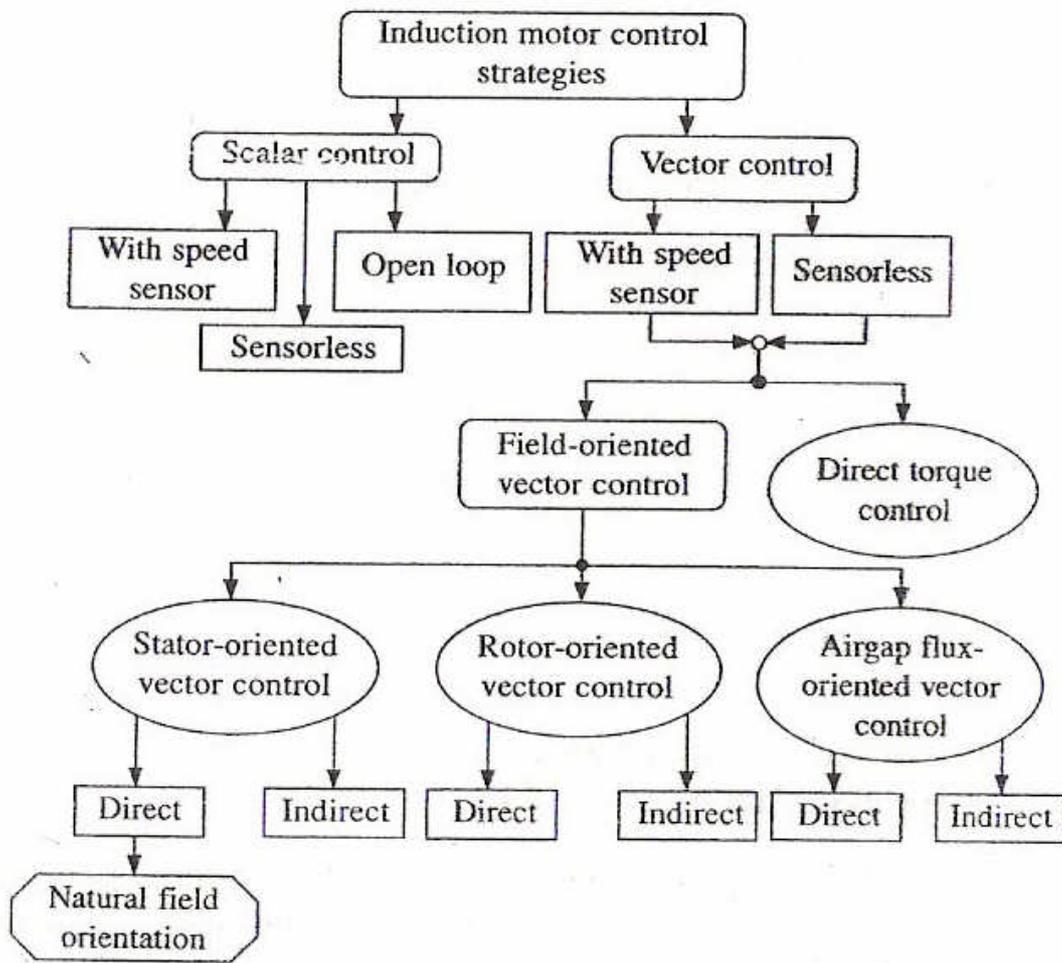


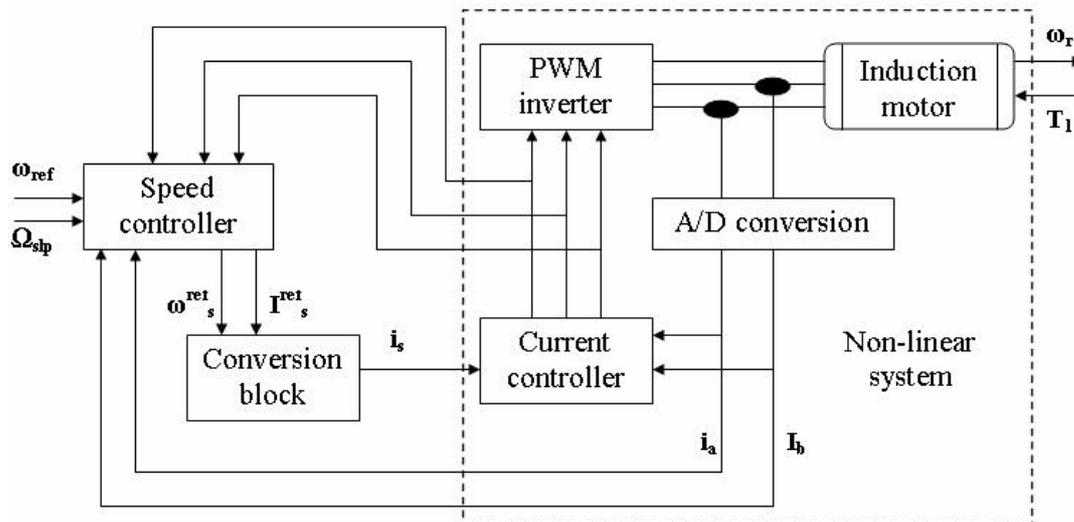
Fig 2.10.1 Classification of induction control strategies

Chapter 3

The sensorless control scheme

3.1 Basic block diagram for the control scheme:

The proposed motor control strategy uses the classical sensorless drive system structure with the motor supplied by a VSI-PWM inverter which is controlled by a digital circuit based only on the stator current feedback information. The predictive current control method uses an



3.1.1 The block diagram of the sensorless control scheme

equivalent $R-L-e$ circuit modeled by the equation.

$$U(t) = Ri(t) + L di(t)/dt + e(t).... (3.1)$$

The $R-L-e$ equivalent circuit parameters for an induction motor can be derived from its general space vector model

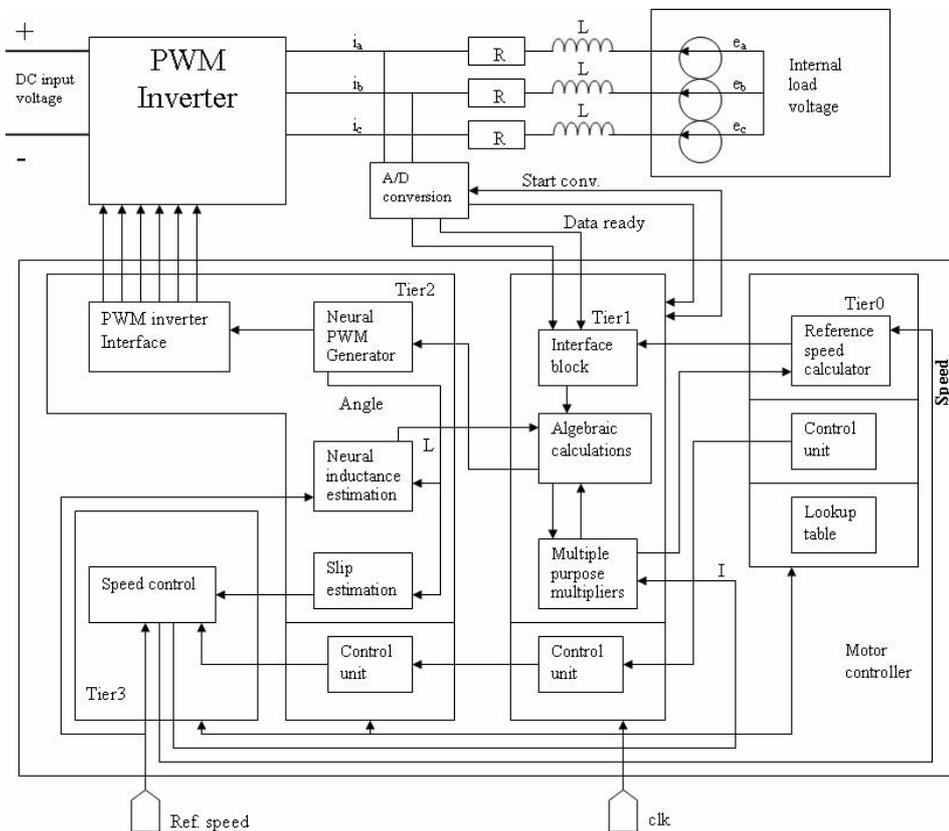
$$\begin{cases} \underline{u}_s^\theta = R_s \underline{i}_r^\theta + \frac{d\Psi_s^\theta}{dt} + j\omega \Psi_s^\theta \\ \underline{u}_r^\theta = R_r' \underline{i}_r'^\theta + \frac{d\Psi_r'^\theta}{dt} + j(\omega_e - \omega_{er}) \Psi_r'^\theta = 0 \\ \Psi_s^\theta = L_s \underline{i}_s^\theta + L_m \underline{i}_r'^\theta \\ \Psi_r'^\theta = L_r' \underline{i}_r'^\theta + L_m \underline{i}_s^\theta \end{cases} \dots(3.2)$$

The complete sensorless induction motor control scheme includes a speed controller, a current controller and a conversion block that interfaces the two controllers. The block diagram is as shown in Fig 3.1. In the control strategy a 3-phase squirrel cage induction motor fed by a VSI-PWM inverter is taken and the control factor stator current is taken in the analog form and converted to digital form by the A/D conversion block, now these converted quantities are fed to the current controller and speed controller as shown in Fig 3.1. The inputs to the speed controller block are the reference angular frequency and the angular slip speed. These quantities are then converted as per the input from the A/D converter and are transferred to the current controller block through the conversion block which acts as the interface between the current controller and speed controller blocks as shown in Fig. 3.1. As no sensors are used so the speed estimation is done by mathematical equations which are developed based on the state space vector model of the 3-phase induction motor.

A VHDL model of three phase induction motor has been created using the mathematical space vector model of the motor. In the VHDL code given in the appendices 1, the motor model is entity having two input ports (the stator voltage and load torque) and two output port(the stator current and rotor angular frequency). All data regarding the motor operation during the simulation is stored in an output ASCII file (motor.txt). The file contains numerical data in matrices format. Each line in the matrix contains the set of quantities that characterize the motor operation at a certain moment: current, voltage, speed and torque.

The design and simulation of the motor controller is carried out using two main software resources Xilinx ISE 7.1 and modelsim which are the sophisticated tools for the design, simulation and testing of FPGA

implemented circuits. The VHDL description of the complete motor controller includes the current control strategy and the sensorless speed control algorithm. The model has been developed using a hierarchical approach and contains four tiers that consists of specialized logic block. The VHDL code related to these blocks utilizes generic parameters to define the size of the register, adders, subtractors, buses and other elements involved, this allows recalling of the controller hardware structure according to the calculation precision imposed by the available types of FPGA circuits.



3.1.2 Functional blocks of the FPGA motor controller

The shaded region in the functional block diagram is implemented using the FPGA chip. The circuitry external to the functional block contains the induction motor, A/D converter, and PWM inverter. The internal

circuitry contains the four tiers 0, 1, 2, and 3. Each of the four tiers performs specific tasks:

Tier0 generates a space vector of constant amplitude and variable angular speed. The angular speed corresponds to stator current frequency. The vector is defined by its real and imaginary parts in the stator reference frame therefore Tier0 is a sine wave generator it produces two variable frequency sine waves shifted with 90° .

Tier1 carries out the algebraic calculations required by the control algorithm and controls the operation of A/D converters that provides information on the motor currents. It determines the reference current space vector by multiplying the unit space vector generated by tier 0 with the amplitude of reference stator current. It also calculates the vectors V_{ni} and V_{Δ} involved in the current control algorithm and in the online inductance estimation process.

Tier2 contains the neural network that generates the PWM switching pattern based on the space vectors I_s and V_{ni} . The angle calculation sub network, which is a part of the complex neural network, is involved in the online inductance estimation algorithm. It also calculates the motor slip angle α equivalent that is used by tier 3.

Tier3 generates the reference stator angular frequency and the reference stator current amplitude using the external reference rotor speed and the motor slip angle as input information.

3.2 Functions of tier1:

Following are some of the important functions performed by the tier1:

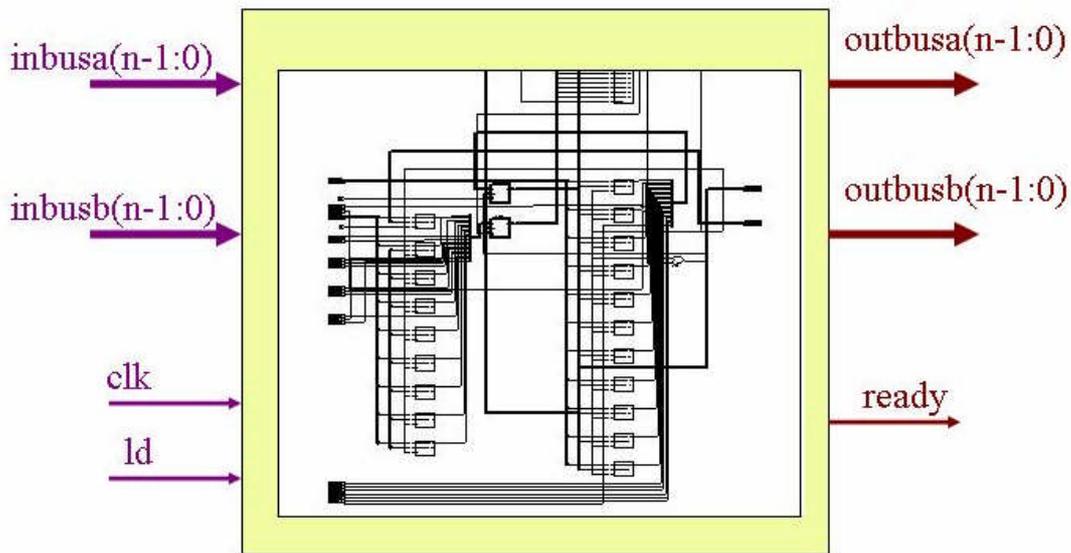
- Controls the operation of the two A/D converters connected to hall sensors that measure the stator currents of the motor.
- It uses the digital output from the A/D converters to calculate the stator current space vectors.
- Determines the derivative of the current space vector and uses this information to calculate the non-inductive space vector V_{ni} .
- Calculates the space vector $V_a = u(k) - V_{ni}(k-1)$ which is used by tier2 to estimate the inductance in equivalent $R-L-e$ circuit.
- Performs adjustment of the numerical values supplied to the neural network located in tier2 so that the network operation speed is maximized.

3.2.1 Adapter block:

The adapter block module designed for the tier1 enhances the operational precision of the angle sub network inside the neural network contained by tier2. The angle subnetwork calculates the argument of space vector based on its rectangular coordinates. The number of input bits of the angle subnetwork is smaller than the total number of bits of the two coordinates. Therefore, it uses only the most significant n bits of these coordinates. If the two values are smaller numbers, the most significant bits are all 0 or all 1 (depending on the sign of the number) and an incorrect result is generated. The adapter simultaneously shifts the two coordinate's values to the left until their leftmost n positions contain significant bits. Shifting a binary number to the left is equivalent to a multiplication by a power of two. As the two coordinates are simultaneously multiplied with the

same power of two, the adapter amplifies the module of the vector but leaves the vector argument unchanged. The VHDL code for the adapter block is given in the appendices 1. The VHDL model of the adapter has two main input ports *inbusa* and *inbusb* (the initial two coordinates) and two main outputs *outbusa* and *outbusb* as shown in code for adapter in appendices 1. The generic parameter *n* defines the width of the input and output buses. It has to be set in accordance with the width of the VHDL signals to which it is connected. All the signals in this tier have correlated widths that are primarily determined by a generic parameters *n* which determines the width of the buses i_a and i_b . The additional input *ld* triggers the input process, while *ready* signals the moment when the process is finished. Each step of the process is synchronized by the clock signal *clk*. The method to determine the end of the process is to test the most significant two bits in each of the two partial results if any of the two pairs contains different bits then the process must stop to avoid an overflow that would change the sign of at least one of the coordinates. The process must also be stopped if all the bits are zero at the same time, this happens when both input coordinates are simultaneously zero which would cause an infinite shifting process.

The architecture of the adapter contains two VHDL processes: The first shifts the two input values in a synchronized manner. The second verifies the existence of nonzero bits and communicated the result through the internal signal *not_all_zero*.



3.2.1.1 The RTL logic schematic of the adapter block

3.3 Field programmable gate array chip (FPGA)

A field programmable gate array (FPGA) is a semiconductor device containing programmable logic components and programmable interconnects. The programmable logic components can be programmed to duplicate the functionality of basic logic gates such as AND, OR, XOR, NOT or more complex combinational functions such as decoders or simple math functions. In most FPGAs, these programmable logic components (or logic blocks, in FPGA parlance) also include memory elements, which may be simple flip-flops or more complete blocks of memories. A hierarchy of programmable



3.3.1 An Altera Stratix II GX FPGA

interconnects allows the logic blocks of an FPGA to be interconnected as needed by the system designer, somewhat like a one-chip programmable breadboard. These logic blocks and interconnects can be programmed after the manufacturing process by the customer/designer (hence the term "field programmable", i.e. programmable in the field) so that the FPGA can perform whatever logical function is needed. FPGAs are generally slower than their application-specific integrated circuit (ASIC) counterparts, can't handle as complex a design, and draw more power. However, they have several advantages such as a shorter time to market, ability to re-program in the field to fix bugs, and lower non-recurring engineering costs. Vendors can sell cheaper, less flexible versions of their FPGAs which cannot be modified after the design is committed. The development of these designs is made on regular FPGAs and then migrated into a fixed version that more resembles an ASIC. Complex programmable logic devices, or CPLDs, are another alternative.

To define the behavior of the FPGA the user provides a hardware description language (HDL) or a schematic design. Common HDLs are VHDL and Verilog. Then, using an electronic design automation tool, a technology-mapped netlist is generated. The netlist can then be fitted to the actual FPGA architecture using a process called place-and-route, usually performed by the FPGA company proprietary place-and-route software. The user will validate the map, place and route results via timing analysis, simulation, and other verification methodologies. Once the design and validation process is complete, the binary file generated (also using the FPGA company's proprietary software) is used to reconfigure the FPGA.

3.3.1 FPGA design and programming:

In an attempt to reduce the complexity of designing in HDLs, which have been compared to the equivalent of assembly languages, there are moves to raise the abstraction level of the design. Companies such as Cadence, Synopsys and Celoxica are promoting SystemC as a way to combine high level languages with concurrency models to allow faster design cycles for FPGAs than is possible using traditional HDLs. Approaches based on standard C or C++ (with libraries or other extensions allowing parallel programming) are found in the Catapult C tools from Mentor Graphics, and in the Impulse C tools from Impulse Accelerated Technologies. Annapolis Micro Systems, Inc.'s CoreFire Design Suite and National Instruments LabVIEW FPGA provide a graphical dataflow approach to high-level design entry. Languages such as SystemVerilog, SystemVHDL, and Handel-C (from Celoxica) seek to accomplish the same goal, but are aimed at making existing hardware engineers more productive versus making FPGAs more accessible to existing software engineers. There is more information on C to HDL and Flow to HDL on their respective pages. To simplify the design of complex systems in FPGAs, there exist libraries of predefined complex functions and circuits that have been tested and optimized to speed up the design process. These predefined circuits are commonly called IP cores, and are available from FPGA vendors and third-party IP suppliers (rarely free and typically released under proprietary licenses). Other predefined circuits are available from developer communities such as OpenCores (typically "free", and released under the GPL, BSD or similar license), and other sources. In a typical design flow, an FPGA application developer will simulate the design at multiple stages throughout the design process. Initially the RTL (register transfer level)

description in VHDL or Verilog is simulated by creating test benches to simulate the system and observe results. Then, after the synthesis engine has mapped the design to a netlist, the netlist is translated to a gate level description where simulation is repeated to confirm the synthesis proceeded without errors. Finally the design is laid out in the FPGA at which point propagation delays can be added and the simulation run again with these values back-annotated onto the netlist.

4. Conclusion

Sensorless position control of induction motors is a promising new technology though in the early state of development. Controlled induction motor drives without mechanical sensors helps in reducing the losses due to mechanical stress and friction between the motor shaft and the speed sensors (tachogenerators) generally used for the purpose. The elimination of the speed sensors also reduces the cost of the overall system as the extra wires used for the sensors are also not required.

The current sensorless control strategies uses microcontroller based DSP chips. The large number of mathematical calculations involved in the estimation algorithms of the system and the total requirement of the presented system easily surpass the calculation power of such a chip , this approach also lacks parallelism and we have to use different DSP chips for carrying out the calculations of different algorithms. This increases the overall cost of the system and also the complexity of the system and such a system is difficult to implement. A field programmable gate array (FPGA) is a semiconductor device containing programmable logic components and programmable interconnects. It has the ability to re-program in the field to fix bugs, and lower non-recurring engineering costs. FPGAs architecture offer massive parallelism which comes handy when used in the algorithms such as the one used in the present system. That's the reason why we are using FPGA chip instead of the conventional DSP chips in the presented system as they take care of the massive and complex mathematical calculations.

5. APPENDICES 1

VHDL codes for the induction motor:

```
library IEEE;
use STD.TEXTIO.all;
use IEEE.math_complex.all;
use IEEE.math_real.all;
use IEEE.numeric_std.all;
use IEEE.std_logic_1164.all;
use IEEE.std_logic_textio.all;

entity motor is
    port(
        us : in complex;
        Tload : in real;
        ist : out complex;
        omegar : out real
    );
end motor;

architecture arch_motor of motor is
    CONSTANT Rs: REAL :=0.371;
    CONSTANT Rr: REAL :=0.415;
    CONSTANT Ls: REAL :=0.08705;
    CONSTANT Lr: REAL :=0.08763;
    CONSTANT Lm: real :=0.08433;
    CONSTANT Jr: REAL :=0.1;
    CONSTANT p: REAL :=2.0;
    CONSTANT deltat: TIME :=50 ns;
    CONSTANT dt: REAL :=5.0e-8;
    SIGNAL next_step: integer :=1;
    FILE outf : TEXT IS OUT "D:\anshul\motor.txt";
BEGIN
    process(next_step)
        variable my_line:LINE;
        variable ist1,ist2,ir1,ir2,Fist1,Fist2,Fir1,Fir2,z:
            complex:= (0.0,0.0);
        variable T,omegar1,omegar2: real := 0.0;
```

```

constant d_space: string := " ";
BEGIN
    if next_step=1 then
        write (my_line,us.re);
        write (my_line,d_space);
        write (my_line,us.im);
        write (my_line,d_space);
        write (my_line,ist1.re);
        write (my_line,d_space);
        write (my_line,ist1.im);
        write (my_line,d_space);
        write (my_line,ir1.re);
        write (my_line,d_space);
        write (my_line,ir1.im);
        write (my_line,d_space);
        write (my_line,T);
        write (my_line,d_space);
        write (my_line,omegar1);
        writeline (outf,my_line);
    end if;
    ist1:=ist2;
    ir1:=ir2;
    Fist1:=Fist2;
    Fir1:=Fir2;
    omegar1:=omegar2;
    Fist2:= Fist1 + (us-Rs*ist1)*dt;
    Fir2:= Fir1+(omegar1*Fir1-Rr*ir1)*dt;
    ist2:= (Lr*Fist2-Lm*Fir2)/(Lr*Ls-Lm*Lm);
    ir2:= (Ls*Fir2-Lm*Fist2)/(Lr*Ls-Lm*Lm);
    z:= ist1*conj(ir1);
    T:= 3.0/4.0*p*Lm*(z.im);
    omegar2:= omegar1 + (T- Tload)/Jr;
    if next_step<1000 then
        next_step<=next_step+1 AFTER deltat;
    else
        next_step<=1 AFTER deltat;
    end if;
    ist<=ist1;
    omegar<=omegar1;
end process;

```

```

end arch_motor;

configuration conf_motor of motor is
    for arch_motor
        end for;
end conf_motor;

```

VHDL code for the adapter module:

```

library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;

entity adapter is
    generic (n: integer :=20);
    port(
        inbusa : in std_logic_vector(n-1 downto 0);
        inbusb : in std_logic_vector(n-1 downto 0);
        outbusa : out std_logic_vector(n-1 downto 0);
        outbusb : out std_logic_vector(n-1 downto 0);
        clk: in std_logic;
        ld: in std_logic;
        ready: out std_logic
    );
end adapter;

architecture adapter_arch of adapter is
    signal int_busa, int_busb: std_logic_vector(n-1 downto 0);
    signal not_all_zero: std_logic;
begin
    process (ld, clk, inbusa, inbusb)
    begin
        if clk = '1' and clk'event then
            if ld = '1' then
                int_busa <= inbusa;
                int_busb <= inbusb;
                ready <= '0';
            end if;
        end if;
    end process;
end adapter_arch;

```

```

        elsif (int_busa(n-1) = int_busa(n-2)) and (int_busa(n-1) =
int_busa(n-2)) and (not_all_zero = '1') then
            int_busa <= int_busa(n-1) & int_busa(n-3 downto 0) & '0';
            int_busb <= int_busb(n-1) & int_busb(n-3 downto 0) & '0';
            else
                ready <= '1';
            end if;
        end if;
    end process;
    process (int_busa, int_busb)
        variable x: std_logic;
    begin
        x:= '0';
        for i in 0 to n-1 loop
            x:= x or int_busa(i);
            x:= x or int_busb(i);
        end loop;
        not_all_zero <= x;
    end process;
    outbusa<= int_busa;
    outbusb<= int_busb;
end adapter_arch;

```

6. APPENDICES 2

VHDL

(Very high scale integrated circuit Hardware Description Language)

WHAT IS VHDL?

VHDL is a language for describing digital electronic systems. It arose out of the United States Government's Very High-Speed Integrated Circuits (VHSIC) program, initiated in 1980. In the course of this program, it became clear that there was a need for a standard language for describing the structure and function of integrated circuits (ICs). Hence the VHSIC Hardware Description Language (VHDL) was developed, and subsequently adopted as a standard by the Institute of Electrical and Electronic Engineers (IEEE) in the US.

VHDL is designed to fill a number of needs in the design process.

- Firstly, it allows description of the structure of a design that is how it is decomposed into sub-designs, and how those sub-designs are interconnected.
- Secondly, it allows the specification of the function of designs using familiar programming language forms.
- Thirdly, as a result, it allows a design to be simulated before being manufactured, so that designers can quickly compare alternatives and test for correctness without the delay and expense of hardware prototyping.

Generally, a synthesis based design flow comprises of a number of stages right from the Design Creation/Verification, Implementation, Programming the device and finally the System Verification. The first step in this process is the "HDL Design Source Entry" wherein you enter your HDL code using a text editor or a context-sensitive HDL editor. We will glance through the

other steps later, and right now concentrate on writing efficient and synthesizable VHDL design descriptions.

VHDL TERMS

Entity:

An entity is the most basic building block in a design. The entity is a module which may be used as a component in a design, or which may be the top-level module of the design. Entities contain the input and output definitions of the design. In VHDL designs that contain a hierarchy of lower-level circuits, the entity functions very much like a block symbol on a schematic. An entity usually has one or more ports, which are analogous to the pins on a schematic symbol. It also has a mode that defines the direction and type of ports.

Here is a simple example of an entity:

```
Entity mux is
    port ( a, b, c, d : in bit;
          s0, s1 : in bit;
          x : out bit);
end mux;
```

The keyword entity specifies the start of an entity statement. The name of the entity is mux. This entity has seven signals in the port clause; six of them are of input type and one of output type. The data type of the i/p and o/p port here is bit. The different modes that define direction are in, out, inout, or buffer.

Architecture:-

The architecture is the actual description of the design. If the entity is a functional block symbol on a schematic, then architecture describes what's inside the block. It contains statements that model the behavior of the entity.

You can freeze the abstract entity which gives the list of the i/o ports and later on choose any architecture that fits in to the entity holder. Architecture consists of two pieces: the architecture declaration section and the architecture body. The declaration section of the architecture is the area between the keyword architecture and the keyword begin. Here the objects that are local to the architecture are declared. After the declaration section comes the architecture body, which is where the behavior of the architecture is specified.

There are three basic kinds of statements in VHDL:

- Declaration Statements
- Concurrent Statements
- Sequential Statements

Declaration Statements:

Declaration statements are used to define constants, types, objects and subprograms that will be used in the design. Declarations can be made in many different locations within a VHDL design, depending on the desired scope of the item being declared.

Concurrent and Sequential Statements:

Concurrent and sequential statements are the fundamental building blocks of a VHDL design description. These statements, which represent the actual logic of a design, include such things as signal assignments, component instantiations, and behavioral descriptions. There are important distinctions to be made between concurrent and sequential statements, as discussed below.

Concurrent Statements:

Concurrent statements are evaluated independently of the order in which they appear. Concurrent statements define logic, typically in the form

of signal assignments (identified by the symbol `<=`) that include combinational logic, which is inherently parallel. With concurrent statements, values are carried on signals, which may be the actual input and output ports of the design or local signals declared using a signal declaration statement.

7. REFERENCES

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